Enhanced Sensitivity Computation for BEM Based Capacitance Extraction Using the Schur Complement Technique

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***Abstract*—This paper presents a useful extension for an existing algorithm of capacitance sensitivity computation with respect to multiple geometric variations. The existing algorithm is applicable for BEM-based capacitance extraction tools and provides good accuracy results. Using the Schur complement technique, the extended algorithm can achieve an even better accuracy at a modest increase of computational cost. With such extension, the enhanced algorithm becomes more flexible in the sense that it is able to provide different solutions for different application requirements: high efficiency with good accuracy or high accuracy with modest time cost.**

I. Introduction

The magnitude of electrical and performance variations due to physically induced process variations is becoming relatively larger with each new technology node. Therefore, modeling the variability of interconnect resistances and capacitances is becoming more and more important.

As the modeling for resistances is relatively simple [1], [2], most work has concentrated on capacitances. The study started with the worst-case corner method, which was acknowledged to be over pessimistic. A statistical approach using empirical capacitance expressions was then proposed to overcome this problem [3]. An enhanced lookup method based on analytical capacitance models was presented in [4], to account for systematic variations by computing the derivatives of capacitances with respect to the thickness and the linewidth deviations. These derivatives were later referred to as *sensitivities* and have been computed using the floating random walk (FRW) method [5] and the boundary element method (BEM) [6].

These sensitivities w.r.t. geometric parameters are useful for diverse variability modeling techniques. They are necessary for establishing the parameterized system description in various variation-aware techniques, such as the moment-based timing analysis [7], the Hermite polynomial based statistic analysis [8] and the parametric Model Order Reduction (pMOR) technique proposed in [9]. The sensitivities have been incorporated in the Standard Parasitic Exchange Format (SPEF) [10]. Based on the 2009 version of the SPEF standard, a netlist consisting of the nominal values of parasitics and their sensitivities could be generated by Layout Parasitic Extraction (LPE) tools for subsequent analysis. Therefore, to be able to compute these sensitivities according to different accuracy and efficiency requirements can be very convenient and useful.

In general, it is very difficult to achieve a good accuracy and a good efficiency at the same time. For instance, although the traditional finite-difference (FD) method is often considered as the accuracy reference for sensitivity computation, it is too slow to be applicable in most cases [5]. On the other hand, some techniques, such as [6], can achieve a very high efficiency as a BEM technique while at the cost of accuracy. In this paper, an extension of this BEM based algorithm is proposed. The enhanced algorithm for sensitivities against multiple parameters provides two options for designers: an extremely fast computation with a good accuracy or a very high accuracy at the cost of a modest computational time.

The rest of this paper is organized as follows. Section II briefly introduces the capacitance extraction using the BEM. Section III presents the cause of the accuracy loss of the existing algorithm [6]. Then an extension of this algorithm that achieves a higher accuracy is proposed. Experiments are shown in Section IV to demonstrate the accuracy of the enhanced algorithm. Finally, Section V concludes the paper.

II. Background

Since our proposed method is an extension of the BEM, this section briefly presents some background on BEM concepts and notations.

Capacitances used in SPICE netlists are called *network capacitances* ($\mathbf{C}$) and can be specified in terms of the so-called *short-circuit capacitances* ($\mathbf{C_s}$) based on the following relationship:

$$\begin{array}{ll} C_{ij} = -C_{sij} & \forall \;\; i \neq j \\ C_{ii} = \sum_{j=1}^{N} C_{sij} & \forall \;\; i = 1, 2, \ldots N \end{array} \tag{1}$$

where C_{ij} is the coupling capacitance between conductors i and j; C_{ii} is the ground capacitance of conductor i. The entry of the short-circuit capacitance matrix C_{sij} equals to the charge on conductor i when conductor j is held at a unit potential and all other conductors are short-circuited to the ground.

