# Relative Velocity Estimation Using Multidimensional Scaling

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***Abstract*—Localization is a fundamental challenge for any wireless network of nodes, in particular when the nodes are mobile. For an *anchorless network of mobile nodes*, we present a relative velocity estimation algorithm based on multidimensional scaling. We propose a generalized two-way ranging model, where the time-varying pairwise distances between the nodes are expressed as a Taylor series for a small observation period. The Taylor coefficients which are obtained by solving a Vandermonde system are in turn used to jointly estimate the initial relative position and the relative velocity of the nodes. Simulations are conducted to evaluate the performance of the proposed solutions and the results are presented.**

***Index Terms*—joint position and velocity estimation, MDS, dynamic ranging, wireless network, anchorless, localization, doppler-free**

## I. Introduction

Wireless network localization is a fundamental challenge for a wide variety of applications. For a network of fixed nodes, range based localization algorithms use the pairwise distances between the nodes to estimate the positions of the nodes. In particular, for anchorless networks i.e., networks without known reference positions, Multi-Dimensional Scaling (MDS) algorithms estimate the relative positions of the nodes up to a rotation and translation [2]. A step further, when the nodes are mobile, then conventionally either the nodes are considered relatively stationary within desired accuracies during all ranging measurements [3] or Doppler measurements are utilized to track the positions. Unfortunately, Doppler measurements are not always available and the assumption on the node positional stability over large time periods is not necessarily practical. Furthermore, for a mobile network, the application of classical MDS-based relative localization at every time instant yields a position matrix with arbitrary rotation, thereby providing no information on the relative velocities of the nodes. To the best of the authors' knowledge, the estimation of relative velocities for an anchorless network has not been investigated in literature.

Our motivation for this work is triggered by *inaccessible* mobile wireless networks, which have partial or no information of absolute coordinates and/or clock references. Such scenarios are prevalent in under-water communication [4], indoor positioning systems [5] and envisioned space-based satellite networks with minimal ground segment capability [6].

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In this article, our quest is to estimate the relative Positions and Velocities (PVs) up to a rotation and translation of an *anchorless network of mobile nodes*, given two-way communication capability between all the nodes.

*Notation:* The element-wise matrix Hadamard product is denoted by $\odot$ and $(\cdot)^{\odot N}$ denotes element-wise matrix exponent. The Kronecker product is indicated by $\otimes$ and the transpose operator by $(\cdot)^T$. $\mathbf{1}_N \in \mathbb{R}^{N\times 1}$ is a vector of ones, $\mathbf{I}_N$ is a $N \times N$ identity matrix, $\mathbf{0}_{M,N}$ is a $M \times N$ matrix of zeros, $\mathrm{diag}(\mathbf{a})$ represents a diagonal matrix with elements of the vector $\mathbf{a}$ on the diagonal and $\|\cdot\|$ is the Euclidean norm. The matrix $\mathbf{A} = \mathrm{bdiag}(\mathbf{A}_1, \mathbf{A}_2, \dots, \mathbf{A}_N)$ consists of matrices $\mathbf{A}_1, \mathbf{A}_2, \dots, \mathbf{A}_N$ along the diagonal and zeros elsewhere.

## II. Relative kinematics

Consider a cluster of $N$ nodes in a $P$-dimensional Euclidean space, whose initial coordinates are given by $\mathbf{X} = [\mathbf{x}_1, \mathbf{x}_2, \dots \mathbf{x}_N] \in \mathbb{R}^{P\times N}$, where $\mathbf{x}_i \in \mathbb{R}^{P\times 1}$ is the position of the $i$th node at time $t = t_0$. At the same time $t_0$, the $i$th node has velocity $\mathbf{y}_i \in \mathbb{R}^{P\times 1}$ and all such velocities are collected in $\mathbf{Y} = [\mathbf{y}_1, \mathbf{y}_2, \dots \mathbf{y}_N] \in \mathbb{R}^{P\times N}$. Without loss of generality, for a small time duration around $t = t_0$ we assume the nodes are in independent linear motion, i.e.,

