

Sensitivity Computation of Interconnect Capacitances with respect to Geometric Parameters

Yu Bi[†], K. van der Kolk[†], D. Ioan[‡], N.P. van der Meijs[†]

[†] EEMCS, Delft University of Technology, Mekelweg 4, Delft, The Netherlands (Email: y.bi@tudelft.nl)

[‡] NML, Politechnica University of Bucharest, Spl. Independentei 313, Bucharest, Romania (Email: daniel@lnm.pub.ro)

Abstract

This paper presents an algorithm that enables an extension of standard 3d capacitance extraction to take into account the effects of small dimensional variations of interconnects by calculating the corresponding capacitance sensitivities. By using an adjoint technique, capacitances and their sensitivities w.r.t. multiple geometric parameters can be obtained with one-time 3d extraction using the Boundary Element Method (BEM).

I. INTRODUCTION

Accurate capacitance extraction is essential for signal integrity analysis of IC interconnects. However, the on-going reduction of feature size goes together with an increase of process variations, which can affect electrical parameters of interconnects (e.g. capacitances) and further influence circuit performance and functionality. Therefore it is very important to capture such effects using an efficient model, which can be integrated in current design flow or verification methodology at a modest computation cost.

Our study focuses on the dimensional variations of interconnects. It has been shown that not all variations are fatal for capacitances [1]. For each capacitance, some geometric variations deserve further study and modeling while others can be simply neglected. First-order capacitance sensitivities w.r.t. these geometric parameters can be used to setup the threshold for making this distinction [1]. Also, given the information of dimensional variations, a linear model of capacitances as a function of geometric parameters can be constructed and placed in the SPICE netlist instead of the nominal capacitances [2]. Subsequent signal integrity analyses can then be conducted, e.g. moment-based timing analysis [3], [4]. Hence we have two tasks to accomplish. One is to generally prove that first-order approximation of capacitance is sufficient enough, which will be shown in the next paragraph. The other is the main goal of this paper: to compute the first-order sensitivities accurately and efficiently.

We have performed several experiments, which show that first-order models have an acceptable accuracy, thus obviating the need for higher-order models. In Figure 1, a histogram of the maximum capacitance approximation-error is shown for different orders of approximations, sampled over a large number of interconnect-structures. These structures have been chosen randomly, but are based on realistic data for a modern technology. We see that a first-order approximation should be acceptable, with errors concentrated below 3%. Clearly, the first-order approximation improves much over the zero-th order approximation (equivalent to the situation in which variability is not accounted for), and it is only slightly improved upon by the second-order approximation. For simplicity reason, all sensitivities mentioned in the following refer to the first-order sensitivities.

We have presented an algorithm based on the Adjoint Field Technique (AFT) to calculate sensitivities of coupling capacitances w.r.t. geometric parameters and verified it with 2d experiments [5]. This paper is a continuation of our previous work. Starting with some necessary background information, Section II derives the sensitivity computation for both ground capacitances and coupling capacitances for completeness and ease of interpretation. Analytical and numerical examples, both in 3d, are given in Section III. Finally, Section IV concludes the paper.

II. AFT FOR SENSITIVITY ANALYSIS

Background The capacitances that are used in SPICE netlists for signal integrity analysis of IC interconnects are actually called the *two-terminal capacitances* or *network capacitances* defined as $C_{ij} = Q_{ij}/(V_i - V_j)$, where $C_{ij} = C_{ji}$ is the coupling capacitance between conductor i and j , V_i is the potential of conductor i and Q_{ij} the charge associated with this capacitance [6].

Assume that there are N conductors in a charge-free domain Ω , the relation between network capacitances, charges and voltages can be expressed in matrix notation: $\mathbf{Q} = \mathbf{C}_s \mathbf{V}$, where \mathbf{Q} and \mathbf{V} are $N \times 1$ vectors, representing the charges and the potentials on N conductors. \mathbf{C}_s is an $N \times N$ matrix whose entry C_{sij} is the so-called *short-circuit capacitance* [6]. It equals the charge on conductor i when conductor j is held at a unit potential and all other conductors are short-circuited to ground. It can be shown that \mathbf{C}_s is symmetrical and positive definite. Multiplying by \mathbf{C}_s^{-1} on both sides of the equation generates $\mathbf{V} = \mathbf{C}_s^{-1} \mathbf{Q} = \mathbf{G} \mathbf{Q}$ where \mathbf{G} is an $N \times N$ matrix of which the entry G_{ij} is given by the Green's function between conductor i and j .

