# Joint ranging and clock synchronization for a wireless network

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***Abstract*—Synchronization and localization are two key aspects for the coherent functioning of a wireless network. Recently, various estimators have been proposed for pairwise synchronization between two nodes based on time stamp exchanges via two way communication. In this paper, we propose a closed form centralized Global Least Squares (GLS) estimator, which exploits the two way communication information between all the nodes in a wireless network. The fusion center based GLS uses a single clock reference and estimates all the unknown clock offsets, skews and pairwise distances in the network. The GLS estimate for clock offsets and skews is shown to outperform prevalent estimators. Furthermore, a new Cramer Rao Lower Bound (CRLB) is derived for the entire network and the proposed GLS solution is shown to approach the theoretical limits.**

***Index Terms*—joint estimation, skew, offset, distance, wireless network, anchorless, global solution, packet based two way communication**

## I. Introduction

Coherent functioning of wireless networks relies heavily on time synchronization among nodes. All the nodes in a network must be synchronized to a global reference, to facilitate accurate time stamping of data and synchronized communication of processed information [1]. Such global time synchronization is achieved by estimating all clock offsets and clock skews of the nodes and compensating the respective clocks aptly. Furthermore, when nodes are mobile and/or arbitrarily deployed in the field, then position estimation is often equally critical as time synchronization. The intermediate distances between all the nodes in the network is one of the key inputs for almost all localization techniques [2].

For a pair of nodes exchanging time packets via two way communication, if one node is assumed to be the reference, then the absolute clock offset and skew of the other node can be estimated using maximum likelihood [3], such as the Gaussian Maximum Likelihood (GMLL) estimate for unknown delay. A step further, Leng et al. presented a Low Complexity Least Squares (LCLS) solution [4] for the joint estimate, ignoring distance as a nuisance parameter. Towards joint localization and synchronization, centralized least squares solutions have been derived for estimating the absolute $2-D$ position and clock parameters of an unknown node [5], given completely synchronized anchors whose positions are known.

### A. Network assumptions

We assume a wireless network of nodes capable of two way communication with each other. Each node is equipped with a light weight atomic clock, which offers sufficient stability during the period for low frequency data collection. Secondly, the nodes within the network are mobile but their positions are sufficiently stable to the required accuracies during the estimation time. Thirdly, every node is equipped with adequate processing and communication capabilities. Our motivation is OLFAR (Oribiting Low Frequency Array for Radio astronomy) [6], an anchorless network of 10-50 satellite nodes in space which is currently being designed. Each satellite in OLFAR has a Rubidium clock and samples the sky at ultra low frequencies of 0.3-30 MHz, thus giving clock coherence up to 30 minutes [7]. In comparison to the raw data exchange and the on board correlation in the satellites, the time stamp exchanges and proposed centralized algorithm are negligible, both in terms of communication and computational power. The satellite nodes will be deployed in locations such that their positions are sufficiently stable during observations of 10-1000 seconds, thus offering adequate computation time for range measurement.

[0]This research was funded in part by the STW OLFAR project (Contract Number: 10556) within the ASSYS perspectief program. A part of the work presented in this paper has been submitted to the SPAMEC 2011 conference.

### B. Contributions

In this paper, we propose a novel centralized Global Least Squares (GLS) estimator to estimate all the unknown clock offsets, skews and pairwise distances in the network using a single clock reference. A new Cramer Rao Lower Bound (CRLB) is derived for the model and the proposed estimator is shown to be optimal asymptotically, in addition to outperforming available clock synchronization estimators.

*Notation:* The element wise matrix Hadamard product is denoted by $\odot$, element wise Hadamard division by $\oslash$, $(\cdot)^{\odot N}$ denotes element-wise matrix exponent. The Kroneker product is indicated by $\otimes$ and the transpose operator by $(\cdot)^T$. $\mathbf{1}_N = [1, 1 \dots, 1] \in \mathbb{R}^{N\times 1}$ is a vector of ones and $\mathbf{I}_N$ is a $N \times N$ identity matrix.