$$\frac{d\mathbf{Y}}{dt} = \mathbf{0}_{P,N}. \tag{1}$$

The absolute PVs $(\mathbf{X}, \mathbf{Y})$ are a linear isometric transformation of the relative PVs $(\underline{\mathbf{X}}, \underline{\mathbf{Y}})$ i.e.,

$$\mathbf{X} = \mathbf{H}_x\underline{\mathbf{X}} + \mathbf{h}_x\mathbf{1}_N^T, \tag{2}$$
$$\mathbf{Y} = \mathbf{H}_y\underline{\mathbf{Y}} + \mathbf{h}_y\mathbf{1}_N^T, \tag{3}$$

where $\underline{\mathbf{X}} \in \mathbb{R}^{P\times N}$ is the relative position matrix, $\mathbf{H}_x \in \mathbb{R}^{P\times P}$ is the unknown rotation and $\mathbf{h}_x \in \mathbb{R}^{P\times 1}$ is the translation of the network at $t = t_0$. We define relative velocity at $t_0$ up to a rotation and translation as $\underline{\mathbf{Y}} \in \mathbb{R}^{P\times N}$ and relative velocity up to a translation as $\mathbf{H}_y\underline{\mathbf{Y}}$, where $\mathbf{H}_y \in \mathbb{R}^{P\times P}$ is an unknown rotation matrix. The relative velocity $\mathbf{H}_y\underline{\mathbf{Y}}$ is relative to the group velocity of the network, i.e., $\mathbf{h}_y \in \mathbb{R}^{P\times 1}$.

In addition, the distances between the nodes are time-varying, which are denoted by the Euclidean Distance Matrix (EDM) $\mathbf{D}(t) \triangleq [d_{ij}(t)] \in \mathbb{R}^{N\times N}$, where $d_{ij}(t)$ is the pairwise distance between the node pair $(i, j)$ at time instant $t$. Finally, we define a double-centered matrix

$$\mathbf{B}(t) \triangleq -0.5\mathbf{P}\big(\mathbf{D}(t)\big)^{\odot 2}\mathbf{P}, \tag{4}$$

where $\mathbf{P} = \mathbf{I}_N - N^{-1}\mathbf{1}_N\mathbf{1}_N^T$ is the centering matrix. *Note* *that* $\mathbf{D}(t)$ *(and subsequently* $\mathbf{B}(t)$*) is a non-linear function of time* $t$*, even when the nodes are in independent linear motion.* Given an estimate of the EDMs $\widehat{\mathbf{D}}(t)$ at time instances $t$, our goal is to find an estimate of the initial relative position $\widehat{\underline{\mathbf{X}}}$ and relative velocity $\widehat{\underline{\mathbf{Y}}}$.

### A. Relative Position

Let $\mathbf{D}(t_0) \triangleq \mathbf{R} = [r_{ij}] \in \mathbb{R}_+^{N \times N}$ be the true EDM containing all the pairwise distances between the nodes at time instant $t_0$, then (4) is

$$\mathbf{B}_x \triangleq -0.5\mathbf{P}\mathbf{R}^{\odot 2}\mathbf{P} = \underline{\mathbf{X}}^T\underline{\mathbf{X}}. \quad (5)$$

The relative position of the nodes is then obtained by the spectral decomposition of $\mathbf{B}_x$ [2], which we describe in brief for the sake of completeness. Let $\widehat{\mathbf{B}}_x \triangleq -0.5\mathbf{P}\widehat{\mathbf{R}}^{\odot 2}\mathbf{P} = \mathbf{U}_x\mathbf{\Lambda}_x\mathbf{U}_x^T$, where $\widehat{\mathbf{R}}$ collects the measured noisy distances between the nodes and $\mathbf{U}_x, \mathbf{\Lambda}_x \in \mathbb{R}^{N \times N}$ are the eigenvectors and eigenvalues of $\widehat{\mathbf{B}}_x$, respectively. An MDS estimate of the relative position $\widehat{\underline{\mathbf{X}}}$ is then

$$\widehat{\underline{\mathbf{X}}} = \underline{\mathbf{\Lambda}}_x^{1/2}\underline{\mathbf{U}}_x^T, \quad (6)$$

where $\underline{\mathbf{\Lambda}}_x \in \mathbb{R}^{P \times P}$ and $\underline{\mathbf{U}}_x \in \mathbb{R}^{P \times N}$ contain the first $P$ nonzero eigenvalues and eigenvectors, respectively.