With the BEM for the capacitance extraction, the surfaces of conductors are discretized into *panels*. Capacitances between these discretized panels are called *partial short-circuit capacitances*, denoted by $\bar{\mathbf{C}}_s$ in this context, i.e., quantities with an overbar relate to *partial* capacitances between panels. An incidence matrix $\mathbf{B}$ is then used to associate $\mathbf{C_s}$ and $\bar{\mathbf{C}}_s$:

$$\mathbf{C_s} = \mathbf{B}^T \bar{\mathbf{C}}_\mathbf{s} \mathbf{B} \tag{2}$$

where $\bar{\mathbf{C}}_s \in \mathbb{R}^{m \times m}$ and $\mathbf{B} \in \mathbb{R}^{m \times N}$, with m being the number of panels and N being the number of conductors. The incidence matrix $\mathbf{B}$ relates the panels to conductors, i.e., B_{ij} equals 1 if panel i lies on conductor j, and 0 otherwise. These partial short-circuit capacitances $\bar{\mathbf{C}}_s$ can be obtained from the inversion of an *influence* matrix $\bar{\mathbf{G}} \in \mathbb{R}^{m \times m}$ ($\bar{\mathbf{C}}_s = \bar{\mathbf{G}}^{-1}$), and thus the short-circuit capacitances $\mathbf{C_s}$ can be computed[1] as

$$\mathbf{C_s} = \mathbf{B}^T \bar{\mathbf{G}}^{-1} \mathbf{B} \tag{3}$$

where $\bar{G}_{ij}$ is given by the evaluation of the Green's function between panel i and panel j.

III. Enhanced Algorithm for Sensitivity Computation

A. Problem statement

As mentioned, the technique proposed in [6] has a drawback in terms of achieving a very high accuracy. The computational error of sensitivities by this method is mostly in the range of $5\% - 25\%$, depending on the structure of conductors and the geometric parameters of interest.

To study the cause of this error, we first review the technique briefly. Without loss of generality, we consider in the theoretical derivations in this paper only a single parameter p. It being a linear sensitivity based model, extension towards more parameters is trivial. As the existing algorithm [6] states, the coupling capacitance sensitivity between two conductors C_{ij} w.r.t. a geometric parameter p can be computed as

$$\frac{\partial C_{ij}}{\partial p} = -\sum_{k \in s_p} \left(\frac{1}{\varepsilon A_k} \sum_{a \in N_i} \sum_{b \in N_j} \bar{C}_{sk,a} \bar{C}_{sk,b} \right) \tag{4}$$

where $\bar{C}_{sk,a}$ and $\bar{C}_{sk,b}$ are entries of the partial short-circuit capacitance matrix, ε is the material permittivity around panel k and A_k is its area. The panel k refers to any panel lying on the *moving plate* s_p incident to parameter p. The moving plate is the surface of which the position is moved slightly due to a small variation in parameter p. For instance, there is a cubic conductor with a parameter of interest p as shown in Fig. 1. The moving plate is hence the rightmost sidewall and the panels lying on it are named *moving panels* indicated as the gray part in the figure. For clarity of discussion in the following, we give a notation of $C'_{plt}(p)$ for the sensitivity $\frac{\partial C_{ij}}{\partial p}$ computed in (4). This description (4) shows that sensitivities w.r.t. different parameters are simply incident to different sets of victim panels. All sensitivities w.r.t. multiple parameters can be computed simultaneously once the associated partial short-circuit capacitances are available, i.e., once the standard BEM extraction is done. This is why such BEM-based algorithm for the sensitivity computation can be highly efficient.

However, these moving panels are not the only *victims* due to the parameter variation Δp and are not the only cause of the capacitance fluctuation. Obviously, the edge panels connected

[1] There are several techniques for speeding up or avoiding this costly matrix inversion operation, but that is not the focus of this paper, and actually indifferent for the proposed methodology.

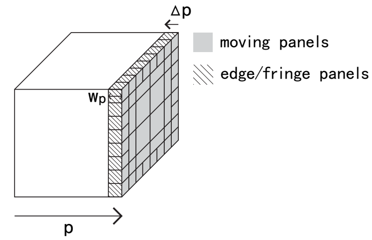

Fig. 1. A cubic conductor to demonstrate two parts of contributions to the capacitance fluctuation due to parameter variation Δp. Partial meshing condition is shown.

to the moving plate are also influenced by Δp, indicated as the shadowed part in the figure. Their sizes (widths) are either growing or shrinking depending on the direction of Δp.