## II. Problem formulation

Consider a network of $N$ nodes equipped with independent clock oscillators which, under ideal conditions, are synchronized to the global time. However, in reality, due to various oscillator imperfections and environment conditions the clocks vary independently. Let $t_i$ be the local time at node $i$, then its divergence from the ideal global time $t$ is to first order given by the affine clock model,

$$t_i \quad = \quad \omega_i t + \phi_i \qquad (1)$$

where $\omega_i \in \mathbb{R}_+$ and $\phi_i \in \mathbb{R}$ are the clock skew and clock offset of node $i$. The clock skew and clock offset parameters for all $N$ nodes are represented by $\boldsymbol{\omega} = [\omega_1, \omega_2, \dots, \omega_N]^T \in \mathbb{R}_+{}^{N\times 1}$ and $\boldsymbol{\phi} = [\phi_1, \phi_2, \dots, \phi_N]^T \in \mathbb{R}^{N\times 1}$ respectively. Alternatively, the translation from local time $t_i$ to the global time $t$ is written as a function of local time,

$$\mathcal{F}_i(t_i) \triangleq t \quad = \quad \alpha_i t_i + \beta_i \qquad (2)$$

where $[\alpha_i, \beta_i] \triangleq [\omega_i^{-1}, -\omega_i^{-1}\phi_i]$ are the calibration parameters needed to correct the local clock of node $i$. Following immediately, for all $N$ nodes in the network, we have $\boldsymbol{\alpha} \triangleq \mathbf{1}_N \oslash \boldsymbol{\omega} \in \mathbb{R}_+{}^{N\times 1}$ and $\boldsymbol{\beta} \triangleq -\boldsymbol{\phi} \oslash \boldsymbol{\omega} \in \mathbb{R}^{N\times 1}$. All $M = \binom{N}{2}$ unique pairwise distances between $N$ nodes are given by $\mathbf{d} = [d_{11}, d_{12}, \dots, d_{(N-1)(N)}] \in \mathbb{R}^{M\times 1}$ and subsequently the propagation delay between nodes is given by $\boldsymbol{\tau} = \mathbf{d}c^{-1} \in \mathbb{R}^{M\times 1}$, where $c$ is the speed of the electromagnetic

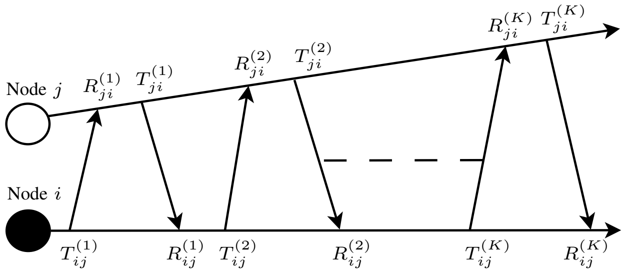


Fig. 1. This figure shows the classical two way communication between a node pair $(i, j)$. Node $i$ is the reference with $[\omega_i, \phi_i] = [1, 0]$ and also initiates the communication with node $j$ whose clock skew ($\omega_j$), clock offset ($\phi_j$) and distance ($d_{ij}$) from node $i$ are unknown. There are $K$ two way communications between the node pair during which $2K$ time markers are recorded at the respective nodes.

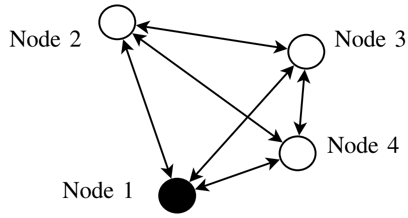


Fig. 2. An illustration of a network with $N = 4$ nodes, each capable of two way communication. Node 1 (shaded in black) is the clock reference with $[\omega_1, \phi_1] = [1, 0]$. The clock skews and clock offsets of node 2, 3 and 4 are unknown and are to be estimated, in addition to all the pairwise distances.

wave in the medium. Given a single reference node (providing the reference clock) and two way communication between all nodes, we intend to efficiently estimate all the absolute clock skews ($\boldsymbol{\omega}$), clock offsets ($\boldsymbol{\phi}$) and pairwise distances ($\mathbf{d}$) in the network.

