

# Convex Optimization for Joint Zero-forcing and Antenna Selection in Multiuser MISO Systems

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**Abstract**—The problem of joint zero-forcing (ZF) beamforming (BF) together with optimal power allocation (PA) and antenna selection (AS) for throughput maximization is considered in this paper for multi-user multiple input single output (MU-MISO) systems. We introduce a new formulation for the joint ZF and PA problem by adapting the algebraic subspace approach which finds a proper set for the optimization variable that inherently satisfies the ZF constraints. Also, the squared group Lasso penalty on the BF matrix is used to linearize (relax) the non-convex, NP-hard problem of joint BF and AS. Extensive simulations show that for the throughput problem, the proposed algorithm performs very closely to the optimal (exhaustive search) joint approach.

**Index Terms**—Multiple input multiple output (MIMO), linear precoding, convex optimization, antenna selection, group Lasso.

## I. INTRODUCTION

ANTENNA selection (AS) for multiple input multiple output (MIMO) systems, both at the transmitter (Tx) and at the receiver (Rx), has been an interesting problem for decades. A critical factor in increasing the number of antennas in MIMO systems is the cost of the radio frequency (RF) chain. AS techniques define the optimal subset of available antennas utilizing the channel state information (CSI) at the Tx or Rx with respect to the objective [1]. The optimal approach (exhaustive search) to solve the AS problem is computationally difficult (NP-hard).

Having the CSI information available at the Tx (or Rx) allows more advanced signal processing techniques to improve the link performance. Beamforming (BF) is the most common and effective technique for boosting the data rate and quality of service (QoS) in MIMO systems [2].

In multi-user (MU) systems, the users are individual entities that in general cannot co-operate, so the BF (interference cancelation) needs to be performed at downlink<sup>1</sup>. Zero forcing (ZF) technique is the capacity maximizing scheme in MU systems with the linear complexity. ZF is feasible only when there exist more antennas at the transmitter than all users. Moreover, power allocation (PA) is used in MU-MIMO to control the signal to interference ratio (SINR) for each user [2] [3]. In the ZF scenario, PA determines each user's SNR, and consequently also the user's rate since the interference is canceled entirely by the beamformer. In this paper, joint BF

and PA at the transmitter is referred to as *precoding* which is handled jointly with transmit (downlink) AS.

There are two problems in the precoding context: power control and performance maximization. Power control is the problem of minimizing the total transmitted power while achieving a pre-specified performance measure such as sum-rate capacity, per user rate or QoS level for each user. Performance maximization is the problem of maximizing the objective measure subject to power constraints. Neither of these problems has a closed form solution for general MU-MIMO [4]. Here we consider the latter with throughput as a performance measure, however, the proposed formulation can readily be extended to other objectives such as fairness for each user or the power control problem.

It is proven in [5] that the problem of receive antenna subset selection for capacity maximization is sub-modular in point to point MIMO system so the greedy algorithm of [6] is guaranteed to achieve the optimal solution. However, the relative antenna selection problem at the transmitter is not sub-modular with uniform power allocation. The greedy algorithms therefore fail for transmit AS. Even so, the capacity expression is monotonic in the total permitted power when CSI is available at the transmitter and the power management scheme (PA) is an option [4]. As a result, the AS problem for downlink transmission is more difficult than the uplink AS task since it affects the overall system gain rather than the individual user's signal quality. In literature, there are few AS techniques for transmitters in point to point MIMO systems [1], [7] and in downlink MU-MIMO [8]. In fact, the first joint precoding and AS approach is introduced in [9] for multicast beamforming and later in [10] for ZF precoding where the sum-rate capacity is set to the maximum achievable rate and the penalty for reducing the number of antennas is imposed on the transmit power.

The main contribution of this work is summarized here as 1) Formulating a first general joint AS and ZF precoding problem for MU-MISO systems. 2) Adapting the subspace approach in [4] to formulate a per antenna power constrained ZF precoding problem. 3) Using the transformation technique to represent the joint problem of AS and ZF precoding as a convex optimization problem by applying the squared group Lasso regularization term.

The squared sum of max-norms as a group sparsity inducing convex regularizer for the joint multicast BF and AS is used in [11] for the first time. Nevertheless, using the infinity norm

This research has been supported in part by STW under contract 10551 (FASTCOM).