Thus these size-changing edge panels also contribute to the capacitance variation induced by the Δp. As will be shown later, neglecting the contribution of these panels is the main reason of the accuracy loss of the capacitance sensitivity computation by the technique presented in [6]. In the next section, an extension of this technique will be proposed to achieve an improved accuracy by taking into account the influence of the size-changing panels.

B. Algorithm extension by the Schur complement technique

As the capacitance fluctuation is a combined result of the contributions of the moving panels and the size-changing panels, it is natural to propose a superposition approach to compute the sensitivity w.r.t. a parameter p:

$$\mathbf{C}'_{tot}(p) = \mathbf{C}'_{plt}(p) + \mathbf{C}'_{frg}(p) \tag{5}$$

where $\mathbf{C}'_{tot}(p)$ is the total or the enhanced capacitance sensitivity to be derived and $\mathbf{C}'_{plt}$, given by (4), refers to the contribution to the sensitivity from the field lines emanating from the moving plate. $\mathbf{C}'_{frg}(p)$ refers to the contribution from the fringe field emanating from the size changing edge panels on the shortened or elongated side of the conductor. Hence, the main task is to compute $\mathbf{C}'_{frg}(p)$.

To proceed, we first study carefully the relation between these panels and the parameter variation Δp. Note that if the variation is in the opposite direction of the positive direction of parameter p, as shown in Fig. 1, and the size (width) of the edge panels is exactly the same as the value of the variation Δp, these panels can thus be considered disappeared or eliminated due to such a parameter variation. They will be referred to as *fringe panels* in the following. In other words, the effect of these fringe panels on capacitances can be captured by eliminating their associated entries from the original partial short-circuit capacitance matrix. To do so, it is necessary to let the fringe panels have an identical width (w_p), which can be done by setting appropriate parameters for the mesh generation. This is the basic idea for computing $\mathbf{C}'_{frg}(p)$.

In the following, we will discuss how to develop such basic idea into an implementable algorithm, using the Schur complement technique. For a system not being subjected to process variations, its partial short-circuit capacitance matrix $\bar{\mathbf{C}}_{s\_o}$ is given by the inverse of the influence matrix $\bar{\mathbf{G}}_o$ for the originally designed dimensions. To distinguish the fringe panels from the rest of the panels, the $\bar{\mathbf{C}}_{s\_o}$ and the $\bar{\mathbf{G}}_o$ matrices can be written as block matrices:

$$\bar{\mathbf{C}}_{s\_o} = \begin{pmatrix} \mathbf{A_c} & \mathbf{B_c} \\ \mathbf{C_c} & \mathbf{D_c} \end{pmatrix} \qquad \bar{\mathbf{G}}_o = \begin{pmatrix} \mathbf{A_g} & \mathbf{B_g} \\ \mathbf{C_g} & \mathbf{D_g} \end{pmatrix} \tag{6}$$

where $\mathbf{A_c}$, $\mathbf{A_g} \in \mathbb{R}^{n \times n}$ correspond to the n fringe panels to be eliminated, $\mathbf{D_c}$, $\mathbf{D_g} \in \mathbb{R}^{(m-n)\times(m-n)}$ correspond to the rest of the panels. $\mathbf{B_c} = \mathbf{C_c}^T$ and $\mathbf{B_g} = \mathbf{C_g}^T$ describe the connection between these two groups of panels.

