

Synchronization for Space Based Ultra Low Frequency Interferometry

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Abstract—Recently, there is a renewed interest in space based arrays for low frequency observations. Orbiting Low Frequency Antenna Array (OLFAR) is one such project which investigates the feasibility of a space based array of ≥ 10 satellites, observing the cosmos in the 0.3 – 30 MHz spectrum. The OLFAR cluster will be deployed far from Earth, with occasional communication with Earth, which presents new challenges in synchronization and localization of the satellites. In this paper, we present first order clock requirements for interferometry in general, by modeling the clock error as a polynomial in time. The bounds on the short term and long term clock requirements of the OLFAR satellite are defined in terms of jitter and Allan deviation respectively. Given a robust clock up to coherence requirements, it is shown that it suffices to assume the clock as a first order model. A list of commercially available clocks which meet OLFAR requirements are discussed and joint localization and synchronization solutions which cater to the needs of OLFAR are suggested.

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1. INTRODUCTION

Recently, new and interesting science drivers have emerged in the spectrum of 0.3-30MHz, such as studying the early cosmos at high hydrogen red shifts, extragalactic surveys, the so-called dark ages, (extra) solar planetary bursts and high energy particle physics [1]. However ground based astronomical observations at these long wavelengths are severely limited by Earth's ionospheric distortions below 50 MHz, complete reflection of radio waves below 10 MHz and man made interference, such as Amplitude Modulated radio broadcasting signal. These limitations of terrestrial arrays can be overcome by a distributed array of radio telescopes in space, far from Earth's atmosphere and terrestrial interference. Until now, such a system in space was financially and technically constrained. However, more recent studies [2] [3] have shown that with extrapolation of current signal processing and satellite technologies, a low frequency radio telescope in space could be feasible in the coming years.

Our motivation, in particular, is OLFAR [4], a project which aims to develop a detailed system concept and to design and build scalable autonomous satellite flight units to be used as an astronomical instrument for low frequencies. The OLFAR satellite will observe the cosmos at 0.3 – 30 MHz and the raw data will be time-stamped and distributed within the network for correlation. All the satellites will be equipped with down-linking capabilities, enabling them to transmit the processed data back to Earth-based ground stations. To maintain *coherence*, during observation, for processing and during communication, the OLFAR network must be synchronized. The OLFAR cluster will be a cooperative network of satellites, which will correct for clock errors independently with minimal help from Earth based ground stations. Furthermore, in addition to synchronization, the OLFAR satellites must also find pairwise distances to estimate relative positions for interferom-

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entry and to avoid collision. In this paper, our focus is on synchronization, in particular, clock requirements for interferometry in general. In addition, we present localization and synchronization solutions which meet OLFAR requirements.

This paper is divided in two sections. The first section discusses the essential clock requirements to maintain coherence for any network. A generic clock model is presented and the clock error is approximated as a polynomial in the presence of random noise. We emphasize the need to minimize clock jitter and its tolerance levels for an OLFAR satellite. The long term deviation of clock from its nominal frequency is described using Allan deviation, which is shown to be a measure of non linear components in a given clock. In the second section, the OLFAR short term and long term clock stability requirements are discussed, in addition to desired position accuracies for interferometry. A few commercially available clocks which meet OLFAR requirements are presented. In addition, joint localization and synchronization algorithms are presented, which suit OLFAR needs and use minimal overhead on the entire system.

2. CLOCK REQUIREMENTS

Clock model

The clock output of a practical oscillator is a time varying periodic signal defined as [5]

$$\begin{aligned} a(t) &= (A + \check{A}(t)) \sin \Psi(t) \\ &= (A + \check{A}(t)) \sin(2\pi\nu(t + \psi(t))) \end{aligned} \quad (1)$$

where t represents the *true* time and $\Psi(t)$ is the total accumulated phase in radians. The nominal amplitude and frequency of the clock are denoted by A and ν respectively and $\check{A}(t)$ is the amplitude error modulated on the ideal amplitude of the signal. The clock source is often hard limited using differential comparison techniques which minimizes this amplitude error. Thus the time varying amplitude $\check{A}(t)$ in (1) can be eliminated, but not ignored.