<sup>1</sup>Even though the terms uplink and downlink are widely used for mobile cellular systems, they are general terms for any bi-directional systems.

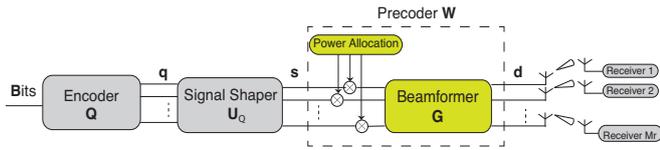


Fig. 1. Block diagram of MU-MISO link with linear precoding including beamformer and power allocation.

gives rise to equal magnitude beamformers so here we use the squared sum of  $\ell_2$ -norms as an alternative sparsity inducing term which allows more flexibility in terms of magnitude.

*Notation:* We use standard bold upper case and bold lower case symbols to indicate matrices and vectors, respectively. We use  $\mathbf{A}(i, j)$  to denote the  $(i, j)$ -entry in the matrix  $\mathbf{A}$ . The  $j$ th column of  $\mathbf{A}$  is denoted  $\mathbf{a}_j$ , and the  $i$ th row is denoted  $\mathbf{a}_{[i]}$ , where both are column vectors. When matrix  $\mathbf{A}$  is partitioned into blocks,  $\mathbf{A}_{i,j}$  denotes the  $(i, j)$ -submatrix. We use  $\mathbf{A}_{:,j}$  to mean the sub-matrix of  $\mathbf{A}$  formed by stacking on top of one another the sub-matrices  $\mathbf{A}_{i,j}$  for all  $i$ . The symbol  $\mathbf{A}_{:,j}(i, j)$  will mean the matrix whose  $(k, l)$ -entry is equal to  $\mathbf{A}_{k,l}(i, j)$ . The  $n$ th component of the vector  $\mathbf{a}$  is denoted  $a_n$ . The conjugate transpose, conjugate, and transpose of  $\mathbf{A}$ , will be denoted respectively as  $\mathbf{A}^H$ ,  $\mathbf{A}^*$ , and  $\mathbf{A}^T$ . Norms for vectors are denoted as follows:  $\ell_2$ -norm  $\|\mathbf{a}\|_2$ ,  $\ell_1$ -norm  $\|\mathbf{a}\|_1$ ,  $\ell_0$ -norm  $\|\mathbf{a}\|_0$  and  $\ell_\infty$ -norm  $\|\mathbf{a}\|_\infty$  or equivalently max-norm. The Kronecker product is denoted by  $\otimes$  and the  $\text{vec}(\mathbf{A})$  operation vectorizes  $\mathbf{A}$  by listing out the columns of the matrix.

## II. SYSTEM MODEL

We use almost the same system model as in [10], except for the uncoordinated receivers. Consider a MU-MISO system with  $M_r$  users each with a single antenna, and an access point with  $M_t$  transmit antennas. We assume  $M_t \geq M_r$ . The received data vector  $\mathbf{r}$  is expressed as  $\mathbf{r} = \mathbf{H}\mathbf{d} + \mathbf{n}$ , where  $\mathbf{H} \in \mathbb{C}^{M_r \times M_t}$  is the MU channel,  $\mathbf{d}$  is the transmit vector, and  $\mathbf{n}$  is a zero-mean Gaussian noise vector. The variance of every entry of  $\mathbf{n}$  is assumed to be 1 and  $\mathbf{H}$  is assumed to be known at Tx.

The encoder unit in Fig.1 determines the inverse covariance matrix of the output signal vector  $\mathbf{q} \in \mathbb{C}^{M_r \times 1}$ . In turn the signal shaper matrix  $\mathbf{U}_Q$  is given by the eigenvectors of the codeword covariance matrix, where the transmit sequence of length  $M_r$  is given by  $\mathbf{s} = \mathbf{U}_Q\mathbf{q}$ . The covariance matrix of the input signal  $\mathbf{s}$  to the precoder block is assumed to be an identity matrix;  $E\{\mathbf{s}\mathbf{s}^H\} = \mathbf{I}_{M_r}$ . This is the optimal design for all precoder criteria [2]. Accordingly, the free design parameters for a linear precoder  $\mathbf{W} \in \mathbb{C}^{M_t \times M_r}$  are shown in the precoder block in Fig. 1. The linear precoder  $\mathbf{W}$  can be decomposed into a BF matrix  $\mathbf{G}$  and a diagonal matrix  $\mathbf{\Upsilon}$ , which is the power allocation unit, so in general we have  $\mathbf{W} = \mathbf{G}\mathbf{\Upsilon}$ .