Preserving the matrix block dimensions, the relation $\bar{\mathbf{C}}_{s\_o} = \bar{\mathbf{G}}_o^{-1}$ can be expressed as

$$\begin{pmatrix} \mathbf{A_c} & \mathbf{B_c} \\ \mathbf{C_c} & \mathbf{D_c} \end{pmatrix} = \begin{pmatrix} \mathbf{A_g} & \mathbf{B_g} \\ \mathbf{C_g} & \mathbf{D_g} \end{pmatrix}^{-1} =$$

$$\begin{pmatrix} \mathcal{S}_{\mathbf{D_g}}{}^{-1} & -\mathcal{S}_{\mathbf{D_g}}{}^{-1}\mathbf{B_g}\mathbf{D_g}{}^{-1} \\ -\mathbf{D_g}{}^{-1}\mathbf{C_g}\mathcal{S}_{\mathbf{D_g}}{}^{-1} & \mathbf{D_g}{}^{-1} + \mathbf{D_g}{}^{-1}\mathbf{C_g}\mathcal{S}_{\mathbf{D_g}}{}^{-1}\mathbf{B_g}\mathbf{D_g}{}^{-1} \end{pmatrix} \tag{7}$$

where

$$\mathcal{S}_{\mathbf{D_g}} = \mathbf{A_g} - \mathbf{B_g}\mathbf{D_g}{}^{-1}\mathbf{C_g} \tag{8}$$

is the Schur complement of the block $\mathbf{D_g}$ [11]. Next, we write down the Schur complement of block $\mathbf{A_c}$, using (7):

$$\begin{aligned} \mathcal{S}_{\mathbf{A_c}} &= \mathbf{D_c} - \mathbf{C_c}\mathbf{A_c}{}^{-1}\mathbf{B_c} \\ &= \mathbf{D_g}{}^{-1} + \mathbf{D_g}{}^{-1}\mathbf{C_g}\mathcal{S}_{\mathbf{D_g}}{}^{-1}\mathbf{B_g}\mathbf{D_g}{}^{-1} - \\ &\quad (-\mathbf{D_g}{}^{-1}\mathbf{C_g}\mathcal{S}_{\mathbf{D_g}}{}^{-1}) \cdot \mathcal{S}_{\mathbf{D_g}} \cdot (-\mathcal{S}_{\mathbf{D_g}}{}^{-1}\mathbf{B_g}\mathbf{D_g}{}^{-1}) \\ &= \mathbf{D_g}{}^{-1} \end{aligned} \tag{9}$$

As addressed, $\mathbf{D_g}$ corresponds to the rest of the panels other than the fringe panels to be eliminated. In other words, it is the influence matrix for the remaining panels after eliminating the fringe panels. Hence, the Schur complement of $\mathbf{A_c}$, being the inverse of $\mathbf{D_g}$, is the updated partial short-circuit capacitance matrix ($\bar{\mathbf{C}}_{s_{\Delta frg}}$) after the fringe panel elimination:

$$\bar{\mathbf{C}}_{s_{\Delta frg}} = \mathbf{D_c} - \mathbf{C_c}\mathbf{A_c}{}^{-1}\mathbf{B_c} \tag{10}$$

It is exactly what needs to be calculated to further derive the supplement sensitivity $\mathbf{C}'_{frg}(p)$ in (5).

From $\bar{\mathbf{C}}_{s_{\Delta frg}}$, we can now first derive the updated short-circuit capacitance matrix $\mathbf{C}_{s_{\Delta frg}}$ with an updated incidence matrix $\mathbf{B}_{\Delta frg}$ analogical to (2). Then using (1), the updated network capacitances $\mathbf{C}_{\Delta frg}$ can be computed. Finally, using the original network capacitances $\mathbf{C}_o$, the supplement sensitivity that accounts for the effect of the fringe panels incident to parameter p can be derived:

$$\mathbf{C}'_{frg}(p) = (\mathbf{C}_{\Delta frg}(p) - \mathbf{C}_o)/(-w_p) \tag{11}$$

where the minus sign comes from the fact that the variation w_p makes the corresponding parameter p smaller (shrinking).