### B. Relative Velocity

Differentiating (4) w.r.t. time $t$, we have

$$\frac{d\mathbf{B}(t)}{dt} \triangleq -\mathbf{P}\Big(\mathbf{D}(t) \odot \dot{\mathbf{D}}(t)\Big)\mathbf{P}\Big|_{t=t_0} = \mathbf{P}\Big(\mathbf{X}^T\mathbf{Y} + \mathbf{Y}^T\mathbf{X}\Big)\mathbf{P}$$

and a step further, differentiating again w.r.t. time under the constraint (1) and substituting $t = t_0$ we have

$$\frac{d^2\mathbf{B}(t)}{dt^2}\Big|_{t=t_0} \triangleq \mathbf{B}_y \triangleq -0.5\mathbf{P}(\mathbf{R}\odot\ddot{\mathbf{R}} + \dot{\mathbf{R}}^{\odot 2})\mathbf{P} = \underline{\mathbf{Y}}^T\underline{\mathbf{Y}} \quad (7)$$

where $\dot{\mathbf{R}} = [\dot{r}_{ij}] \in \mathbb{R}^{N \times N}$ and $\ddot{\mathbf{R}} = [\ddot{r}_{ij}] \in \mathbb{R}_+^{N \times N}$ are the respective first-order and second-order derivatives of the pairwise distances w.r.t. time at $t = t_0$. Now given the corresponding measurement matrices $\widehat{\dot{\mathbf{R}}}$ and $\widehat{\ddot{\mathbf{R}}}$, let the eigenvalue decomposition of the measurement $\widehat{\mathbf{B}}_y \triangleq -0.5\mathbf{P}(\widehat{\mathbf{R}} \odot \widehat{\ddot{\mathbf{R}}} + \widehat{\dot{\mathbf{R}}}^{\odot 2})\mathbf{P} = \mathbf{U}_y\mathbf{\Lambda}_y\mathbf{U}_y^T$, where $\mathbf{\Lambda}_y, \mathbf{U}_y \in \mathbb{R}^{N \times N}$ are the corresponding eigenvalues and eigenvectors, then an MDS-based relative velocity estimate $\widehat{\underline{\mathbf{Y}}}$ is given by

$$\widehat{\underline{\mathbf{Y}}} = \underline{\mathbf{\Lambda}}_y^{1/2}\underline{\mathbf{U}}_y^T, \quad (8)$$

where $\underline{\mathbf{\Lambda}}_y \in \mathbb{R}^{P \times P}$ are the first $P$ nonzero eigenvalues and $\underline{\mathbf{U}}_y \in \mathbb{R}^{P \times N}$ the corresponding eigenvectors.

While MDS based relative localization (6) is well understood [2], our fundamental contribution is the definition of relative velocities (3) and its estimation (8).

## III. Dynamic ranging

In essence the required range coefficients can be obtained from Doppler measurements. Note that, $\mathbf{R}$ is the ToA (Time of Arrival), the range rate $\dot{\mathbf{R}}$ is the radial velocity (observed from Doppler shift) and the second-order range parameter $\ddot{\mathbf{R}}$ is the rate of radial velocity (observed from Doppler spread) between the nodes at $t_0$. Hence, if available these measurements can be readily used to estimate the relative velocity from (8). Alternatively, we here propose an elegant time-based Taylor series approximation of the measured time-varying pairwise distances to obtain these range coefficients.