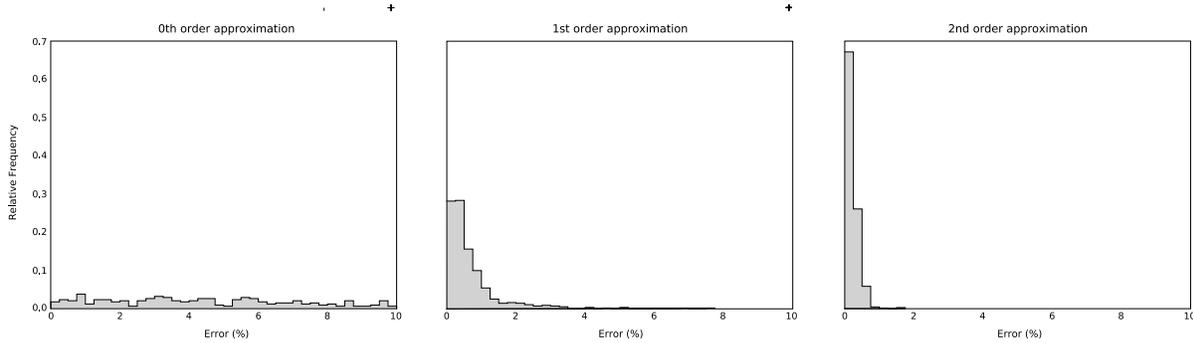


Fig. 1. Error-distribution for different orders of approximations of capacitance models.

In fact, network capacitances are specified in terms of the short-circuit capacitances based on the following relationship:

$$C_{ij} = -C_{sij} \quad \forall i \neq j; \quad C_{ii} = \sum_{j=1}^N C_{sij} \quad \forall i = 1, 2, \dots, N \quad (1)$$

where C_{ii} is the ground (network) capacitance and the short-circuit capacitances are given by the inverse of the Green's function matrix ($\mathbf{C}_s = \mathbf{G}^{-1}$).

When the Boundary Element Method (BEM) is used for capacitance extraction, conductors are discretised into panels. Each panel forms effectively a separate conductor assumed not to have galvanic (DC) coupling to any other panel. The short-circuit capacitances associated to the discretised panels before their association to conductors are called *partial short-circuit capacitances* [6], denoted by \mathbf{c}_s in this paper. If there are in total m panels in the domain, \mathbf{c}_s becomes an $m \times m$ (usually $m \geq N$) matrix. Correspondingly, other electrical quantities for panels, instead of \mathbf{V} , \mathbf{Q} and \mathbf{G} , are denoted in lower case as well: $\mathbf{q} = \mathbf{c}_s \mathbf{v} = \mathbf{g}^{-1} \mathbf{v}$.

Algorithm Derivation Our algorithm [5] is based on an adjoint technique derived from an application of Tellegen's theorem to the electrostatic (ES) field, which is expressed as

$$(\hat{\mathbf{V}}, (\Delta \mathbf{C}_s) \mathbf{V}) = \langle (\Delta \epsilon) \mathbf{E}, \hat{\mathbf{E}} \rangle \quad (2)$$

where we assume there are N conductors in domain Ω and use a notation “ $\hat{\cdot}$ ” for the adjoint field quantities. $\Delta \mathbf{C}_s$ and $\Delta \epsilon$ are the effective changes of \mathbf{C}_s and ϵ (the permittivity of the medium in domain Ω) induced by the variation in geometric parameter p (Δp).

Equation (2) shows that the effects of the geometric variations on capacitances can be measured by the effects on the permittivity. It is necessary to mention that (2) is derived under the condition that $\Delta \mathbf{V} = 0$, which means the excitation voltages are considered constant in our study.