The phase error plaguing the clock is represented by $\psi(t)$, which is frequency normalized and has units of time. For an ideal clock source, $\psi(t) = 0$ and the output is a pure sinusoid of frequency ν and amplitude A . In reality however, time varying errors $\psi(t)$ exists on t and the challenge is to minimize it within acceptable limits depending on system requirements. This phase error of the clock $a(t)$ can be understood, by expanding $\psi(t)$ as a polynomial of *true* time t

$$\begin{aligned} \psi(t) &= \phi + \frac{d\phi}{dt}t + \frac{1}{2} \frac{d^2\phi}{dt^2}t^2 + q(t) \\ &= \phi + \dot{\phi}t + \ddot{\phi}\frac{t^2}{2} + q(t) \end{aligned} \quad (2)$$

where $\psi(t)$ is the frequency normalized phase shift in

seconds, ϕ indicates the time or phase offset and $\dot{\phi} = \frac{d\phi}{dt}$ represents rate of change of phase i.e., the frequency offset. The rate of change of frequency offset, i.e., the frequency drift with time is denoted by $\ddot{\phi} = \frac{d^2\phi}{dt^2}$ and $q(t)$ is a random deviation which is non-deterministic. Other systematic deviations which affect the clock performance include shock, vibrations, humidity, temperature and radiation. These environmental effects have been ignored and thus not explicitly incorporated into the model. However, more generally, these errors can be assumed to be contained within the indeterministic $q(t)$. The phase parameter ϕ and its higher order derivatives are real valued and deterministic, whose slow variations with *true* time t are neglected in the above model. For an ideal clock with no phase errors, we have $\psi(t) = 0$ and subsequently $[\phi, \dot{\phi}, \ddot{\phi}] = [0, 0, 0]$.

Dynamic range and Sampling jitter

Prior to investigating the relevance of these clock parameters, we look at impact of short time ($t \ll 1$ second) clock variations on the signal processing system. Traditionally, the first stage comprises of two process, i.e., *sampling* and *quantization*. The Sample and Hold (S/H) block decimates the input signal periodically along the time axis and the Analog to Digital Converter (ADC) quantizes the input signal along the amplitude scale. Fluctuations in the sampling time causes phase modulation of the incoming analog signal and results in an additional noise component in the signal. These unwanted variations in time, called jitter, lead to uncertainty as to when the analog input is actually sampled. The Signal to Noise Ratio (SNR) of the S/H block with sampling jitter Δt_{jitter} is given as [6]

$$\text{SNR}(dB) = -20 \log_{10}(2\pi\nu_{in}\Delta t_{\text{jitter}}) \quad (3)$$

where ν_{in} is the frequency of a pure sinusoid input signal. The Δt_{jitter} is the total RMS jitter from the clock source and the ADC circuitry, i.e., $\Delta t_{\text{jitter}} = \sqrt{\Delta t_{ADC}^2 + \Delta t_{CLK}^2}$ where Δt_{ADC} and Δt_{CLK} are RMS jitters due to ADC and clock source respectively. Now, since the ADC follows the S/H, the best achievable SNR post quantization is limited by the SNR of the S/H. Hence assuming a high SNR ADC i.e., $\Delta t_{ADC} \ll \Delta t_{CLK}$, we can approximate the total jitter (3) as

$$\Delta t_{\text{jitter}} \approx \Delta t_{CLK} = \frac{10^{\frac{-\text{SNR}}{20}}}{2\pi\nu_{in}} \quad (4)$$

Thus, given the desired dynamic range, (4) solves for the tolerable jitter. Observe that the jitter sampling error is not a function of the clock frequency, but instead only dependent on the desired dynamic range and the frequency of the input signal. Figure 1 shows limiting cases of the SNR versus input frequencies, which illustrates the fact that the dynamic range of system deteriorates with increase in frequency due to jitter error. Further more, assuming a best performance scenario (i.e., with a