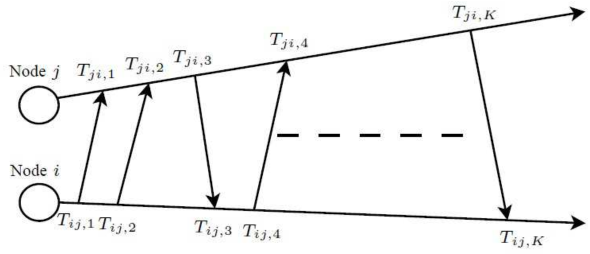


Fig. 1: A generalized two-way ranging between a pair of *mobile* nodes, where the nodes transmit and receive and record $K$ time stamps independently. Similar to [3], [7], we levy no pre-requisite on the sequence, direction of communications.

### A. Taylor series approximation

Let $\tau_{ij}(t_0) \equiv \tau_{ji}(t_0) = c^{-1}d_{ij}(t_0)$ be the propagation delay between the node pair $(i, j)$ at time instant $t_0$ where $d_{ij}(t_0)$ is the corresponding pairwise distance and $c$ is the speed of the electromagnetic wave in the medium. Now, for a small interval $\Delta t = t - t_0$, we consider the relative distance to be a smoothly varying polynomial of time, which empowers us to describe the propagation delay $\tau_{ij}(t)$ at $t$ as an infinite Taylor series in the neighborhood of $t_0$. Thus, we have

$$\tau_{ij}(t_0 + \Delta t) \triangleq c^{-1}d_{ij}(t_0 + \Delta t) \triangleq c^{-1}d_{ij}(t), \quad (9)$$

where $d_{ij}(t)$ is the distance at $t = t_0 + \Delta t$, given by

$$d_{ij}(t) = r_{ij} + \frac{\dot{r}_{ij}}{1!}\Delta t + \frac{\ddot{r}_{ij}}{2!}\Delta t^2 + \ldots, \quad (10)$$

where $[r_{ij}, \dot{r}_{ij}, \ddot{r}_{ij}, \ldots] \in \mathbb{R}^{L \times 1}$ are the derivatives of the pairwise distance at $t_0$, which are limited to $L - 1$th order. Without loss of generality, assuming $t_0 = 0$, we have $t = \Delta t$ and subsequently (9) and (10) simplify to the Maclaurian series as

$$\tau_{ij}(t) = c^{-1}\Big(r_{ij} + \dot{r}_{ij}t + \frac{\ddot{r}_{ij}}{2!}t^2 + \ldots\Big). \quad (11)$$

The *unique* pairwise ranges between all the $N$ nodes are collected in a vector $\mathbf{r} \in \mathbb{R}^{\bar{N} \times 1}$, where $\bar{N} = \binom{N}{2}$ is the number of unique pairwise baselines. Along similar lines, we have $\dot{\mathbf{r}} \in \mathbb{R}^{\bar{N} \times 1}$, $\ddot{\mathbf{r}} \in \mathbb{R}^{\bar{N} \times 1}$ and corresponding higher-order terms. The polynomial range basis is simplified further by introducing

$$\Big[\underline{r}_{ij}, \dot{\underline{r}}_{ij}, \ddot{\underline{r}}_{ij}, \ldots\Big]^T = \text{diag}(\mathbf{f})^{-1}\Big[r_{ij}, \dot{r}_{ij}, \ddot{r}_{ij}, \ldots\Big]^T, \quad (12)$$

where $\mathbf{f} = c[1,\ 1!,\ 2!,\ \ldots]^T \in \mathbb{R}^{L \times 1}$, such that (11) is

$$\tau_{ij}(t) = c^{-1}d_{ij}(t) \triangleq \underline{r}_{ij} + \dot{\underline{r}}_{ij}t + \ddot{\underline{r}}_{ij}t^2 + \ldots. \quad (13)$$

Following the definition of $[\mathbf{r}, \dot{\mathbf{r}}, \ddot{\mathbf{r}}, \ldots]$, we define $\underline{\mathbf{r}} \in \mathbb{R}^{\bar{N} \times 1}$, $\dot{\underline{\mathbf{r}}} \in \mathbb{R}^{\bar{N} \times 1}$, $\ddot{\underline{\mathbf{r}}} \in \mathbb{R}^{\bar{N} \times 1}$ and similarly higher order terms.