First of all, let's study the left-hand side of (2). Given the relationship between network capacitances and short-circuit capacitances described in (1), if we want to calculate:

- 1) ΔC_{ii} , the change of the ground capacitance due to Δp , we need to define the excitation voltages of the original system and the adjoint system as $V_k = 1$ ($k = 1, 2, \dots, N$) and $\hat{V}_i = 1, \hat{V}_k = 0$ ($k \neq i$). Therefore the left-hand side of (2) becomes

$$(\hat{\mathbf{V}}, (\Delta \mathbf{C}_s) \mathbf{V}) = \sum_{j=1}^N \Delta C_{sij} = \Delta C_{ii}. \quad (3)$$

- 2) ΔC_{ij} , the change of the coupling capacitance due to the geometric variation Δp , we need to define $V_j = 1, V_k = 0$ ($k \neq j$) and $\hat{V}_i = 1, \hat{V}_k = 0$ ($k \neq i$). In this case, the left-hand side of (2) turns to be

$$(\hat{\mathbf{V}}, (\Delta \mathbf{C}_s) \mathbf{V}) = \Delta C_{sij} = -\Delta C_{ij}. \quad (4)$$

Now let's look at the right-hand side of (2): $\langle (\Delta \epsilon) \mathbf{E}, \hat{\mathbf{E}} \rangle$, which implies that we need to study how the variation of geometric parameter Δp influences the ϵ in Ω .

We know that the electric displacement field \mathbf{D} represents how much an electric field \mathbf{E} influences the organization of electrical charges in a medium characterized by the permittivity ϵ . In other words, the effect of \mathbf{E} is represented by \mathbf{D} via ϵ , and ϵ describes the relationship between \mathbf{E} and \mathbf{D} . While a perfect metal is placed in Ω , the inner part of the metal is actually shielded out, so there is no \mathbf{E} -field, nor its effect (\mathbf{D}) and their relationship (ϵ) exist anymore.

Figure 2 schematically shows the cross-section of a conductor when there is a variation in p (Δp). S_p is the influenced surface due to Δp and we call it the *victim surface* incident to parameter p . In fact, the dimensional variation Δp is

exactly the displacement of the victim surface. The influence on ϵ lies on the shadowed area $\Delta\Omega$ where the effect of E-field is shielded out. Therefore $\Delta\epsilon\Delta\Omega = \epsilon(\vec{n}_p\Delta p)(\vec{n}_s\Delta s)$, where \vec{n}_s is pointing from the victim surface to the medium (characterized by ϵ) and \vec{n}_p is the direction of the geometric parameter we defined. Thus (2) becomes

$$(\hat{\mathbf{V}}, (\Delta\mathbf{C}_s)\mathbf{V}) = \langle (\Delta\epsilon)\mathbf{E}, \hat{\mathbf{E}} \rangle = \int_{\Omega} (\Delta\epsilon)\mathbf{E}\hat{\mathbf{E}}d\Omega = \int_{s_p} \epsilon\mathbf{E}\hat{\mathbf{E}}(\vec{n}_p\Delta p)\vec{n}_s ds \quad (5)$$

which will be used to calculate capacitance sensitivities categorized in two cases: sensitivity of ground capacitances and sensitivity of coupling capacitances.

Case-1 Ground capacitance sensitivity $\frac{\partial C_{ii}}{\partial p}$.

$$\frac{\partial C_{ii}}{\partial p} = \lim_{\Delta p \rightarrow 0} \sum_{j=1}^N \Delta C_{sij} / \Delta p = \lim_{\Delta p \rightarrow 0} \int_{s_p} \frac{\epsilon\mathbf{E}\hat{\mathbf{E}}}{\Delta p} (\Delta p \vec{n}_p) \vec{n}_s ds = \epsilon \vec{n}_p \vec{n}_s \int_{s_p} \mathbf{E}\hat{\mathbf{E}} ds \quad (6)$$