### Zero Forcing Beamforming

Since  $M_t \geq M_r$ , and since the channel is known at the transmitter, a linear precoder can be designed to force the

interference between users to zero, and to pre-equalize the channel in fading channels. This ZF precoder is expressed as  $\mathbf{H}\mathbf{G} = \mathbf{I}$ , so

$$\mathbf{H}\mathbf{W} = \mathbf{\Upsilon} =: \mathbf{P}^{1/2}, \quad (1)$$

where  $\mathbf{P}$  is the unknown diagonal power matrix  $\mathbf{P} = \text{diag}(p_1, p_2, \dots, p_{M_r})$ , and  $p_j$  ( $p_j \geq 0$ ) is the signal to noise ratio (SNR) on the  $j$ th receive antenna for each  $j = 1, 2, \dots, M_r$  (assuming a unit variance noise on the receiver). We define  $\mathbf{P}^{1/2}$  to be the component-wise square-root of  $\mathbf{P}$ .

One can write the zero forcing constraint as

$$\mathbf{h}_j^H \mathbf{w}_l = 0, \quad \forall j \neq l, \quad j, l = 1, 2, \dots, M_r. \quad (2)$$

We follow the same approach as in [4] to form a complementary matrix that describes the interference between users. The symbol received by the  $l$ th user can be specified as

$$r_j = \sum_{l=1}^{M_r} \mathbf{h}_j^H \mathbf{w}_l s_l + n_j = \mathbf{h}_j^H \mathbf{w}_j s_j + \mathbf{h}_j^H \bar{\mathbf{W}}_j \bar{\mathbf{s}}_j + n_j, \quad (3)$$

where,  $\bar{\mathbf{W}}_j \in \mathbb{C}^{M_t \times (M_r - 1)}$  and  $\bar{\mathbf{s}}_j \in \mathbb{C}^{M_r - 1}$  are the concatenated BF matrix and data vector respectively, which contain the BF vectors and data for all users except  $j$ . In ZF BF, we constrain the second term (interference) in (3) to be zero. This forces  $\bar{\mathbf{W}}_j$  to lie in the null space of  $\bar{\mathbf{H}}_j \in \mathbb{C}^{M_r - 1 \times M_t}$  which is defined as  $\bar{\mathbf{H}}_j = [\mathbf{h}_1, \dots, \mathbf{h}_{j-1}, \mathbf{h}_{j+1}, \dots, \mathbf{h}_{M_r}]^T$ .

Theoretically, it can be proven that data can be sent to user  $j$  without interference if the nullity of  $\bar{\mathbf{H}}_j$  is greater than zero, which is true in our system since  $M_t \geq M_r$ ; the size of nullity is equal to  $S_n = M_t - M_r + 1$ . Furthermore,  $\bar{\mathbf{H}}_j$  can be decomposed as

$$\bar{\mathbf{H}}_j = \bar{\mathbf{U}}_j \bar{\mathbf{\Sigma}}_j [\bar{\mathbf{V}}_j \quad \bar{\mathbf{V}}_j^\perp]^H, \quad (4)$$

where  $\bar{\mathbf{V}}_j^\perp \in \mathbb{C}^{M_t \times S_n}$  holds an orthonormal basis for the null space of  $\bar{\mathbf{H}}_j$ . Therefore any linear combination of its columns gives a ZF BF matrix. This approach is introduced in [4] for ZF beamformer with total power constraint. We make use of it to further specify the optimization variables for our ZF problem which maximize some performance measure like throughput subject to per antenna power constraints and later to formulate the joint AS and ZF precoding.

## III. PROPOSED FORMULATION FOR ZF PRECODING

In this section we present a new formulation for the conventional ZF BF problem based on the subspace approach explained earlier. It is proved in [12] that the ZF precoding problem with total power constraint is rather easy to solve as it boils down to a simple power allocation problem where  $\mathbf{G} = \mathbf{H}^\dagger$  is the pseudo-inverse of the channel matrix. However, this is no longer the case when per antenna power constraints are added. Note that the total power constraint is a relaxation of the optimization problem when each antenna has its own power limit, so any feasible solution for the latter is also feasible for the former.

