Discrete Recursive Algorithm for Estimation of Non-Stationary Noise in Deep-Submicron Integrated Circuits

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Abstract

This paper reports a new recursive algorithm for efficient estimation of the noise content in time-domain noise analysis of non-linear dynamic integrated circuits with arbitrary excitations. Statistical simulation of specific circuit fabricated in 65 nm CMOS process shows that the proposed algorithm offers accurate and efficient solution.

1. Introduction

The steady increase in the number of systems based on mixed-technology designs, combined with the continuing trend in the decrease of their physical dimensions, is bringing renewed attention to algorithms [1], and correspondingly, CAD tools [2]-[4] capable of assessing circuit or system performance in the presence of noise. The noise performance of a circuit can be analyzed in terms of small-signal equivalent circuits by considering each of the uncorrelated noise sources in turn and separately computing their contribution at the output. Unfortunately, this method is only applicable to circuits with fixed operating points and is not appropriate for noise simulation of circuits with changing bias conditions, such as dynamic digital circuits and dynamic latch comparators. Additionally, in several applications the noise influences the system behavior in a nonlinear way such that linear noise analysis is no longer satisfactory and transient noise analysis, i.e., the simulation of noisy systems in the time domain, becomes necessary.

In this paper, we treat the noise as a non-stationary stochastic process, and introduce an Itô system of stochastic differential equations (SDE) as a convenient way to represent such a process. In order to accommodate noise considerations with varying *dc* operating points, circuits have been analyzed in the time domain. A stochastic noise source is associated with each of the circuit elements, generating a stochastic noise current for the associated element. This stochastic noise current represents a stochastic random process comprised of a white noise source scaled by the standard deviation of the physical noise process that exists within the associated element. Since sample data for such processes are available, we apply a discrete recursive algorithm to accurately estimate the noise content contribution for any particular node in the circuit.

2. Discrete Recursive Algorithm

In general, for time-domain analysis, modified nodal analysis (MNA) leads to a nonlinear ordinary differential equation (ODE) or differential algebraic equation (DAE) system that, in most cases, is transformed into a nonlinear algebraic system by means of linear multistep integration methods [5] and, at each integration step, a Newton-like method is used to solve this nonlinear algebraic system. Therefore, from a numerical point of view, the equations modeling a dynamic circuit are transformed to equivalent linear equations at each iteration of the Newton method and at each time instant of the time-domain analysis. Consider MNA and circuits embedding, besides voltage-controlled elements, independent voltage sources, the remaining types of controlled sources and noise sources. Combining Kirchhoff's Current law with the element characteristics yields a stochastic differential equation of the form

$$F(x',x,t;\theta)+B(x,t;\theta)\cdot\lambda=0 \quad (1)$$

where x is the vector of stochastic processes which represents the state variables (e.g. node voltages) of the circuit, θ is finite-dimensional parameter vector, λ is a vector of white Gaussian processes and $B(x,t)$ is state and time dependent modulation for the vector of noise sources. Every column of $B(x,t)$ corresponds to λ, and has normally either one or two nonzero entries. The rows correspond to either a node equation or a branch equation of an inductor or a voltage source.

We will interpret (1) as an Itô system of stochastic differential equations

$$dX_t = f(t,X_t;\theta)dt + g(t,X_t;\theta)dW_t \quad X_0=x_0, \quad t\geq 0 \quad (2)$$

where we substituted $dW(t)=\chi(t)dt$ with a vector of Wiener process W. If the functions $f(t)$ and $g(t)$ are measurable and bounded on the time interval of interest, there exists a unique solution for every initial value $\lambda(t_0)$ [6], f:$[0,+\infty) \times \mathbb{R}^d \times \Theta \rightarrow \mathbb{R}^d$ and g: $[0,+\infty) \times \mathbb{R}^d \times \Theta \rightarrow \mathbb{R}^{d\times d}$ are known functions depending on an unknown finite-dimensional parameter vector $\theta\in\Theta$. We assume that the initial value x_0 is deterministic and that $x_0, x_1, \ldots, x_n$ is a sequence of observations from the deterministic process X sampled at non-stochastic discrete time-points $t_0 < t_1 < \ldots < t_n$. Since X is Markovian, the maximum likelihood estimator (MLE) of θ can be calculated if the transition densities $p(x_t; x_s, \theta)$ of X are known, $s<t$. A simulated maximum likelihood approach is considered in [7]; here we suggest modifications with respect to the postulated algorithm and introduce this approach in the circuit simulation.