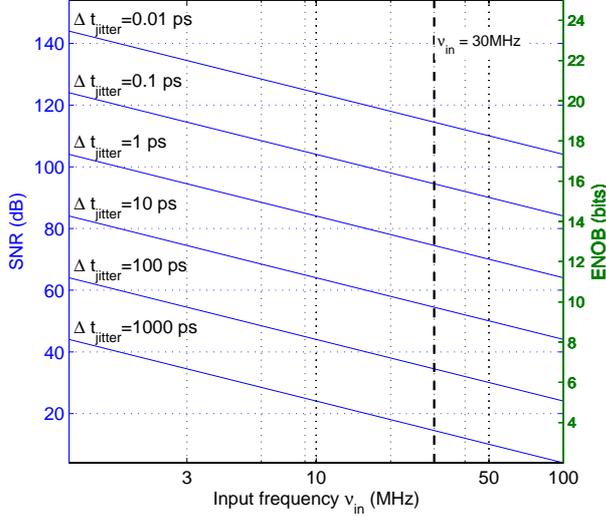


Figure 1. Limiting cases of the SNR (and corresponding Effective Number of Bits) due to jitter t_{jitter} versus input frequency ν_{in} . A demarkation line shows the maximum input frequency of the OLFAR system at $\nu_{\text{in}} = 30\text{MHz}$

distortion-less High-SNR ADC), the SNR from (4) can be represented as the Effective Number Of Bits (ENOB) using (5) required to achieve SNR

$$\text{ENOB} = \frac{\text{SNR} - 1.76}{6.02} \quad (5)$$

The satellites of the OLFAR cluster will employ direct sampling of the entire observation bandwidth (0.3MHz-30MHz) [4]. Furthermore, the required dynamic range will depend on the Radio Frequency Interference (RFI) level at the deployment locations. Some of the potential deployment locations include, moon orbit, Earth-moon L2 point and Earth- leading/trailing, where the interference levels at these wavelengths are little understood [7]. Hence, although only 1-2 bits are sufficient for radio astronomy imaging, the OLFAR system will sample at >8 bits and the remaining bits will be discarded by a RFI mitigation stage[4]. Refereing to Figure 1, to achieve quantization resolution of 16 bits at $\nu_{\text{in}} = 30\text{MHz}$, the required sampling jitter must be $< 0.1\text{ps}$ and for 12 bits/cycle sampling $\Delta t_{\text{jitter}} < 1\text{ps}$.

Coherence time and Allan deviation

OLFAR will comprise of ≥ 10 nodes individually sampling the observational bandwidth 0.3MHz - 30MHz. The observed data is accurately time stamped with respective onboard clocks and exchanged with other satellite nodes for correlation. To make correlation products between any two independent satellite nodes, the clocks systems within these nodes must remain coherent within a certain *coherence time* τ_c . To quantify this *coherence time*, we use *Allan variance*, which is defined as the

second moment of dispersion of frequency from the nominal value. The normal standard deviation of the phase error $\psi(t)$ in (1) does not converge due to the accumulation of phase errors with increase in time t . However, since the *Allan variance* converges as compared to the normal standard deviation, it is a recognized clock specification parameter to estimate the long term stability ($t \gg 1$ second) of a practical clock [5]. For the sake of completeness and to illustrate its relation with the phase error $\psi(t)$, the Allan deviation [8] is briefly derived for time domain measurements of the clock source.

We start with determining the instantaneous frequency deviation of the clock signal $a(t)$ in (1) from its ideal frequency ν , which is obtained by differentiating the total accumulated phase $\Psi(t)$ from the clock equation eq(1) and dividing by 2π

$$\nu(t) = \frac{1}{2\pi} \frac{d\Psi}{dt} = \nu + \nu \frac{d\psi(t)}{dt} \quad (6)$$