From (4), one sees that the real design variables to form the desired BF matrix are vectors  $\mathbf{m}_j$  of length  $S_n$ , defined

by  $\mathbf{w}_j = \bar{\mathbf{V}}_j^\perp \mathbf{m}_j$  for all  $j$ , and  $\mathbf{M} = [\mathbf{m}_1, \mathbf{m}_2, \dots, \mathbf{m}_{M_r}] \in \mathbb{C}^{S_n \times M_r}$ . To continue, we need to redefine some widely used terms in BF context. First, we introduce per antenna transmit power  $p_{t_i}$  in terms of the matrices  $\mathbf{m}_j$ . For this, we use some properties of Kronecker product and vectorization operators:

$$p_{t_i} = \mathbf{w}_{[i]}^T \mathbf{w}_{[i]}^* = \sum_{j=1}^{M_r} \bar{\mathbf{v}}_{j[i]}^{\perp T} \underbrace{\mathbf{m}_j \mathbf{m}_j^H}_{\mathbf{Y}_j} \bar{\mathbf{v}}_{j[i]}^{\perp *} = \sum_{j=1}^{M_r} \underbrace{(\bar{\mathbf{v}}_{j[i]}^{\perp H} \otimes \bar{\mathbf{v}}_{j[i]}^{\perp T})}_{\mathbf{g}_{j[i]}^T \in \mathbb{C}^{1 \times S_n^2}} \text{vec}(\mathbf{Y}_j) = \sum_{j=1}^{M_r} \mathbf{g}_{j[i]}^T \mathbf{y}_j, \quad (5)$$

where  $\bar{\mathbf{v}}_{j[i]}^\perp$  is the  $i$ th row of matrix  $\bar{\mathbf{V}}_j^\perp$ . The total transmit power  $P_t = \sum_{i=1}^{M_t} p_{t_i}$  is therefore the sum of the expressions (5) over all transmit antennas. The expression in (5) can be readily verified by noting that  $\mathbf{w}_{[i]}^H \in \mathbb{C}^{1 \times M_r} = [\bar{\mathbf{v}}_{1[i]}^{\perp T} \mathbf{m}_1, \bar{\mathbf{v}}_{2[i]}^{\perp T} \mathbf{m}_2, \dots, \bar{\mathbf{v}}_{M_r[i]}^{\perp T} \mathbf{m}_{M_r}]^H$ .

Then, SNR for  $j$ th user can be defined as  $p_j = \mathbf{h}_j^H \bar{\mathbf{V}}_j^\perp \mathbf{Y}_j \bar{\mathbf{V}}_j^{\perp H} \mathbf{h}_j$ . So the  $p_j$ s are not independent variables and are used to clarify the power allocation part of the precoder design process.

The conventional precoding optimization problem with per antenna power constraints w.r.t.  $\mathbf{w}_j$  is defined as

$$\begin{aligned} & \text{maximize} && f(p_j) \\ & p_j, \mathbf{w}_j \\ & \text{s.t.} && \mathbf{h}_j^H \mathbf{w}_l = 0, \quad \forall j \neq l; \\ & && \mathbf{h}_j^H \mathbf{w}_j = \sqrt{p_j}; \\ & && \sum_j [\mathbf{w}_j \mathbf{w}_j^H](i, i) \leq P_i^*, \quad \forall i. \end{aligned} \quad (6)$$

where  $P_i^*$  is the total permitted transmit power for the  $i$ th antenna and  $f(p_j)$  is a concave objective function such as throughput. optimization problem in (6) is clearly not convex due to the nonlinear equality constraint and hard to solve in general. However, such quadratically constrained problems can sometimes be relaxed to linear programs which can be solved efficiently [13].

The relaxation and solution for (6) is thoroughly explained in [12], however, we summarize it here for the sake of self-containedness. The quadratic variable in (6) is written as  $\mathbf{w}_j \mathbf{w}_j^H = \mathbf{Z}_j \in \mathbb{C}^{M_t \times M_t}$  where  $\mathbf{Z}_j$  is a Hermitian positive semidefinite matrix. To extract  $\mathbf{w}_j$  from the relaxed solution,  $\mathbf{Z}_j$  needs to be of rank one. We drop this non-convex constraint in order to obtain the relaxation. It is known that if there exists a rank one solution for  $\mathbf{Z}_j$  then the relaxation is tight and the exact BF matrix can be obtained. Otherwise, there are approximate and probabilistic techniques to solve for the  $\mathbf{w}_j$  BF vectors [13]. Here is the relaxation of (6):