Let $p(t_i, x_i; (t_{i-1}, x_{i-1}), \theta)$ be the transition density of x_i starting from x_{i-1} and evolving to x_i, then the maximum likelihood estimate of θ will be given by the value maximizing the function

$$L(\theta) = \prod_{i=1}^{n} p(t_i, x_i; (t_{i-1}, x_{i-1}), \theta) \tag{3}$$

with respect to θ. To evaluate the contribution of the parameter θ, analysis of the likelihood function requires computing an expectation over the random parameter vector. Even if the likelihood function can be obtained analytically off line, it is invariably a nonlinear function of θ, which makes the maximization steps (which must be performed in real time) computationally infeasible. The proposed algorithm provides a solution, albeit iterative, to such estimation problem:

1. Consider the time interval $[t_{i-1}, t_i]$ and divide it into M subintervals of length $h = (t_i - t_{i-1})/M$: then (2) is integrated on this discretization by using a standard algorithm (e.g. Euler-Maruyama, Milstein) by taking x_{i-1} at time t_{i-1} as the starting value, thus obtaining an approximation of X at t_i. This integration is repeated R times, thereby generating R approximations of the X process at time t_i starting from x_{i-1} at t_{i-1}. We denote such values with $X_{ti}^{1}, \ldots, X_{ti}^{R}$, i.e. X_{ti}^{r} is the integrated value of (2) at t_i starting from x_{i-1} at t_{i-1} in the rth simulation ($r= 1, \ldots, R$).

2. The simulated values $X_{ti}^{1}, \ldots, X_{ti}^{R}$ are used to construct a kernel density estimate of the transition density $p(t_i, x_i; (t_{i-1}, x_{i-1}), \theta)$

$$p^R(t_i, x_i; t_{i-1}, x_{i-1}, \theta) = \frac{1}{Rh_i} \sum_{r=1}^{R} K\left(\frac{x_i - X_{t_i}^{r}}{h_i} \right) \tag{4}$$

where h_i is the kernel bandwidth at time t_i and $K(.)$ is a suitable symmetric, non-negative kernel function. However, as the number of nodes in the observed circuit increase, the convergence rate of the estimator (4) to their asymptotic distribution deteriorates exponentially. As a consequence, unlike [7], for the circuits with large number of nodes, we construct an estimate of the transition density $p^R(t_i, x_i; (t_{i-1}, x_{i-1}), \theta)$ by

$$p^R(t_i, x_i; t_{i-1}, x_{i-1}, \theta) = \frac{1}{R} \sum_{r=1}^{R} \phi(x_i; mean_i^R, variance_i^R) \tag{5}$$

where

$$\begin{aligned} mean_i^R &= X_{t_{i-1}}^{r} + hf(t_{i-1} + (M-1)h, X_{t_{i-1}}^{r}; \theta), \\ variance_i^R &= h \sum (t_{i-1} + (M-1)h, X_{t_{i-1}}^{r}; \theta), \end{aligned} \tag{6}$$

$\varphi(x; ., .)$ denoting the multivariate normal density at x and $\Sigma(t,x;\theta) = g(t,x;\theta)\, g(t,x;\theta)^T$, where T denotes transposition.

3. The previous procedure is repeated for each x_i and the $p^R(t_i, x_i; t_{i-1}, x_{i-1}, \theta)$ to construct (3).

4. In contrast to [7], we maximize $L^R(\theta)$ with respect to θ to obtain the approximated MLE θ^R of θ. The correct construction of $L^R(.)$ requires that the Wiener increments, which once created, are kept fixed for a given optimization procedure. Notice that, for numerical reasons, it is normally more convenient to minimize the negative log-likelihood function

$$-\log L^R(\theta) = -\sum_{i=1}^{n} \log p^R(t_i, x_i; (t_{i-1}, x_{i-1}), \theta) \tag{7}$$

and the approximated MLE is given by $\theta^R = \arg\min_\theta -\log(L^R(\theta))$.

3. Experimental Results

All the experimental results are carried out on a single processor Linux system with Intel Core 2 Duo CPUs with 2.66 Ghz and 3 GB of memory. The effectiveness of the proposed approach was evaluated on several circuits exhibiting different distinctive features in a variety of applications. As a representative example of the results that can be obtained, we show noise characterization of the continuous-time biquad filter fabricated in standard 65 nm CMOS technology [8] (Figure 1 and Figure 2). We assumed that the time series x are composed of a smoothly varying function, plus additive Gaussian white noise λ (Figure 3), and that at any point x can be represented by a low order polynomial. This is achieved by trimming off the tails of the distributions and then using percentiles to reverse the desired variance. The estimation method is based on the maximization of an approximation of the likelihood function. Thus, the obtained (approximated) maximum likelihood estimates θ^R of the freely varying parameters $\hat{\theta} \subseteq \theta$ are asymptotically normally distributed as $n \to \infty$ with mean $\hat{\theta}$ and variance given by the inverse of the expected Fisher information matrix [9]. The latter is often unknown, thus we considered the observed Fisher information in place of the expected Fisher information, since it often makes little difference numerically (e.g. [10]) (Figure 4). The observed Fisher information at θ^R is given by $-H(\theta^R)$, where $H(\theta^R)$ is the Hessian matrix of the log-likelihood function $l(\theta^R)$ computed using the central approximation. The noise variance estimation at the output of the filter is illustrated in Figure 5. For G_{m1}-G_{m4}, OTA_{1-2} and $Opamp_{1-2}$, in comparison with 2000 Spice Monte Carlo iterations, the difference is within 1.1% for mean and 4.2%, 3.9%, 4.5%, 4.3% 3.4%, 3.7%, 5.1% and 4.6% for variance, respectively, while achieving 16-fold *cpu*-time reduction (1241.7s vs 77.6 s).