In fact, we can consider the above approach for calculating the sensitivity $\mathbf{C}'_{frg}$ as an enhanced FD method. The cost of the sensitivity extraction is the sum of the cost of extracting $\mathbf{C}'_{plt}(p)$ and $\mathbf{C}'_{frg}(p)$. Note that the cost of computing $\mathbf{C}'_{plt}(p)$ was established to be negligible in [6]. We now show that the computational cost of computing $\mathbf{C}'_{frg}(p)$ is also small compared to that of the nominal extraction.

In essence, two configurations must be computed as shown in (11). The delta configuration is computed by a fast update

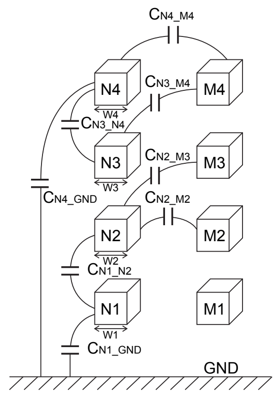

Fig. 2. 3-D representation of 8 cubic conductors on 4 layers.

of the nominal configuration, using a much smaller system of which evaluation of (10) forms the main cost. The cost of the update can be estimated as follows. Note that n is the number of fringe panels and m is the total number of panels. In practical cases, $n \ll m$. Since $\mathbf{A_c}{}^{-1} \in \mathbb{R}^{n \times n}$ and $\mathbf{B_c} = \mathbf{C_c}{}^T \in \mathbb{R}^{n \times (m-n)}$, the evaluation of (10) is much faster than the evaluation of (3), which has a size of $m \times m$ and is the main cost of nominal extraction. In case of n_p parameters, the computational cost is linear in n_p. Next section gives a comparison of the accuracy and the efficiency between the proposed algorithm and the traditional FD method.

IV. Experiment and Results

This section presents an experiment for verifying the accuracy and the efficiency of the enhanced algorithm. The algorithm has been implemented in C/C++ and the experiment has been conducted on a 3.00GHZ Intel 2 Core CPU.

As shown in Fig. 2, the example has 4 layers with 8 cubic conductors. The cubic conductors present some kind of practical worst case situation as for the relevance of the *fringe* terms. Typically, the width of conductors is changed by process variability, and not so much the length. If the length is changed, it can be relevant only if the conductor is very short, which means an almost cubic conductor. Hence this experiment studies the effects of the width variations of cubic conductors. Since the structure is symmetrical, only the widths of the left side cubes (w_1, w_2, w_3, w_4) are studied. Sensitivities given by the traditional FD method are used as references.

Table I compares the C'_{plt} and C'_{tot} sensitivities to C'_{ref} as obtained by the traditional FD method. For each parameter, 2 capacitances with the largest sensitivities to that parameter are selected to demonstrate the accuracy improvement of the enhanced algorithm. As shown, the errors of the sensitivities C'_{plt} may reach 26%, compared to C'_{ref}. While it is acceptable for many cases since sensitivity itself is a second order effect, the proposed method can be used when greater accuracy is needed. Indeed, the C'_{tot} rows of Table I show errors of less than 6%, providing a substantial accuracy improvement.

Using the computed sensitivity, it is also interesting to conduct a variational study on the same structure to show how much is the effect of parameter variations on capacitances ($\mathbf{C}$), and whether a linear model can capture such effects.

We assume $\pm 30\%$ variations of the nominal value of each parameter. As for the reference, the dimension of the structure

TABLE I

RESULTS-I. COMPARISON OF THE SENSITIVITY COMPUTATION GIVEN BY DIFFERENT TECHNIQUES

	parameter: w1		parameter: w2	
	N1-GND	N1-N2	N2-M2	N2-M3
C_{nom} (fF)	0.1969	0.0746	0.0666	0.0194
C'_{ref} (fF/um)	0.1665	0.0548	0.0804	0.0188
C'_{plt} (fF/um)	0.1297	0.0427	0.0717	0.0148
error	-22.08%	-22.05%	-10.48%	-21.30%
C'_{tot} (fF/um)	0.1687	0.0572	0.0849	0.0191
error	1.34%	4.35%	5.57%	1.58%
	parameter: w3		parameter: w4	
	N3-N4	N3-M4	N4-GND	N4-M4
C_{nom} (fF)	0.0798	0.0228	0.1496	0.0844
C'_{ref} (fF/um)	0.0594	0.0217	0.1207	0.0901
C'_{plt} (fF/um)	0.0440	0.0170	0.0927	0.0789
error	-25.99%	-21.63%	-23.20%	-12.44%
C'_{tot} (fF/um)	0.0603	0.0220	0.1209	0.0943
error	1.44%	1.30%	0.17%	4.64%