## III. Pairwise Synchronization and Distance estimation

Prior to investigating the entire network, we focus our attention on a single pair of nodes. Consider the classical two way communication between a node pair $(i, j)$ as shown in Figure 1. Node $i$ initiates the communication and up-links a message to node $j$ and node $j$ responds by downlinking a message back to node $i$. The two nodes communicate messages back and forth, and the transmission and reception times are recorded independently at the respective nodes. $T_{ij}^{(k)}$ denotes the local time recorded at node $i$ for the $k$ th message departing to node $j$ and $R_{ji}^{(k)}$ is the corresponding local time marker recorded by the node $j$ on receiving the message from node $i$. Similarly during downlinking, $T_{ji}^{(k)}$ and $R_{ij}^{(k)}$ are the local timings recorded at node $j$ and $i$ respectively. There are $K$ such two way communications between the node pair, during which we assume that the propagation delay between the two nodes $\tau_{ij} = d_{ij}/c \equiv d_{ji}/c$ is fixed. The transmission and reception markers are then related as [8]

$$\begin{aligned} T_{ij}^{(k)} + q_1^{(k)} &= \omega_i(\mathcal{F}_j(R_{ji}^{(k)} + q_2^{(k)}) - \tau_{ij}) + \phi_i, \\ R_{ij}^{(k)} + q_3^{(k)} &= \omega_i(\mathcal{F}_j(T_{ji}^{(k)} + q_4^{(k)}) + \tau_{ij}) + \phi_i \end{aligned} \quad (3)$$

where $\{q_1^{(k)}, q_2^{(k)}, q_3^{(k)}, q_4^{(k)}\} \sim \mathcal{N}(0, 0.5\sigma^2)$ are Gaussian i.i.d noise variables plaguing the timing measurements. Rearranging the terms and from (1) and (2) we have

$$\begin{aligned} \alpha_i T_{ij}^{(k)} &= \alpha_j R_{ji}^{(k)} - \beta_i + \beta_j - \tau_{ij} - \alpha_i q_1^{(k)} + \alpha_j q_2^{(k)}, \\ \alpha_i R_{ij}^{(k)} &= \alpha_j T_{ji}^{(k)} - \beta_i + \beta_j + \tau_{ij} - \alpha_i q_3^{(k)} + \alpha_j q_4^{(k)} \end{aligned} \quad (4)$$

For all $K$ two way communications, a generalized model for a pair of nodes is

$$\begin{bmatrix} \mathbf{t}_{ji} & -\mathbf{t}_{ij} & \mathbf{1}_{2K} & -\mathbf{1}_{2K} & \mathbf{e} \end{bmatrix} \begin{bmatrix} \alpha_j \\ \alpha_i \\ \beta_j \\ \beta_i \\ \tau_{ij} \end{bmatrix} = \mathbf{q}_{ij} \quad (5)$$

where $\mathbf{t}_{ij}, \mathbf{t}_{ji} \in \mathbb{R}^{2K\times 1}$ are time markers recorded at node $i$ and node $j$ respectively while communicating with each other and are given by

$$\begin{aligned} \mathbf{t}_{ij} &= [T_{ij}^{(1)}, R_{ij}^{(1)}, T_{ij}^{(2)}, \ldots, R_{ij}^{(K)}]^T, \\ \mathbf{t}_{ji} &= [R_{ji}^{(1)}, T_{ji}^{(1)}, R_{ji}^{(2)}, \ldots, T_{ji}^{(K)}]^T \end{aligned} \quad (6)$$

$\mathbf{e} = [-1, +1 \ldots, +1]^T \in \mathbb{R}^{2K\times 1}$ and $\mathbf{q}_{ij}$ is the i.i.d noise vector, which is modeled as $\mathbf{q}_{ij} \sim \mathcal{N}(0, 0.5\sigma^2(\beta_i^2 + \beta_j^2)) \in \mathbb{R}^{2K\times 1}$. In reality, the clock skews $\omega_i, \omega_j$ are very close to 1 and the errors are of the order of $10^{-4}$. Hence the noise vector could be approximated by

$$\mathbf{q}_{ij} \sim \mathcal{N}(0, \sigma^2) \in \mathbb{R}^{2K\times 1} \quad (7)$$