$$\begin{aligned} & \text{maximize} && f(p_j) \\ & p_j, \mathbf{Z}_j \succeq 0 \\ & \text{s.t.} && \mathbf{h}_j^H \mathbf{Z}_l \mathbf{h}_j = 0, \quad \forall j \neq l; \\ & && \mathbf{h}_j^H \mathbf{Z}_j \mathbf{h}_j = p_j; \\ & && \sum_j [\mathbf{Z}_j](i, i) \leq P_i^*, \quad \forall i. \end{aligned} \quad (7)$$

Having the key BF parameters translated to  $\mathbf{M}$ , we can rewrite the precoding optimization problem (7) in terms of  $\mathbf{m}_j$ .

$$\begin{aligned} & \text{maximize} && f(p_j) \\ & p_j, \mathbf{Y}_j \succeq 0 \\ & \text{s.t.} && \mathbf{y}_j = \text{vec}(\mathbf{Y}_j), \quad \forall j; \\ & && \mathbf{h}_j \bar{\mathbf{V}}_j^\perp \mathbf{Y}_j \bar{\mathbf{V}}_j^{\perp H} \mathbf{h}_j^H = p_j, \quad \forall j; \\ & && \sum_{j=1}^{M_r} \mathbf{g}_{j[i]}^T \mathbf{y}_j \leq P_i^*, \quad \forall i. \end{aligned} \quad (8)$$

The same linearization technique as in (7) is used in our proposed formulation where  $\mathbf{m}_j \mathbf{m}_j^H$  is taken as  $\mathbf{Y}_j \succeq 0$ .

To summarize, problem (6) is the original non-convex ZF precoding problem with per antenna power constraints. We linearize (6) to obtain the relaxation (7). In [12] it is proven that there always exist rank one solutions ( $\mathbf{Z}_j$ ) for (7) and that they can be found efficiently. The first constraint of (7) says that the  $\mathbf{Z}_j$  should belong to the interference null space; (7) can therefore be simplified since the interference null space has a straightforward parameterization. Problem (8) is equivalent to (7) and consequently equivalent to (6). We conclude that the relaxation is tight and that exact ZF beamformers exist.

#### IV. CONVEX SPARSITY INDUCING REGULARIZER

So far we studied a pure ZF precoding problem. In this section we formulate the joint ZF and antenna selection problem by appending a convex sparsity inducing regularizer to (8) that can simultaneously force all the elements in an arbitrary row of  $\mathbf{W}$  to zero as explained in [10]. However this is not an intuitive task when there are quadratic terms involved in the optimization problem. For the sake of space we refer to the feasible set (including the non rank-1 as well as rank-1 solutions) that satisfies the constraints in (8) as  $\mathcal{C}$ . We first introduce the original joint problem:

$$\begin{aligned} & \text{maximize} && f(p_j) \\ & p_j, \mathbf{w}_j, \mathbf{Y}_j \in \mathcal{C} \\ & \text{s.t.} && \|\mathbf{W}\|_{0,2} = L_s; \\ & && \mathbf{w}_j \mathbf{w}_j^H = \bar{\mathbf{V}}_j^\perp \mathbf{Y}_j \bar{\mathbf{V}}_j^{\perp H}, \end{aligned} \quad (9)$$

where  $\|\mathbf{W}\|_{0,2}$  denotes the cardinality of rows in  $\mathbf{W}$  and  $L_s$  the required number of selected antennas. Obviously (9) is not convex as the zero norm is not a proper norm and also because the rank constraint (quadratic term) is not convex. Note that the zero norm is defined on  $\mathbf{W}$  and not on  $\mathbf{Y}_j$ 's, that is why we need to put the second constraint in (9).

The tightest convex relaxation for  $\|\mathbf{W}\|_{0,2}$  is known to be a group Lasso regularization [14] which is defined as  $\|\mathbf{W}\|_{1,2} = \sum_i \|\mathbf{w}_{[i]}\|_2$ . Requiring this norm to be small encourages the  $\ell_2$ -norm of the rows to be zero, typically resulting in group sparsity. Looking at (9), replacing the  $\ell_0$ -norm with its convex surrogate, does not make the problem any easier as the optimization parameter is a quadratic term. So, we need to rewrite our sparsity regularizer in terms of the quadratic variable in order to be able to solve the relaxed version of (9). We use the transformation technique to write the sparsity inducing term as a linear function of  $\mathbf{w}_j \mathbf{w}_j^H$ . Our

simple transformation function is the square operator on the sum of non-negative  $\ell_2$ -norms of the precoding matrix rows which is a monotone increasing function and can preserve the convexity [15].

