Transactions Papers

Balanced Capacity of Wireline Multiuser Channels

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***Abstract*—This paper analyzes the multiuser capacity of Gaussian frequency-selective wireline multiaccess channels. Both the uplink (multiple-access channel) and downlink (broadcast channel) capacity regions are considered. The concept of balanced capacity is introduced to characterize the multiuser channel performance. Algorithms for the computation of the balanced multiuser capacity (and the associated power allocations) are proposed for an arbitrary number of users. The optimal power allocation in a K-user memoryless Gaussian channel is analyzed in detail, and an extension to intersymbol interference channels is given with various kinds of power constraints. Results are provided for a wireline access network with 20 users.**

***Index Terms*—Broadcast channel (BC), multiple-access (MA) channel, multiuser capacity, multiuser waterfilling, wireline channel.**

I. Introduction

THE wireline multiple-access (MA) channel is an example of a multiuser channel with intersymbol interference (ISI). It provides simultaneous duplex connections between the cable head-end (access point to external services) and a number of subscribers spatially distributed along the common physical cable. Some practical applications are the outdoor powerline channel and the CATV network. Functionally, it can be modeled as an MA channel for the uplink and a broadcast channel (BC) for the downlink, and each user channel can be modeled as a linear time-invariant frequency-selective channel with Gaussian noise.

Some simple criteria are needed to characterize the performance of the MA channel. The maximum error-free transmission rate for a given power budget is a commonly adopted criterion, although the minimum required power for a given target rate could also be investigated (see [1]). The list of single-user Shannon capacities versus the position of the remote modem along the cable gives a first idea of the channel potential. Single-user capacities are sufficient to characterize the MA channel when time-domain multiplexing (TDM) is used in the downlink and time-domain MA (TDMA) or random-access methods are used in the uplink. If more elaborate multiuser signaling techniques are used (involving simultaneous transmission of the different user signals), the full characterization of the MA channel requires the determination of the capacity region. Each point of the capacity region is associated with a given power allocation, and successive decoding (with a specific decoding order) is generally necessary to obtain the corresponding data rates. The boundary of the capacity region can be traced out by maximizing a weighted sum of the individual data rates with variable priority coefficients α_k. All points of the capacity region, however, are not desirable working points. Many papers focus on the maximum sum rate, but this solution does generally not guarantee a fair data rate to each subscriber. On the opposite, the common rate strategy is not necessarily a good idea, especially when the attenuation levels associated with the various channels in competition are very different. As a tradeoff, the idea of balanced capacity is introduced, and the set of maximum balanced user rates is proposed to characterize the performance of the MA channel.

The *balanced capacity* of a multiuser system is defined as the distribution of maximum simultaneously achievable bit rates that are proportional to the single-user rates. It is a specific point of the boundary of the capacity region for which the coexistence with the other users has the same relative cost for every user. The computation of maximum balanced rates basically involves the computation of the right priority coefficients α_k associated with these balanced rates. The objective of this paper is to propose practical algorithms for the computation of the uplink/downlink balanced capacity of an MA network with K users, where K may be much larger than two. Different types of power constraints are investigated, leading to distinct capacity regions. The problem of optimal duplexing is not considered here; the multiuser channel is supposed to be operated in uplink OR downlink at a given time.

The capacity of multiuser networks has been extensively studied in the past few years, but the issue of a fair rate distribution was generally ignored. In [2], a limiting expression for the capacity regions of MA channels with memory was obtained.