### B. Data model

Now, consider a pair of nodes $(i, j)$ communicating with each other, where $T_{ij,k}$ and $T_{ji,k}$ are respectively the time of departure (or arrival) of the message at node $i$ and node $j$ for the $k$th time instant. As shown in Fig.1, the nodes exchange $K$ messages where similar to [3], [7] we impose no restrictions on the direction or sequence of communication. The propagation delay between the node pair at the $k$ th time instant is $|T_{ij,k} - T_{ji,k}|$, and in conjunction with the polynomial approximation (13) we have

$$\tau_{ij}(T_{ij,k}) = \underline{r}_{ij} + \underline{\dot{r}}_{ij} T_{ij,k} + \underline{\ddot{r}}_{ij} T_{ij,k}^2 + \ldots = |T_{ij,k} - T_{ji,k}|, \quad (14)$$

where without loss of generality we have replaced $t$ with $T_{ij,k}$. By replacing the *true* time with $T_{ij,k}$, we inherently assume $T_{ij,k}$ is in the neighborhood of $t_0 = 0$ and $\tau_{ij,k}$ is measured as a function of the local time at node $i$. Furthermore, we also assume the network to be synchronized, which is a valid assumption since for an asynchronous network, the clock parameters (up to first order) can be decoupled from the range parameters and estimated efficiently [7].

In practice, the time measurements are also corrupted with noise and hence (14) is

$$\begin{aligned}\underline{r}_{ij} + \underline{\dot{r}}_{ij}(T_{ij,k} + q_{i,k}) + \underline{\ddot{r}}_{ij}(T_{ij,k} + q_{i,k})^2 + \ldots \\ = |(T_{ji,k} + q_{j,k}) - (T_{ij,k} + q_{i,k})|, \quad (15)\end{aligned}$$

where $q_{i,k} \sim \mathcal{N}(0, \Sigma_i)$, $q_{j,k} \sim \mathcal{N}(0, \Sigma_j)$ are Gaussian i.i.d. noise by assumption, plaguing the timing measurements at node $i$ and node $j$, respectively. Rearranging the terms, we have

$$\underline{r}_{ij} + \underline{\dot{r}}_{ij} T_{ij,k} + \underline{\ddot{r}}_{ij} T_{ij,k}^2 + \ldots = |T_{ji,k} - T_{ij,k}| + q_{ij,k}, \quad (16)$$

where $q_{ij,k} = |q_{j,k} - q_{i,k}| - (2\underline{\ddot{r}}_{ij} T_{ij,k} q_{i,k} + \underline{\ddot{r}}_{ij} q_{i,k}^2 + \ldots)$. For wireless communication with $c = 3 \times 10^8$m/s, note that the modified range parameters are scaled by $c^{-1}$ (12). Furthermore, since the dynamic range model is proposed for a small time interval, the term $(2\underline{\ddot{r}}_{ij} T_{ij,k} q_{i,k} + \underline{\ddot{r}}_{ij} q_{i,k}^2 + \ldots)$ is relatively small and subsequently the noise vector plaguing the measurements can be approximated as $q_{ij,k} \approx |q_{j,k} - q_{i,k}|$ which begets

$$q_{ij,k} \sim \mathcal{N}(0, \Sigma_i + \Sigma_j). \quad (17)$$

Aggregating all $K$ packets, we have

$$\overbrace{\begin{bmatrix} \mathbf{1}_K & \mathbf{t}_{ij} & \mathbf{t}_{ij}^{\odot 2} & \ldots \end{bmatrix}}^{\mathbf{A}_{ij}} \overbrace{\begin{bmatrix} \underline{r}_{ij} \\ \underline{\dot{r}}_{ij} \\ \underline{\ddot{r}}_{ij} \\ \vdots \end{bmatrix}}^{\boldsymbol{\theta}_{ij}} = \boldsymbol{\tau}_{ij} + \mathbf{q}_{ij}, \quad (18)$$