We introduce a new variable  $\mathbf{X} \in \mathbb{C}^{M_t M_r \times M_t M_r}$  which is defined as  $\mathbf{X} = \mathbf{x}\mathbf{x}^H$ , where  $\mathbf{x} = \text{vec}(\mathbf{W})$  is the vectorized version of  $\mathbf{W}$ .

$$\mathbf{X} = \begin{bmatrix} \mathbf{X}_{1,1} & \mathbf{X}_{1,2} & \cdots & \mathbf{X}_{1,M_r} \\ \mathbf{X}_{2,1} & \mathbf{X}_{2,2} & \cdots & \mathbf{X}_{2,M_r} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{X}_{M_r,1} & \mathbf{X}_{M_r,2} & \cdots & \mathbf{X}_{M_r,M_r} \end{bmatrix}, \quad (10)$$

where  $\mathbf{X}_{j,l} = \mathbf{w}_j \mathbf{w}_l^H \in \mathbb{C}^{M_t \times M_t}$ . Then the squared mixed norm can be rewritten as

$$\begin{aligned} \left( \sum_{i=1}^{M_t} \|\mathbf{w}_{[i]}\|_2 \right)^2 &= \sum_{i_1=1}^{M_t} \sum_{i_2=1}^{M_t} \|\mathbf{w}_{[i_1]}\|_2 \cdot \|\mathbf{w}_{[i_2]}\|_2 \\ &= \sum_{i_1=1}^{M_t} \sum_{i_2=1}^{M_t} \|\mathbf{X}_{:,:(i_1, i_2)}\|_2. \end{aligned} \quad (11)$$

where  $\mathbf{X}_{:,:(i_1, i_2)}$  groups the  $(i_1, i_2)$  elements of all  $\mathbf{X}_{j,l}$ ,  $j, l = 1, \dots, M_r$ . Since the rank relaxation constraint  $\mathbf{Y}_j \succeq 0$  applies only on the diagonal matrices  $\mathbf{X}_{j,j}$ , these are the only variables that contribute to the solution; and the off-diagonal matrices are unbounded variables that do not appear in the constraint so they can be dropped. We further denote the diagonal matrices by  $\mathbf{X}_j$  for simplicity. Now we rewrite (11) in terms of the reduced size parameter  $\mathbf{m}_j$ , introduced previously in section II:

$$\begin{aligned} \text{vec}(\mathbf{X}_j) &= \text{vec}(\mathbf{w}_j \mathbf{w}_j^H) = \text{vec}(\bar{\mathbf{V}}_j^\perp \mathbf{Y}_j \bar{\mathbf{V}}_j^{\perp H}) \\ &= (\bar{\mathbf{V}}_j^{\perp *} \otimes \bar{\mathbf{V}}_j^\perp) \text{vec}(\mathbf{Y}_j) = \mathbf{G}_j \mathbf{y}_j, \end{aligned} \quad (12)$$

where  $\mathbf{G}_j \in \mathbb{C}^{M_t^2 \times S_n^2}$ , introducing  $\mathbf{G} = [\mathbf{G}_1, \mathbf{G}_2, \dots, \mathbf{G}_{M_r}] \in \mathbb{C}^{M_t^2 \times S_n^2 M_r}$  and  $\mathcal{Y} = \text{diag}(\mathbf{y}_1, \mathbf{y}_2, \dots, \mathbf{y}_{M_r}) \in \mathbb{C}^{S_n^2 M_r \times M_r}$  is a tall block diagonal matrix with  $\mathbf{y}_j$ 's as diagonal blocks (vectors). The group sparsity promoting term in (11) can be replaced by

$$\sum_{k=1}^{M_t^2} \|\mathbf{g}_{[k]} \mathcal{Y}\|_2. \quad (13)$$

Also the power expression in (5) and the total transmit power can be written in terms of  $\mathcal{Y}$  and  $\mathbf{G}$ , as  $p_{t_i} = \|\mathbf{g}_{[(i-1)M_t+i]}\|_1$ , since  $\mathbf{g}_{[i]}^T$  in (5) is the  $[(i-1)M_t+i]$ th row in  $\mathbf{G}_j$ .