When the BEM is applied, \mathbf{E} is a piecewise constant (PWC) quantity on the set of panels. Hence, the integration over s_p becomes a summation. When also using $\mathbf{D} = \epsilon\mathbf{E}$ and $\nabla \cdot \mathbf{D} = \rho$ (Gauss law), the sensitivity expression becomes $\vec{n}_p \vec{n}_s \sum_{k \in s_p} \rho_k \hat{\rho}_k a_k / \epsilon$, where a_k is the corresponding area of panel k on the victim surface s_p . For panel k , since PWC is used for the BEM, the charge density ρ_k can be related to its charge q_k with the corresponding area a_k as $q_k = \rho_k a_k$. Consequently, the sensitivity expression transforms into $\vec{n}_p \vec{n}_s \sum_{k \in s_p} q_k \hat{q}_k / (a_k \epsilon)$. We know that q_k and \hat{q}_k , as entries in \mathbf{q} , are certain combinations of some entries of the partial short-circuit capacitance matrix \mathbf{c}_s which is given by the inverse of the Green's function matrix \mathbf{g}^{-1} . According to (3), we have

$$q_k = \sum_{j=1}^N \sum_{a \in N_j} c_{sk,a}; \quad \hat{q}_k = \sum_{a \in N_i} c_{sk,a}. \quad (7)$$

with N_j being conductor j and $c_{sk,a}$ the short-circuit capacitance between panel k and a .

For ease of discussion, we introduce a short-hand notation: $C_{kj}^* = \sum_{a \in N_j} c_{sk,a}$. Therefore equation (6), the sensitivity of ground capacitance w.r.t. p becomes

$$\frac{\partial C_{ii}}{\partial p} = \frac{\vec{n}_p \vec{n}_s}{\epsilon} \sum_{k \in s_p} C_{ki}^* \left(\sum_{j=1}^N C_{kj}^* \right) / a_k. \quad (8)$$

Case-2 Coupling capacitance sensitivity $\frac{\partial C_{ij}}{\partial p}$. Similar to Case-1, according to (4), we finally obtain

$$\frac{\partial C_{ij}}{\partial p} = -\frac{\vec{n}_p \vec{n}_s}{\epsilon} \sum_{k \in s_p} \left(\sum_{a \in N_i} \sum_{b \in N_j} c_{sk,a} c_{sk,b} \right) / a_k = -\frac{\vec{n}_p \vec{n}_s}{\epsilon} \sum_{k \in s_p} \frac{C_{ki}^* C_{kj}^*}{a_k}. \quad (9)$$

From the above derivation we can see, similar to network capacitances (1), their sensitivities can also be computed from (partial) short-circuit capacitances (8), (9). In other words, with only one 3d capacitance extraction using the BEM (e.g. SPACE layout-to-circuit extractor [7]), we can obtain network capacitances as well as their sensitivities. Furthermore, multiple geometric parameters can be studied simultaneously. This is because they are associated to various victim surfaces, containing various sets of panels. Thus the sensitivities w.r.t. different parameters simply means different combinations of partial short-circuit capacitances in (7) and (9).

III. EXAMPLES

Analytical Example In this subsection, we give an example of a system with two concentric spheres as is shown in Figure 3. We define the inner sphere as conductor 1 and the outer sphere conductor 2, while q_i , v_i ($i = 1, 2$) are the corresponding charges and applied voltages on them. Analytically, the capacitance between the two spheres is $C_{12} = 4\pi\epsilon / (\frac{1}{r_1} - \frac{1}{r_2})$ and its sensitivities or derivatives w.r.t. r_1 and r_2 are $\frac{\partial C_{12}}{\partial r_1} = 4\pi\epsilon(r_2)^2 / (r_2 - r_1)^2$ and $\frac{\partial C_{12}}{\partial r_2} = -4\pi\epsilon(r_1)^2 / (r_2 - r_1)^2$.

Now we compute the sensitivities with our algorithm. Without loss of generality, we consider the inner sphere and the outer sphere to be a single panel each. So the area of each panel is $a_i = 4\pi r_i^2$ ($i = 1, 2$). According to the above analysis, the sensitivity of C_{12} against r_1 can be computed as $-q_1 \hat{q}_1 / (\epsilon a_1)$, where q_1 and \hat{q}_1 are the charges on the inner sphere when $V_1 = 0$, $V_2 = 1$ and $\hat{V}_1 = 1$, $\hat{V}_2 = 0$ respectively. Knowing that $q_1 = -\hat{q}_1 = -4\pi\epsilon / (\frac{1}{r_1} - \frac{1}{r_2})$, it is trivial to calculate the sensitivity of the coupling capacitance w.r.t. r_1 : $4\pi\epsilon(r_2)^2 / (r_2 - r_1)^2$. Similarly the sensitivity of C_{12}