where $\mathbf{t}_{ij} = [T_{ij,1}, T_{ij,2}, \ldots, T_{ij,K}] \in \mathbb{R}^{K \times 1}$ and $\boldsymbol{\tau}_{ij} = |\mathbf{t}_{ij} - \mathbf{t}_{ji}| \in \mathbb{R}^{K \times 1}$ and $\boldsymbol{\theta}_{ij} \in \mathbb{R}^{L \times 1}$ is a vector containing the unknown range parameters. The known Vandermonde matrix $\mathbf{A}_{ij} \in \mathbb{R}^{K \times L}$ contains the measured time stamps and is invertible if $T_{ij,k}$ is unique for all $k \leq K$. The noise vector on the linear system is $\mathbf{q}_{ij} = [q_{ij,1}, q_{ij,2}, \ldots \; q_{ij,K}]^T \in \mathbb{R}^{K \times 1}$, where $q_{ij,k}$ is given by (17) and the corresponding covariance matrix is

$$\boldsymbol{\Sigma}_{ij} \triangleq \mathbb{E}\left[\mathbf{q}_{ij}\mathbf{q}_{ij}^T\right] = (\Sigma_i + \Sigma_j)\mathbf{I}_K \;\in \mathbb{R}^{K \times K}. \quad (19)$$

For a network of $N$ nodes, the normal equations (18) can be extended to

$$\overbrace{\begin{bmatrix} \mathbf{I}_{\bar{N}} \otimes \mathbf{1}_K & \mathbf{T} & \mathbf{T}^{\odot 2} & \ldots \end{bmatrix}}^{\mathbf{A}} \overbrace{\begin{bmatrix} \underline{\mathbf{r}} \\ \underline{\dot{\mathbf{r}}} \\ \underline{\ddot{\mathbf{r}}} \\ \vdots \end{bmatrix}}^{\boldsymbol{\theta}} = \boldsymbol{\tau} + \mathbf{q}, \quad (20)$$

where $\mathbf{T} = \text{bdiag}(\mathbf{t}_{12}, \mathbf{t}_{13}, \ldots \mathbf{t}_{1N}, \; \mathbf{t}_{23}, \ldots) \in \mathbb{R}^{\bar{N}K \times \bar{N}}$, $\boldsymbol{\tau} = [\boldsymbol{\tau}_{12}^T, \boldsymbol{\tau}_{13}^T, \ldots \boldsymbol{\tau}_{1N}^T, \; \boldsymbol{\tau}_{23}^T, \ldots]^T \in \mathbb{R}^{\bar{N}K \times 1}$ contain the time stamp exchanges of the $\bar{N}$ *unique* pairwise links in the network, and $\boldsymbol{\theta} \in \mathbb{R}^{\bar{N}L \times 1}$ contains the unknown range parameters for the entire network. The noise vector is $\mathbf{q} = [\mathbf{q}_{12}^T, \mathbf{q}_{13}^T, \ldots \mathbf{q}_{1N}^T, \; \mathbf{q}_{23}^T, \ldots]^T \in \mathbb{R}^{\bar{N}K \times 1}$ and the covariance matrix is

$$\boldsymbol{\Sigma} \triangleq \mathbb{E}\left[\mathbf{q}\mathbf{q}^T\right] \in \mathbb{R}^{\bar{N}K \times \bar{N}K}. \quad (21)$$

**Remark**: *Mobility of the nodes*: In traditional two-way ranging, for a fixed pair of nodes (i.e., $L = 1$), the pairwise distance $d_{ij,k}$ is classically assumed to be invariant for the total measurement period $T_{ij,K} - T_{ij,1}$. However, when the nodes are mobile, the distance at each time instance $k$ is dissimilar. Hence, instead of the classical assertion [3], we suppose that the nodes are relatively stable over a much smaller time period of $|T_{ij,k} - T_{ji,k}|$, i.e., the propagation time of the message.