Such an approximation is satisfactory and is implicitly employed in various cases such as [3] and [4]. Now, by asserting node $i$ as the reference node with $[\alpha_i, \beta_i] = [1, 0]$, equation (5) is simplified to

$$\mathbf{A}_{ji}\boldsymbol{\theta}_j = \mathbf{t}_{ij} + \mathbf{q}_{ij} \quad (8)$$

where

$$\begin{aligned} \mathbf{A}_{ji} &= [\mathbf{t}_{ji} \quad \mathbf{1}_{2K} \quad \mathbf{e}] \in \mathbb{R}^{2K\times 3} \\ \boldsymbol{\theta}_j &= [\beta_j \quad \alpha_j \quad \tau_{ij}]^T \in \mathbb{R}^{3\times 1} \end{aligned}$$

The Pairwise Least Squares (PLS) solution for estimating the clock skew ($\omega_j \triangleq 1/\alpha_j$), the clock offset ($\phi_j \triangleq -\beta_j/\alpha_j$) of node $j$ and its distance ($d_{ij} \triangleq c\tau_{ij}$) is obtained by minimizing the least squares norm, i.e.,

$$\hat{\boldsymbol{\theta}}_j = \arg\min_{\boldsymbol{\theta}_j} \ \|\mathbf{A}_{ji}\boldsymbol{\theta}_j - \mathbf{t}_{ij}\|_2^2 = (\mathbf{A}_{ji}^T\mathbf{A}_{ji})^{-1}\mathbf{A}_{ji}^T\mathbf{t}_{ij} \quad (9)$$

provided the number of two way communications $K \geq 2$. This Pairwise Least Squares (PLS) solution is a minor extension of the Low Complexity Least Squares (LCLS) by Leng et al. [4], where distance was considered a nuisance parameter and consequently not estimated. Note that in the two way communication model, there is no assumption that the messages have to be alternating regularly. Hence the measured time stamps are valid as long as the distance between the nodes and the clock parameters are stable within reasonable limits during the estimation process. Secondly, if the two way link is replaced with one way communication then matrix $\bar{\mathbf{A}}_{ji}$ is rank deficient and hence there is no optimal solution to jointly estimate the clock parameters and pairwise distances.

## IV. Network synchronization and ranging

Our aim is to extend the pairwise model in (5) to the entire network and find a global optimal solution for all unknown clock parameters and pairwise distances using a single reference. As an illustration, Figure 2 shows a network consisting of $N = 4$ nodes, all capable of

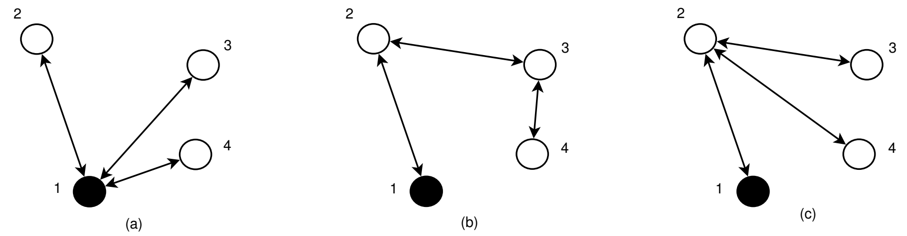


Fig. 3. An illustration of 3 networks with $N = 4$ nodes each capable of two way communication. The node shaded in black is the clock reference. The 3 networks are illustrative examples where GLS algorithm can be applied for network wide clock synchronization, despite missing communication links.

two way communication with each other. Without loss of generality, we assume node 1 is the reference node in this wireless network and that all the links are present. Rearranging the terms in (5), for all $\{i, j\}$, we have

$$[\mathbf{T} \quad \mathbf{E}_1 \quad \mathbf{E}_2] \begin{bmatrix} \boldsymbol{\alpha} \\ \boldsymbol{\beta} \\ \boldsymbol{\tau} \end{bmatrix} = \mathbf{q} \tag{10}$$

where $\mathbf{T} \in \mathbb{R}^{2KM \times N}$ contains all the timing vectors from all the $N$ nodes , $\mathbf{E}_1 \in \mathbb{R}^{2KM \times N}$, $\mathbf{E}_2 = -\mathbf{I}_M \otimes \mathbf{e} \in \mathbb{R}^{2KM \times M}$. For $N = 4$, $\mathbf{T}$ and $\mathbf{E}_1$ are of the form