Rearranging the terms and dividing both sides by the expected frequency ν we have

$$\zeta(t) \triangleq \frac{d\psi(t)}{dt} = \frac{\nu(t) - \nu}{\nu} \quad (7)$$

where $\zeta(t)$ is the normalized fractional frequency deviation of $a(t)$ and $\psi(t)$ is the phase error in seconds. Now, the average fractional frequency deviation over a period, say τ , is then

$$\bar{\zeta}(t, \tau) = \frac{1}{\tau} \int_t^{t+\tau} \zeta(t) dt \quad (8)$$

Given a discrete set of time deviations $\psi(t_k)$ at time t_k such that the nominal spacing between adjacent measurements is $\tau = t_{k+1} - t_k$, the average fractional frequency offset during the k th measurement interval of item length τ is

$$\bar{\zeta}_{k, \tau} = \frac{\psi(t_k + \tau) - \psi(t_k)}{\tau} \quad (9)$$

Finally Allan variance ($\sigma_{\zeta}^2(\tau)$) is defined as one half of the time average of the squares of the differences between successive readings of the averaged normalized frequency deviation $\bar{\zeta}_{k, \tau}$ sampled over the sampling period i.e.,

$$\sigma_{\zeta}^2(\tau) = \left\langle \frac{(\bar{\zeta}_{k+1, \tau} - \bar{\zeta}_{k, \tau})^2}{2} \right\rangle \quad (10)$$

where $\langle \cdot \rangle$ is the expectancy operator and $\sigma_{\zeta}(\tau)$ is the Allan deviation for the time duration τ .

Now substituting for $\psi(t_k)$ from (2) in (9), we have

$$\bar{\zeta}_{k, \tau} = \frac{(\phi + \dot{\phi}(t_k + \tau) + \ddot{\phi} \frac{(t_k + \tau)^2}{2} + q(t_k + \tau))}{\tau} - \frac{(\phi + \dot{\phi}t_k + \ddot{\phi} \frac{t_k^2}{2} + q(t_k))}{\tau} \quad (11)$$

$$= \dot{\phi} + \ddot{\phi}(t_{k+1} - 0.5\tau) + \dot{q}(t_k) \quad (12)$$

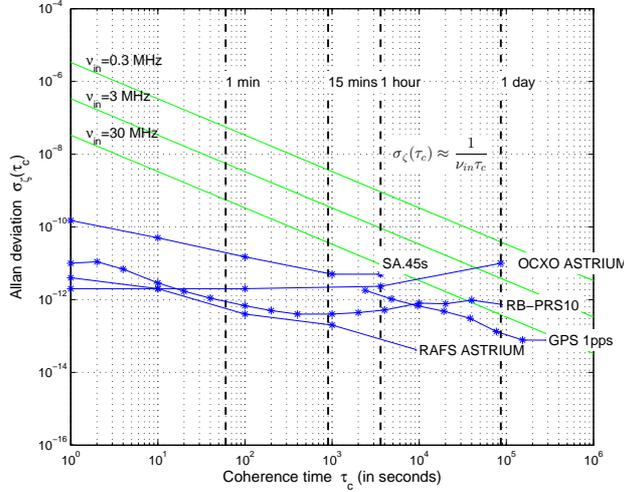


Figure 2. The plot shows desired Allan deviations of free running clocks versus the coherence time (in green) as per (16) for various input frequencies ν_{in} . The map is overlaid with Allan deviations of potential clocks (in blue) for OLFAR namely PRS-10 Rubidium [9], RAFS ASTRIUM [10], GPS 1pps [11], OXCO ASTRIUM [12] and SA.45s CSAC [13]

where

$$\dot{q}(t_k) = \frac{q(t_k + \tau) - q(t_k)}{\tau} \quad (13)$$

Subsequently, the Allan variance in (10) is then

$$\begin{aligned} \sigma_\zeta^2(\tau) &= \frac{1}{2} \left\langle (\bar{\zeta}_{k+1,\tau} - \bar{\zeta}_{k,\tau})^2 \right\rangle \\ &= \frac{1}{2} \left\langle (\dot{\phi} + \ddot{\phi}(t_{k+1} - 0.5\tau) + \dot{q}(t_{k+1}) \right. \\ &\quad \left. - (\dot{\phi} + \ddot{\phi}(t_k - 0.5\tau) + \dot{q}(t_k)))^2 \right\rangle \\ &= \frac{1}{2} \left\langle (\ddot{\phi}\tau + \ddot{q}(t_k))^2 \right\rangle \end{aligned} \quad (14)$$