### C. Dynamic ranging algorithm

Under the assumption that the covariance matrix $\boldsymbol{\Sigma}$ is known, a Weighted Least Squares (WLS) solution $\hat{\boldsymbol{\theta}}$ is obtained by minimizing the $l_2$ norm of the linear system (20), leading to

$$\hat{\boldsymbol{\theta}} = (\mathbf{A}^T \boldsymbol{\Sigma}^{-1} \mathbf{A})^{-1} \mathbf{A}^T \boldsymbol{\Sigma}^{-1} \boldsymbol{\tau}, \quad (22)$$

which is a valid solution if $K \geq L$ for each of the $\bar{N}$ pairwise links. Given an estimate of $\boldsymbol{\theta}$, the range coefficients $[\mathbf{r}, \dot{\mathbf{r}}, \ddot{\mathbf{r}}, \ldots]$ can be directly obtained from (12). More generally, when $L$ is unknown, an order recursive least squares can be employed to obtain the range coefficients [8]. Furthermore, the Cramér Rao lower Bound (CRB) for the least squares model (20) in combination with the range translation (12) is given by [8]

$$\boldsymbol{\Sigma}_{crb} \triangleq \; \mathbf{F}(\mathbf{A}^T \boldsymbol{\Sigma}^{-1} \mathbf{A})^{-1} \mathbf{F}, \quad (23)$$

where $\boldsymbol{\Sigma}_{crb}$ is the lowest variance attained by any unbiased estimate of the range parameters $[\mathbf{r}^T, \dot{\mathbf{r}}^T, \ddot{\mathbf{r}}^T, \ldots]^T$ and $\mathbf{F} = \text{diag}(\mathbf{f}) \otimes \mathbf{I}_{\bar{N}} \in \mathbb{R}^{\bar{N}L \times \bar{N}L}$. It is worth noting that (22) achieves this lower bound.

## IV. Simulations

To evaluate the performance of the proposed solutions, we consider a cluster of $N = 5$ nodes in $P = 2$ dimensions,

$$\mathbf{X} = \begin{bmatrix} -382 & 735 & 959 & 630 & 800 \\ 9 & 7 & 727 & 366 & -858 \end{bmatrix}, \qquad \mathbf{Y} = \begin{bmatrix} -6 & 8 & -1 & -10 & 3 \\ 8 & -9 & -7 & -2 & -8 \end{bmatrix} \tag{24}$$

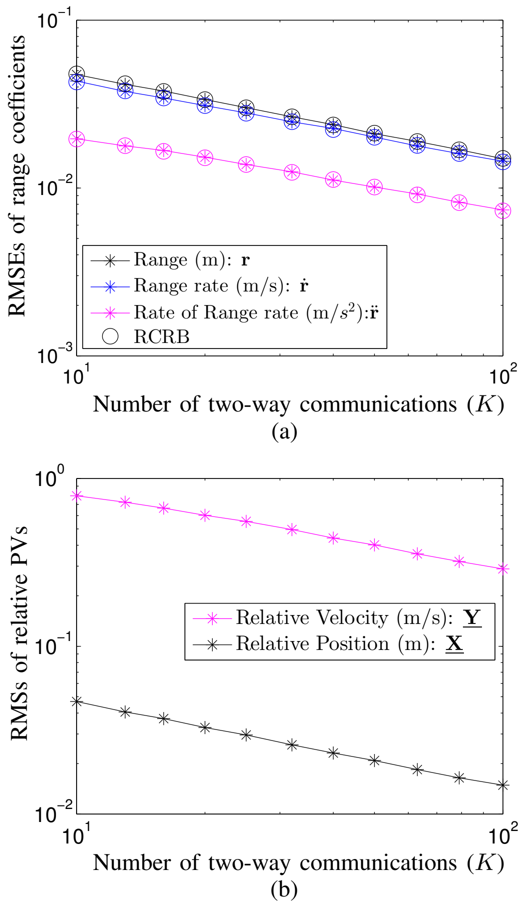


Fig. 2: (a) RMSEs of range parameters and (b) RMSs relative position, relative velocity for a varying number of communications ($K$) between the nodes for $\sigma = 0.1$ meters