TABLE II

RESULTS-II. VARIATIONAL STUDY OF CAPACITANCES USING THE LINEAR MODEL

	parameter variations: -30%			
	N1-N2	N2-M2	N3-N4	N4-M4
C_{nom} (fF)	0.0746	0.0666	0.0798	0.0844
C_{var} (fF)	0.0660	0.0548	0.0706	0.0712
variation in C	-11.53%	-17.72%	-11.53%	-15.64%
C_{plt} (fF)	0.0682	0.0558	0.0733	0.0726
error	3.29%	1.82%	3.81%	2.02%
C_{tot} (fF)	0.0660	0.0539	0.0708	0.0703
error	0.00%	-1.79%	0.38%	-1.22%
	parameter variations: +30%			
C_{var} (fF)	0.0822	0.0808	0.0882	0.0999
variation in C	10.19%	21.32%	10.53%	18.36%
C_{plt} (fF)	0.0810	0.0773	0.0864	0.0963
error	-1.41%	-4.29%	-1.94%	-3.63%
C_{tot} (fF)	0.0832	0.0793	0.0889	0.0986
error	1.23%	-1.84%	0.84%	-1.32%

is modified manually by $\pm 30\%$ and a standard extraction is performed to obtain the capacitance in the varied case (C_{var}). As shown in Table II, the resulting variation in capacitance (compared to the nominal capacitance C_{nom}) goes easily beyond 15%, and even 20%. It indicates that process variations can not be simply neglected. An appropriate modeling method needs to be found and applied, for instance, a linear model. With the computed sensitivities (C'_{plt} and C'_{tot}), it is very easy to build the linear model of capacitances, obtaining C_{plt} and C_{tot} (see Table II). It shows that the linear model using C'_{plt} can already capture the variational effect nicely, with an error of less than 5%. With the enhanced sensitivity C'_{tot}, the linear model is able to further decrease the error. Note that in general, the variation hardly goes up to $\pm 30\%$ for back-end-of-line (BEOL) processing. Thus, for realistic variations, the error of the sensitivity using the proposed algorithm can be even less.

Regarding the CPU time, the linear model is much faster than the FD method, as shown in Table III. Note that the CPU time of the linear model (C_{plt} or C_{tot}) includes both the computation of the nominal capacitance (C_{nom}) and the sensitivities (C'_{plt} or C'_{tot}). It indicates that the algorithm for the basic sensitivity computation is extremely fast and results in only a little overhead. And the proposed algorithm for the enhanced sensitivity also provides a competitive efficiency, especially compared to the FD method which requires n_p extra full capacitance extractions given n_p parameters of interest.

TABLE III

RESULTS-III. CPU TIME COMPARISON OF THE CAPACITANCE VARIATIONAL STUDY

	C_{nom}	C_{plt}	C_{tot}	C_{FD}
CPU Time	$37.33''$	$37.69''$	$41.25''$	$186.69''$
	$(1\times)$	$(1.01\times)$	$(1.11\times)$	$(5.00\times)$

V. CONCLUSION

This paper proposes an extension for an existing algorithm of BEM based capacitance sensitivity computation w.r.t. multiple geometric parameters. The extended algorithm is much faster than the traditional FD approach while providing a similarly high accuracy. The extension serves as a useful and sometimes necessary supplement for the existing algorithm which features high speed in generating good accuracy results. The enhanced algorithm thus is able to offer users various solutions for various requirements and applications. As such, it provides a flexible tool for BEM-based capacitance extractors subject to geometric process variations.

VI. ACKNOWLEDGMENT

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