## V. THROUGHPUT PROBLEM

Now we can formulate the joint AS and ZF precoding problem as a convex optimization problem. Here we look at the sum-rate capacity (throughput) of the MU-MISO system, so problem (9) can be relaxed as

$$\begin{aligned} &\text{minimize} && (\lambda \sum_{k=1}^{M_t^2} u_k \|\mathbf{g}_{[k]} \mathcal{Y}\|_2) - t \\ &t, p_j, \mathbf{Y}_j \in \mathcal{C} \\ &\text{s.t.} && \mathcal{Y} = \text{diag}(\mathbf{y}_1, \mathbf{y}_2, \dots, \mathbf{y}_{M_r}); \\ &&& \sum_{j=1}^{M_r} \log_2(1 + p_j) \geq t. \end{aligned} \quad (14)$$

In the throughput problem, the selection process is performed based on manipulating the sum-rate capacity subject to rigid power constraints. This minimizes the capacity loss ( $-t$ ) by eliminating a subset of antennas. In this paper we use a sparsity enhancing technique [16] to control the number of zero rows and converge faster to the desired number of chosen antennas. The regular sum in (13) is replaced by a weighted sum which is controlled by the vector  $\mathbf{u} = [u_1, u_2, \dots, u_{M_t^2}]$ . Also,  $\lambda$  is a regularization parameter which defines the favor of sparsity to throughput loss and is found together with  $\mathbf{u}$  iteratively for each particular  $L_s$ .

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### Algorithm 1 pseudocode for AS in throughput problem

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Initialize the regularization parameter;  $\lambda, \lambda_L, \lambda_U > 0$  and number of chosen antennas to  $L = M_t$ .

**while**  $L \neq L_s$  **do**

- Initialize the weight as  $\mathbf{u} = \mathbf{1}$  which is an all one vector of length  $M_t^2$  and the iteration counter  $n = 0$ .

**repeat**

- Solve (14) and update  $L = \|\mathbf{1}\|_0$  where  $\mathbf{1}_i = \sum_j \mathbf{g}_{j[i]}^T \mathbf{y}_j$ , is the Tx power for  $i$ th antenna.
  - Increment  $n$  and update the weight vector as  $u_k^{(n)} = 1/(\|\mathbf{g}_{[k]}\|_2^{(n-1)} + \epsilon)$ .
- until**  $n > n_{max}$  **or**  $L = L_s$
- if**  $L > L_s$ ,  $\lambda_L = \lambda$ ; **else if**  $L < L_s$ ,  $\lambda_U = \lambda$ ; **end if**.
  - Update  $\lambda = (\lambda_U - \lambda_L)/2 + \lambda_L$ .

**end while**

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The algorithm can be summarized as follows: there exist two loops in Algorithm 1; the inner loop for finding the proper  $\mathbf{u}$  with maximum iteration number  $n_{max}$  and the outer loop to balance the weight for the sparsity inducing term against the throughput. At each inner iteration, the sparsity enhancing vector  $\mathbf{u}$  is updated by penalizing the rows with the smaller norms. Then, the convex problem (14) is solved to force zero rows on  $\mathbf{W}$ , or equivalently to put the transmit power to zero for the eliminated antenna as expressed above. If in this step  $M_t - L_s$  antennas are eliminated, then the algorithm stops, and otherwise the weighting vector  $\mathbf{u}$  is reset and  $\lambda$  is updated to repeat the inner loop again. This goes on until exactly  $L_s$  antennas are chosen. The convergence is very fast and most of the time it needs only a few inner iterations with the initial  $\lambda$ . Here we use the Matlab package CVX [17] to solve (14), but alternative off-the-shelf solvers such as YALMIP, MAXDET and MOSEK can be used instead.