whose coordinates $\mathbf{X}$ and velocities $\mathbf{Y}$ are arbitrarily chosen to be (24). Without loss of generality, we assume that all nodes employ one way communication and communicate with each other within the same time interval $[T_{ij,1}, T_{ij,K}] = [-3, 3]$ seconds, where the transmit time markers are spaced equidistantly. We consider a classical pairwise communication scenario, where all the pairwise communications are independent of each other and thus $\mathbf{\Sigma} = \sigma^2 \mathbf{I}_{\bar{N}K}$.

The metric used to evaluate the performance of the range parameters is Root Mean Square Error (RMSE), given by $\text{RMSE}(\mathbf{z}) = \sqrt{N_{exp}^{-1} \sum_{n=1}^{N_{exp}} \|\hat{\mathbf{z}}(n) - \mathbf{z}\|^2}$, where $\hat{\mathbf{z}}(n)$ is the $n$th estimate of the unknown vector $\mathbf{z} \in \mathbb{R}^{\bar{N} \times 1}$ during $N_{exp} = 1000$ Monte Carlo runs. To qualify these estimates, the square Root of the Cramér Rao Bound (RCRB) is plotted along with the respective RMSE. Furthermore, since the relative PVs are known only upto a rotation, we use a metric based on the mean of *raw stress* [2] for evaluation. Let $\mathbf{Z} = [\mathbf{z}_1, \mathbf{z}_2, \ldots, \mathbf{z}_N] \in \mathbb{R}^{P \times N}$ be the matrix under evaluation, then the average *raw stress* of $\mathbf{Z}$ for each $n$th Monte carlo realization is defined as $\mathcal{S}_{\mathbf{Z}}(n) = \bar{N}^{-1} \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} (\widehat{\mathcal{D}}_{ij}(n) - \mathcal{D}_{ij})^2$, where $\mathcal{D}_{ij} = \|\mathbf{z}_i - \mathbf{z}_j\|$ is the Euclidean distance between the vectors $(\mathbf{z}_i, \mathbf{z}_j)$ and $\widehat{\mathcal{D}}_{ij}(n) = \|\hat{\mathbf{z}}_i(n) - \hat{\mathbf{z}}_j(n)\|$ its corresponding estimate. Subsequently, the Root Mean Stress (RMS) for relative PVs are $\text{RMS}(\mathbf{Z}) = \sqrt{N_{exp}^{-1} \sum_{n=1}^{N_{exp}} \mathcal{S}_{\mathbf{Z}}(n)}$, where $N_{exp} = 1000$.

The dynamic ranging algorithm (22) is implemented for $L = 4$, where the number of communications $K$ is varied from 10 to 100. The noise on the propagation delays is $\sigma = 0.1$ meters, which is typically considered for conventional anchored MDS-based velocity estimation using Doppler measurements [9]. Fig. 2a shows the RMSE of the first 3 range coefficients (which are relevant for estimating the relative velocities) achieving the RCRB asymptotically. The PVs are obtained using these range coefficients via (6), (8) and the corresponding RMSs are plotted in Fig. 2b.

## V. Conclusion

A closed-form solution is proposed for relative velocity estimation for an *anchorless network of mobile nodes*. Under a linear velocity assumption, we show that the solution for relative velocities up to a rotation can be obtained from the derivatives of the time-varying pairwise distance. In the absence of Doppler measurements, a least squares based dynamic ranging algorithm is proposed, which employs a classical Taylor series based approximation to efficiently estimate pairwise distance derivatives at a given time instant. In practice, over longer durations the estimated parameters can be readily extended to both relative and absolute tracking of mobile nodes.

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