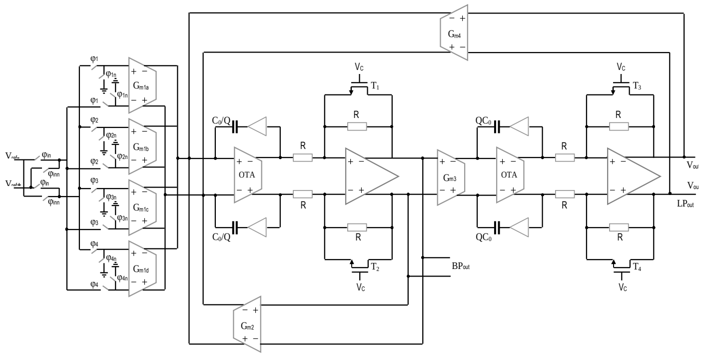

Figure 1: Gm-C-OTA biquad filter [8]

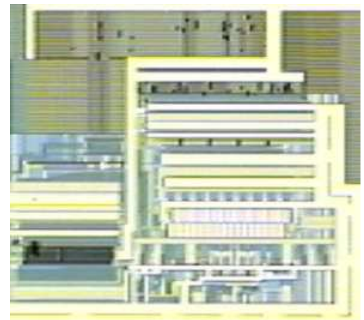

Figure 2: Prototype micrograph

Contribution of each instance G_{m1}-G_{m4}, OTA_{1-2} and $Opamp_{1-2}$, to the total filter noise is 29%, 10%, 14%, 10%, 9%, 7%, 12% and 9%, respectively. The measured noise figure at the filter output node (measured across 25 prototype samples) is within 5% of the simulated noise figure obtained as average noise power calculated over the periodic noise variance waveform.

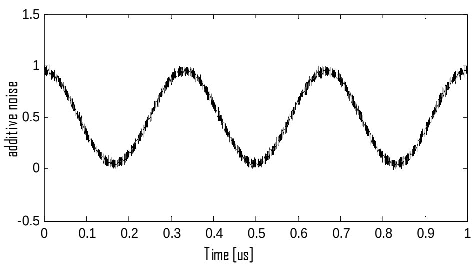

Figure 3: Time series with additive Gaussian noise

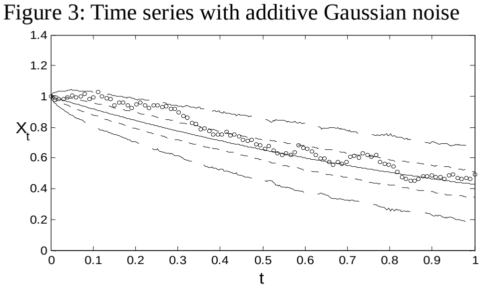

Figure 4. Data () vs the empirical mean (solid line), the 95% confidence bands (dashed lines) and the first-third quartile (dotted lines) of (2)

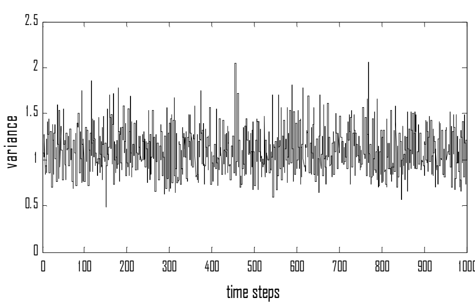

Figure 5: Estimation of noise variance

4. Conclusions

Statistical simulation is one of the foremost steps in the evaluation of successful high-performance IC designs due to circuit noise which strongly affects device behavior in today's deep submicron technologies. In this paper, an discrete recursive algorithm is proposed to accurately estimate noise contributions of individual electrical quantities. This makes it possible for the designer to evaluate the devices that most affect a particular performance, so that design efforts can be addressed to the most critical section of the circuit. As the results indicate, the suggested numerical method provides an accurate and efficient solution.

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