where

$$\ddot{q}(t_k) = \frac{\dot{q}(t_k + \tau) - \dot{q}(t_k)}{\tau} \quad (15)$$

Note that $\ddot{\phi}$ is the frequency drift, $q(t_k)$ is the random deviation and more generally, contains other higher order terms of the polynomial in (2). Thus, in essence, the Allan deviation alleviates the linear trend of the phase error $\psi(t)$ in (2) by eliminating $\{\phi, \dot{\phi}\}$ and gives a measure of noise contributed by the higher order non linear components of the clock for an integration time τ .

Stability requirement

Now, following [14] [15], we pose a rough stability requirement on the clock and define the *coherence time*

τ_c , such that the RMS phase error of the clock remains less than 1 radian

$$\nu_{in} \sigma_\zeta(\tau_c) \tau_c \lesssim 1 \quad (16)$$

where ν_{in} is the observational frequency and $\sigma_\zeta(\tau_c)$ is the Allan deviation as a function of τ_c . The product $\sigma_\zeta(\tau_c) \tau_c$ can be visualized as the time drift due to non linear components of the clock after τ_c seconds. Figure 2 shows expected Allan deviations of the clock versus the coherence time as per (16) for various input frequencies ν_{in} . The clocks should be stable at least for the integration time τ_c , during which it suffices to estimate and correct $\{\phi, \dot{\phi}\}$ for synchronization.

In an OLFAR node the maximum input frequency is 30MHz and the minimum integration time desired is 1 second [4]. Thus for the minimum *coherence time* $\tau_c^{min} = 1s$, the Allan deviation must satisfy $\sigma_\zeta^{max}(\tau_c^{min} = 1s) \leq 10^{-8}$. Ideally, the maximum coherence time τ_c^{max} must be as large as possible since it defines the calibration interval available to correct for the phase error.

3. OLFAR: ORBITING LOW FREQUENCY ANTENNAS FOR RADIO ASTRONOMY

Requirements

For interferometry at wavelengths $10 - 10^3$ meters, the positions of the satellites must be known accurately up to a fraction of the smallest observational wavelength. This implies that the distance between the OLFAR satellites must be known with an accuracy $\ll 1$ meter. Relative positions of the satellites are sufficient for both radio astronomy and to avoid collision, which in turn can be estimated by measuring pairwise ranges between the satellites. In conjunction with the clock accuracies discussed in the previous section, the requirements on position and time of an OLFAR satellite can be briefly summarized as

1. Clock: Sampling jitter Δt_{jitter}
 - (a) $\Delta t_{jitter} < 10ps$ for 8 bit sampling
 - (b) $\Delta t_{jitter} < 1ps$ for 12 bit sampling
2. Clock: Allan deviation $\sigma_\zeta(\tau_c)$
 - (a) Short term $\sigma_\zeta(\tau_c) \leq 10^{-8}$ for $\tau_c = 1$ second
 - (b) Long term $\sigma_\zeta(\tau_c) \leq 10^{-11}$ for $\tau_c = 1000$ seconds
3. Range: $< 1m$ accuracy

The time-position requirements of OLFAR scale with the needs of the ground based low frequency radio telescope LOFAR (LOW Frequency Antenna aRray) which observes the sky at 30 – 120MHz. Here, the stations are equipped with Rubidium standard PRS-10 and long term stability is ensured by correcting the station based rubidium clocks using the 1 pulse per second (1pps) output of a GPS receiver [16]. It is observed that for longer antenna separations at lower frequencies the phase errors are dominated by ionospheric disturbances than clock errors. Furthermore, the antennas are fixed and

their positions are measured up centimeter accuracies on each dimension.