$$\mathbf{T} = \begin{bmatrix} \mathbf{t}_{12} & -\mathbf{t}_{21} & & \\ \mathbf{t}_{13} & & -\mathbf{t}_{31} & \\ \mathbf{t}_{14} & & & -\mathbf{t}_{41} \\ & \mathbf{t}_{23} & -\mathbf{t}_{32} & \\ & \mathbf{t}_{24} & & -\mathbf{t}_{42} \\ & & \mathbf{t}_{34} & -\mathbf{t}_{43} \end{bmatrix} \tag{11}$$

$$\mathbf{E}_1 = \begin{bmatrix} +\mathbf{1}_{2K} & -\mathbf{1}_{2K} & & \\ +\mathbf{1}_{2K} & & -\mathbf{1}_{2K} & \\ +\mathbf{1}_{2K} & & & -\mathbf{1}_{2K} \\ & +\mathbf{1}_{2K} & -\mathbf{1}_{2K} & \\ & +\mathbf{1}_{2K} & & -\mathbf{1}_{2K} \\ & & +\mathbf{1}_{2K} & -\mathbf{1}_{2K} \end{bmatrix} \tag{12}$$

and a similar structure can be generalized for $N \geq 4$. The global noise vector is $\mathbf{q} = [\mathbf{q}_{12}, \mathbf{q}_{13}, \dots, \mathbf{q}_{(N-1)(N)}] \in \mathbb{R}^{2KM \times 1}$ where each $\mathbf{q}_{ij}$ is given by (7). Since node 1 is the reference node, *i.e.*, $[\alpha_1, \beta_1] = [1, 0]$, rearranging the terms in (10) we have

$$\bar{\mathbf{A}}\boldsymbol{\theta} = -\bar{\mathbf{t}}_1 + \mathbf{q} \tag{13}$$

where

$$\bar{\mathbf{A}} = [\bar{\mathbf{T}} \quad \bar{\mathbf{E}}_1 \quad \mathbf{E}_2] \in \mathbb{R}^{2KM \times L}$$
$$\boldsymbol{\theta} = [\bar{\boldsymbol{\alpha}} \quad \bar{\boldsymbol{\beta}} \quad \boldsymbol{\tau}]^T \in \mathbb{R}^{L \times 1}$$

where $L = 2N + M - 2$ and $\bar{\mathbf{T}}, \bar{\mathbf{E}}_1 \in \mathbb{R}^{2KM \times (N-1)}$ are submatrices of $\mathbf{T}$ and $\mathbf{E}_1$ respectively, excluding the corresponding first columns. $\bar{\boldsymbol{\alpha}}, \bar{\boldsymbol{\beta}} \in \mathbb{R}^{(N-1) \times 1}$ represent the unknown clock parameters of all the nodes excluding node 1. $\bar{\mathbf{t}}_1 \in \mathbb{R}^{2KM \times 1}$ is the first column of matrix $\mathbf{T}$ which contains the timing markers recorded at node 1, whilst communicating with the other nodes in the network. Analyzing the components of matrix $\bar{\mathbf{A}}$, both $\bar{\mathbf{T}}, \bar{\mathbf{E}}_1$ are full rank, since the respective first $2K(N-1)$ rows are formed by block diagonal matrices. Note that all columns of $\mathbf{E}_2$ are also independent. In addition, if $K \geq 2$, then a Global Least Squares (GLS) solution is feasible and is obtained by minimizing the least squares norm, i.e.,

$$\hat{\boldsymbol{\theta}} = \arg\min_{\boldsymbol{\theta}} \quad \|\bar{\mathbf{A}}\boldsymbol{\theta} + \bar{\mathbf{t}}_1\|_2^2 = (\bar{\mathbf{A}}^T\bar{\mathbf{A}})^{-1}\bar{\mathbf{A}}^T\bar{\mathbf{t}}_1 \tag{14}$$