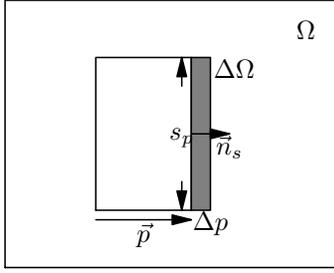


Fig. 2. Illustration of $\Delta\Omega$

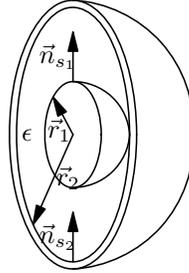


Fig. 3. Diagram of concentric spheres.

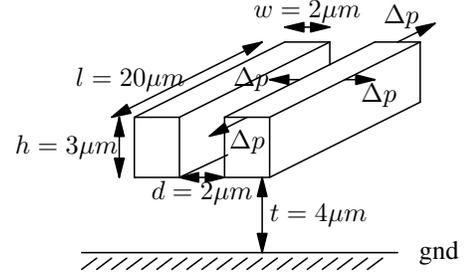


Fig. 4. Two wires above the ground plane.

w.r.t. r_2 can be calculated using our algorithm, resulting in $-4\pi\epsilon(r_1)^2/(r_2 - r_1)^2$. Note that when r_2 is the geometric parameter, $\vec{n}_p\vec{n}_s = \vec{n}_{r_2}\vec{n}_{s_2} = -1$. Apparently, our algorithm gives identical sensitivities as the analytical results.

We also studied a special case where there is one isolated sphere with a radius of r (we can also consider the radius of the outer sphere infinitely large). Analytically, the capacitance to the infinity (the reference ground) is $C_{gnd} = 4\pi\epsilon r$ and its sensitivity w.r.t. r is $4\pi\epsilon$.

The sensitivity of C_{gnd} computed by our algorithm is consistent with the analytical result:

$$\frac{dC_{gnd}}{dr} = \frac{\vec{n}_p\vec{n}_s}{\epsilon} \frac{q\hat{q}}{a} = \frac{1}{\epsilon} \frac{C_{gnd}^2}{a} = \frac{1}{\epsilon} \frac{(4\pi\epsilon r)^2}{4\pi r^2} = 4\pi\epsilon \quad (10)$$

Numerical Example An experiment is conducted on a system with two parallel wires above the ground plane, as shown in Figure 4. Assume there is a 10% variation in the layout ($\Delta p = 0.1\mu m$). Hence all the sidewalls of both conductors are victim surfaces.

The relative variability of the coupling capacitance C_{12} and the ground capacitance C_{gnd} can be computed via sensitivities given by our algorithm:

$$\frac{\Delta C_{12}}{C_{12}} = \frac{\partial C_{12}}{\partial p} \times \frac{\Delta p}{C_{12}} = 9.35\%; \quad \frac{\Delta C_{gnd}}{C_{gnd}} = \frac{\partial C_{gnd}}{\partial p} \times \frac{\Delta p}{C_{gnd}} = 1.43\%. \quad (11)$$

To verify the accuracy of our algorithm, we change the layout by Δp and extract the capacitances again, denoting the results as C_{12}^p and C_{gnd}^p . Thus the actual relative variability of C_{12} and C_{gnd} are $(C_{12}^p - C_{12})/C_{12} = 10.94\%$ and $(C_{gnd}^p - C_{gnd})/C_{gnd} = 1.66\%$ respectively.

This experiment shows that for a $0.1\mu m$ geometric variation, our algorithm very nicely captures its effect on network capacitances. The coupling capacitance is very susceptible to variations in the layout, which needs to be modeled and integrated in the design flow. On the other hand, the ground capacitance, in this particular case, is not sensitive to layout variations.

IV. CONCLUSION

We addressed an algorithm to compute first-order capacitance sensitivities w.r.t. geometric parameters. Both the network capacitances and their sensitivities can be obtained under only one time 3d extraction using the BEM. Analytical and numerical examples have been shown to verify our algorithm.

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