Potential clocks

To achieve Allan deviations of order $10^{-8} - 10^{-11}$, the usual solutions are Rubidium standards and Oven controlled Crystal oscillators (OCXO) are considered. Although cesium and maser families can offer orders of magnitude lower Allan deviations, they are also very expensive, in terms of mass and power for an OLFAR satellite and hence not considered in this survey. The Allan-deviations σ_ζ of potential (currently available) clocks are plotted (in blue) in Figure 2 against the coherence time τ_c . For the sake of reference, the plot is overlaid with the desired Allan-deviation (16) for various input frequencies ν_{in} (in green). Table 1 shows additional features of these clocks such as mass and power consumption.

Rubidium standards: RB-PRS10 is a rubidium based frequency standard with Allan deviations of $\sigma_{\zeta,prs} \leq 10^{-11}$ for 1-100 seconds[9] which is currently employed in LOFAR radio telescope [17], but not space qualified. A secondary solution is the Astrium RAFS (Rubidium Atomic Frequency Standard) which is a space qualified robust clock system, used in the Galelio satellite navigation [10] with $\sigma_{\zeta,rafs} \leq 10^{-11}$ for $1 < \tau_c < 1000$ seconds. Both these systems have low clock jitters < 0.1 picosecond for input frequencies of $\nu_{in} = 30\text{MHz}$. The RAFS is marginally equivalent to the PRS10-RB in terms of Allan deviations however achieves a far better noise floor at 3×10^{-14} for ($t > 10^3$ seconds). However, referring to Table 1, RAFS is not a practical solution due to limitation of space and mass in an OLFAR satellite.

Oven controlled crystal oscillator (OCXO) standards: An alternative to Rubidium standard is the oven controlled crystal oscillator or OCXO. More specifically a space qualified variant is offered by EADS ASTRUM [12], which weighs 220 grams, consumes 2 Watts during steady state operation and which can be easily incorporated in the OBC module [2]. The Allan variances of OCXO are better than its Rubidium counterparts for only up to $\tau_c=10$ seconds, beyond which the Rubidium standards fair better.

Vertical Cavity Surface Emitting Laser (VCSEL) based Rubidium: Beyond the above mentioned clocks, there has been consistent research in developing chip scale atomic clocks in the past decade [18][19] based on Vertical Cavity Surface Emitting Lasers, which enable orders of magnitude reduction in size and power. SA.45s is a Rubidium Chip Scale Atomic Clock (CSAC) which is based on VCSEL and meets the Allan deviation requirements of OLFAR upto 1000seconds, as shown in Figure 2. This CSAC weighs < 35 grams and has a steady power consumption of 125 mW, which suit the needs of a OLFAR satellite.

However, although the commercially available SA.45s meets the requirements of OLFAR, it is not space qualified.

4. JOINT SYNCHRONIZATION AND RELATIVE LOCALIZATION

Unlike terrestrial radio astronomy arrays, the OLFAR nodes are mobile and unsynchronized, which presents a unique challenge to jointly synchronize and locate the nodes up to desired accuracies. Furthermore, localization and synchronization solutions must use limited resources to keep the mass and power budget to minimum. The OLFAR cluster will employ *Distributed correlation-Distributed Down-link* architecture for processing and communication (Figure 3), where in, within the network, all the satellites are capable of two way communication with one other. All the observed data are accurately time stamped by the local oscillator before transmitting to another satellite for correlation. Furthermore, all the satellites are independently capable of down-linking correlated data to Earth and which provides an occasional link with Earth based ground stations.

As shown in (14), the Allan deviation is a measure of non linear components in a given clock over a coherence time τ_c . Hence, it is vital to choose a clock (such as the SA.45s discussed in the previous section), which offers a small Allan deviation for a coherence time period as long as possible. More generally, this empowers us to approximate the total phase error $\psi(t)$ in (2) as

$$\psi(t) \approx \phi + \dot{\phi}t + q(t) \quad (17)$$

for a period of τ_c , during which the average fractional frequency deviation and the higher order terms are minimal. Thus, given a robust clock meeting Allan deviation requirements, it suffices to estimate the clock offset ϕ and clock drift $\dot{\phi}$ to achieve network wide synchronization.