Hence, the unknown clock skews ($\bar{\boldsymbol{\omega}} \triangleq \mathbf{1}_N \oslash \bar{\boldsymbol{\alpha}}$), the unknown clock offsets ($\bar{\boldsymbol{\phi}} \triangleq -\bar{\boldsymbol{\beta}} \oslash \bar{\boldsymbol{\alpha}}$) of the nodes and the pairwise distances ($\mathbf{d} \triangleq \boldsymbol{\tau}c$) in the network can be estimated by solving (14). Note that the LS solution proposed in the previous section is a special case of GLS when $N = 2$. The closed form solution (14) is for a full mesh network. More in general, if some pairwise communications links are missing then corresponding rows in matrix $\bar{\mathbf{A}}$ are dropped. Consequentially, the pairwise distances between those particular nodes cannot be optimally estimated. However, despite missing links network wide synchronization is still feasible (example Figure 3) using (14) if and only if $\bar{\mathbf{A}}$ is full rank. In other words, every node has at least single two way communication link with any other node in the network.

## V. Cramer Rao lower bound

The Cramer Rao Lower Bound (CRLB) on the error variance for any unbiased estimator states [9]

$$\varepsilon\left\{(\hat{\boldsymbol{\theta}} - \boldsymbol{\theta})(\hat{\boldsymbol{\theta}} - \boldsymbol{\theta})^T\right\} \geq \mathbf{F}^{-1} \tag{15}$$

where $\mathbf{F}$ is the Fisher information matrix. The error vector $\mathbf{q}$ in (13) is Gaussian by assumption and the corresponding Fisher information matrix is [9]

$$\mathbf{F} = \frac{1}{\sigma^2}\mathbf{J}^T\mathbf{J} \tag{16}$$

where $\mathbf{J} \in \mathbb{R}^{2KM \times L}$ is the Jacobian matrix. For jointly estimating the clock skew $\bar{\boldsymbol{\omega}}$, clock offset $\bar{\boldsymbol{\phi}}$ and all the pairwise distances $\mathbf{d}$, we have

$$\mathbf{J} = \left[\frac{\partial \bar{\mathbf{A}}\boldsymbol{\theta}}{\partial \boldsymbol{\theta}^T}\right] \triangleq \left[\mathbf{J}_{\bar{\boldsymbol{\omega}}} \quad \mathbf{J}_{\bar{\boldsymbol{\phi}}} \quad \mathbf{J}_{\boldsymbol{\tau}}\right] \tag{17}$$

where the independent components can be shown as

$$\begin{aligned} \mathbf{J}_{\bar{\boldsymbol{\omega}}} &= -(\bar{\mathbf{T}} + \bar{\mathbf{E}}_1 \odot \mathbf{1}_{2KM}\bar{\boldsymbol{\phi}}^T) \oslash (\mathbf{1}_{2KM}\bar{\boldsymbol{\omega}}^T)^{\odot 2} \\ \mathbf{J}_{\bar{\boldsymbol{\phi}}} &= \bar{\mathbf{E}}_1 \oslash \mathbf{1}_{2KM}\bar{\boldsymbol{\omega}}^T \\ \mathbf{J}_{\boldsymbol{\tau}} &= \mathbf{E}_2 \end{aligned} \tag{18}$$

## VI. Simulations

Simulations are conducted to evaluate the performance of the proposed estimator. We consider a network of $N = 4$ nodes, as shown in Figure 2, wherein all the nodes are located within $10Km$ of each other and consequently $\mathbf{d}$ is a random vector in the range (0,10Km].

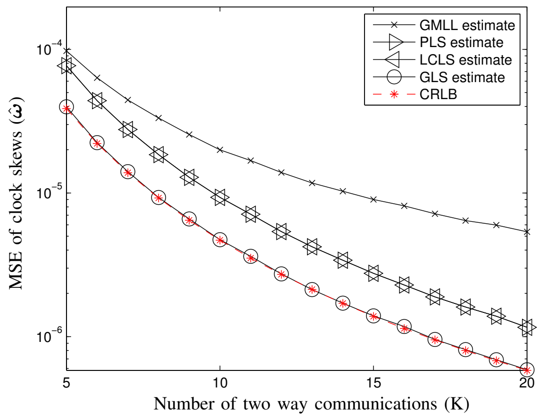


Fig. 4. Mean Square Error (MSE) plot of estimated clock skews ($\hat{\omega}$) for a network of $N = 4$ nodes, where noise is Gaussian with $\sigma = 0.1$