After this selection process the throughput can still be improved as it is compromised for sparsity in (14), so in the next step the pure BF problem is solved for the reduced size problem to find the best possible solution. This forms the new channel matrix  $\mathbf{H}_s$  where the corresponding columns of  $\mathbf{H}$  for the eliminated antennas are removed and (8) is solved for  $\mathbf{Y}_j \in \mathbb{C}^{S_n \times S_n}$  where  $S_n = L_s - M_r$ ,  $L_s > M_r$ , and  $p_j$  with known  $\mathbf{H}_s$ . This means that the decompositions of the  $\mathbf{Y}_j$ 's are done after this second step since we do not need  $\mathbf{w}_j$ 's for counting the eliminated antennas as this can be done by looking at the antennas with zero transmit power. Note

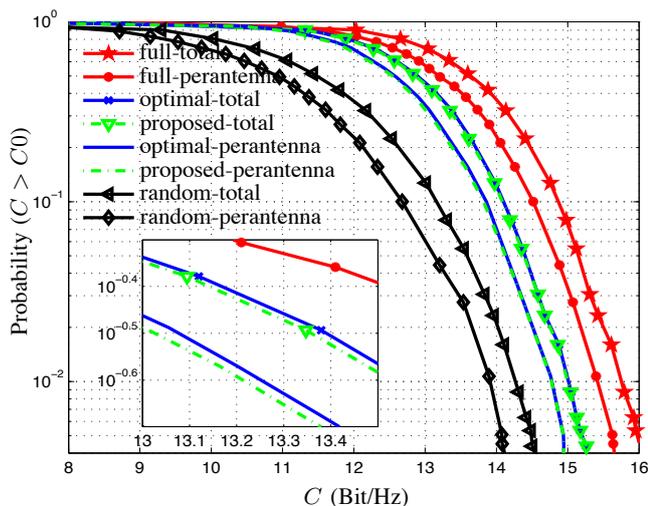


Fig. 2. CCDF curves for the full (all transmit antennas), optimal and the proposed throughput problem when  $M_t = 8$ ,  $M_r = 4$  and  $L_s = 6$ . The optimization problem is solved for both the total power constraint of  $P^* = 25$  dB and the equal per antenna power constraint for  $P_i^* = \frac{P^*}{L_s}$ . The maximum number of inner iterations is set to  $n_{max} = 5$ .

that there is always a rank one solution for (8), so there exist beamformers for the relaxed problem [12].

Here we verify the proposed algorithm with the aid of Matlab simulations. The complementary cumulative density function (CCDF) is chosen to show the statistical behavior of the random channel. The CCDF curves show the probability that the throughput is less than a certain value  $C$ . Globally optimal solutions are found via exhaustive search to find the best subset in terms of throughput, and then performing the beamforming on the selected subset. The optimal approach is simulated so that the pure beamforming problem (8) is solved for all  $\binom{M_t}{L_s}$  possible choices, which for Fig. 2 is  $\binom{8}{6} = 28$ . Then, the beamformer maximizing the objective is chosen. The performance of the proposed algorithm is very close to the optimal selection. CCDF curves are always better in terms of throughput (towards the right in Fig. 2) for the full-size set which shows the monotonicity of throughput w.r.t. the number of involved antennas at Tx. Random selection leads to a relatively poor capacity.

## VI. REMARKS

- Once the joint problem of AS and precoding is formulated as a convex optimization problem, it can be solved by relatively efficient tools that are widely developed for this sort of problem, for instance interior point methods [15].
- The gap between the optimal combinatorial joint AS and ZF BF problem and the proposed convex problem comes from replacing the cardinality norm in (9) by its convex surrogate, namely the squared mixed norm in (13); otherwise the linearization relaxation is tight.
- The linearization technique of expanding the matrix dimension has two important practical ramifications: First, the complexity of the problem is lifted up as the optimization variable becomes  $M_t$  times bigger. Second, the solu-

tions need not be of rank 1 for general QCQC programming problems due to the relaxation of rank constraint. Even though the rank relaxation is tight in ZF BF problem, the complexity is an issue for large scale problems.

- The throughput criteria can be easily replaced by the fairness criteria when the minimum SNR among the users is being considered in both the throughput and the power control problem. This is because the  $\min(p_j)$  is a concave function, so it is treated the same as the capacity function.
- Alternatively, the price for reducing the number of antennas can be paid by increasing the transmit power, so the power control problem deals with finding a subset of antennas that minimizes the total transmit power subject to having a certain pre-defined rate  $C^*$ . However, the mixed norm squared in (13) is a tighter constraint in terms of power, so by minimizing this term, we minimize the power as well. The results for the power control problem are presented in the following work by the authors.

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