Without the loss of generality, using one of the satellites as a clock reference, the clock parameters $\{\phi, \dot{\phi}\}$ can be efficiently estimated using a time of arrival based Global Least Squares (GLS) solution[20]. The GLS uses the *transmission time* and the *reception time* of data between the satellites as measurements to estimate the clock parameters, under the assumption of Gaussian noise for a small time period. In addition, the GLS also estimates the pairwise distances between the satellites up to desired accuracies of 1 meter. Since the OLFAR cluster is a network of mobile satellites, the pairwise distances i.e., ranges are time varying. Hence, beyond the range, the range rate can be estimated using an Extended Global Least Squares (EGLS) algorithm [21] and further more higher order of range parameters can be found efficiently using Extended² Global Least Squares (*E²GLS*) [22]. Relative positions can be estimated by applying Multi-Dimensional Scaling algorithms on the range estimates [23].

The joint synchronization and relative localization solutions are for a full mesh network, with each satellite communicating with every other satellite in the cluster. Alternatively, instead of a full mesh network, a dynamically configurable Master-Slave inter satellite communication reduces communication overload [24]. Note that the GLS (and its extended revisions) can be applied

Clock	Manufacturer	Technology	Mass (grams)	Power (watts)	Space qualified
PRS-10	SRS	Rubidium	600	14	No
RAFS	EADS Astrium	Rubidium	3300	30	Yes
OCXO-F	EADS Astrium	OCXO	220	2	Yes
SA.45s	Symmetricon	Rubidium	< 35	< 0.125	No

Table 1. List of potential clocks for an OLFAR satellite which are commercially available. All the clocks output a 10MHz reference frequency and power indicates steady state consumption

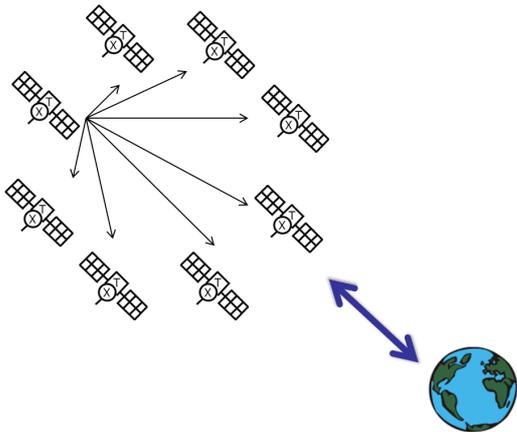


Figure 3. The OLFAR cluster in the proposed *Distributed Correlation - Distributed Down-link* architecture for correlation [4] and clock synchronization

to OLFAR network, despite missing links (as shown in Figure 3, [20]), which ensures synchronization as long as each satellite has at least 1 communication link with ANY other satellite in the network.

5. CONCLUSIONS

The clock requirements of an OLFAR satellite have been investigated, by modeling the clock errors as polynomial in time. Each observation satellite needs to maintain a short term clock stability ($t \ll 1s$) of ≤ 1 ps to ensure sampling with 16 bits. Long term stabilities ($t \geq 1$) of clocks are given by Allan deviations $\sigma_{\zeta}(\tau_c)$ as a function of the coherence time τ_c . It is shown, using a simple derivation, that the Allan deviation indicates the noise contributed by the non linear components of the clock. Furthermore, given a space qualified clock which meets the Allan deviation requirements of $\sigma_{\zeta}(1) \leq 10^{-8}$ and $\sigma_{\zeta}(1000) \leq 10^{-11}$, the clock error can be approximated as a linear function of *true* time, during the coherence period. The VCSEL based Rubidium SA.45s is a non-space qualified clock with low mass and minimal power consumption, which meets the long term stability requirements of OLFAR but only for coherence time up to 1000 seconds. This limits the maximum calibration time for each satellite to re-synchronize, i.e., estimate and correct for their respective clock offset (ϕ) and clock drift

($\dot{\phi}$). Joint synchronization and relative localization can be achieved using GLS and extended revisions, which exploit the two way inter-satellite communication and efficiently estimate the clock and range parameters.

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BIOGRAPHY



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