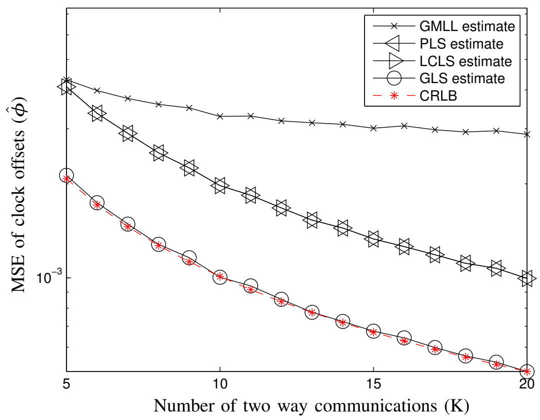


Fig. 5. Mean Square Error (MSE) plot of estimated clock offsets ($\hat{\phi}$) for a network of $N = 4$ nodes, where noise is Gaussian with $\sigma = 0.1$

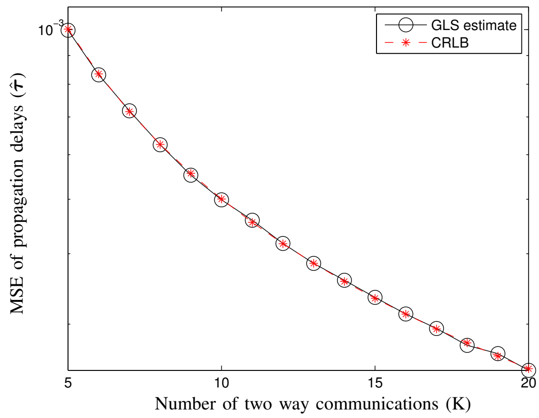


Fig. 6. Mean Square Error (MSE) plot of estimated propagation delays ($\hat{\tau}$) for a network of $N = 4$ nodes, where noise is Gaussian with $\sigma = 0.1$

The clock offsets ($\bar{\phi}$) and clock skews ($\bar{\omega}$) are uniform randomly distributed in the range $[-1, 1]$ seconds and $[0.998, 1.002]$ respectively. The transmission time markers $\mathbf{t}_{ij}$ are linearly distributed between 1 to 100 seconds, for a number of two way communication links $K$ from 5 to 20. The noise variance on the timing markers is $\sigma = 0.1$ and all results presented are averaged over 10,000 independent Monte Carlo runs.

Figure 5 and 4 show the Mean Square Errors (MSEs) of clock skews and offsets against the number of two way communications $K$ for various estimators. The Low Complexity Least Squares (LCLS) [4], the proposed Pairwise Least Squares (LS) and the Maximum Likelihood GMLL [3] algorithms are independently applied, pairwise from node 1 to every other node, to estimate all the unknown skews $\bar{\omega}$ and offsets $\bar{\phi}$. As shown by the plots, the PLS solution matches the LCLS performance. Secondly, the proposed Global Least Squares (GLS) solution, which exploits information from all the pairwise two way communications, outperforms the (LCLS) for both clock skew and clock offset estimation in addition to achieving the theoretical Cramer Rao Lower Bound. In addition to clock skews and offsets, the pairwise distances $\mathbf{d}$ are also estimated in terms of propagation delays $\boldsymbol{\tau}$. Figure 6 shows the proposed Global Least Squares (GLS) solution for $\boldsymbol{\tau}$ which achieves the Cramer Rao Lower Bound. To the best of the author's knowledge, there are no other distance estimators available for this data model for comparison.

## VII. Conclusions

In this paper, an efficient and novel closed form Global Least Squares (GLS) estimator for network wide synchronization is proposed. The GLS utilizes a single reference node and exploits all two way communication information between nodes in the network. The proposed estimator is shown to improve available solutions for clock skews and offsets, in addition to estimating the pairwise distances between all nodes in a closed form. A new CRLB has been derived and the proposed solution achieves the bound, which shows that the linearized model is sufficiently accurate. As a trivial extension, Multi-Dimensional Scaling (MDS) [2] can be applied on the estimated distances to obtain all relative positions of the nodes, thereby achieving absolute clock synchronization and relative localization for an anchorless wireless network.

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