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In [3], this result was used to derive the capacity region of a two-user Gaussian MA channel with ISI, and the well-known single-user waterfilling algorithm (that provides the optimal power allocations) was extended to the two-user case. The authors of [4] extended these results to channels with an arbitrary number of users in the context of MA fading channels. All these papers assume individual power constraints at the transmitters: the K-user channels need to be scaled (concept of *equivalent channel*), so that the multiuser waterfilling algorithm provides K power allocations that satisfy the K power constraints simultaneously. The K scaling coefficients have to be computed iteratively. A general algorithm is proposed in [4], while the authors of [5] and [6] propose practical algorithms well suited to specific situations (two-user channel and/or maximum sum-rate solution). An alternative method for the computation of the maximum sum-rate solution was proposed in [7], and basically involves the iterative application of the single-user waterfilling algorithm. A detailed analysis of this method is proposed in [8]. [9] addresses the problem of computing the frequency-division MA (FDMA) capacity region (which is useful when successive decoding is undesired at the receiver), and proposes suboptimal algorithms for the two-user case in the context of optimal duplex in digital subscriber lines (DSL). In the general case, the optimal user rates can be achieved through the use of multicarrier MA [10], [11] with appropriate coding on each carrier. The multiuser waterfilling process is highly simplified when the individual power constraint is relaxed and a single constraint is put on the transmission power sum. In that case, the uplink capacity region is known to be the same as the downlink one [12], [13]. The capacity of Gaussian BCs is analyzed in [14] and [15]. The issue of a fair rate allocation in the uplink was addressed in [16] with the same constraint relaxation.

This paper is organized as follows. In Section II, a general model of a wireline MA channel is proposed. Section III describes the notion of capacity region and introduces the concept of balanced capacity. In Section IV, the optimal power allocation for a Gaussian memoryless channel with K users is analyzed in detail for the broadcast and MA channels, respectively. A simple allocation algorithm is given, as well as explicit expressions for the optimal allocated powers and the associated user rates. An iterative method is derived to obtain balanced user rates. The problem of optimal spectrum allocation in multiuser frequency-selective channels is then addressed in Section V. Different types of power constraints are first introduced, and the spectrum-allocation problem is analyzed in three specific situations. Practical algorithms for the computation of balanced user rates are finally proposed. Section VI presents detailed results obtained with a 20-user wireline access network.

II. Wireline MA Channels

The practical application that motivated this study is the broadband low-voltage powerline access network with typically 20–30 users and a bandwidth in the range of 500 kHz–10 MHz [17].

The general MA channel considered in this paper is composed of a main cable (the *distribution cable*) and $\overline{K}$ derivations (the *connection cables*) whose terminations are connected to the $\overline{K}$ user modems. The end of the distribution cable is connected to the cable head-end, which is supposed to provide a common access to the broadband services. Any subset of $K \leq \overline{K}$ user modems should be able to establish a duplex connection with the cable head-end at a given time. Fig. 1 illustrates the access network topology, where d_k and d'_k denote the length of the different cable segments. Unlike the DSL access networks (where each user has its own twisted pair), the physical medium has to be shared by all the users who wish to establish a connection simultaneously.

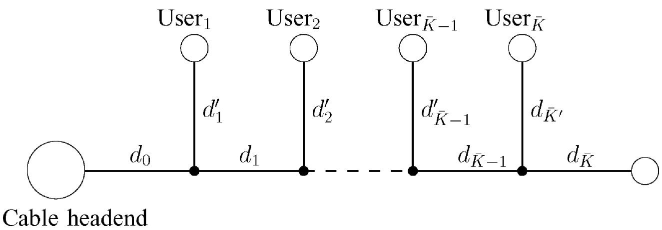

Fig. 1. Wireline MA network.

A major feature of the wave propagation in these networks is that multiple reflections may happen on the cable derivations and unmatched terminations. The resulting channels are multipath. Each path is characterized by its weight a_{lk} and its length L_{lk}, and the channel frequency responses are [18]

$$H_k(\omega) = \sum_l a_{lk} \exp\left[-\gamma(\omega) L_{lk}\right] \tag{1}$$

where the propagation factor $\gamma(\omega)$ depends on the cable characteristics. The weights a_{lk} do not depend on the frequency for standard cable derivations and terminations (matched or open).

For lossy cables, each path is also frequency-selective. A convenient model for the propagation factor $\gamma(\omega)$ is [19]

$$\gamma(2\pi f) = \frac{2\pi f \sqrt{\kappa}}{c} \sqrt{(j+\delta_d)\left[\frac{(1+j)\delta_{c1}}{\sqrt{f}} + j\right]} \tag{2}$$

where c is the light velocity in the vacuum, κ is the dielectric constant of the cable, δ_d is the dielectric loss angle, and δ_{c1} is the conductor loss angle at 1 Hz (representing the skin effect in the conductors). The resulting channel responses $H_k(\omega)$ are highly frequency-selective. According to the specific combination of the respective propagation paths, the global channel responses do not necessarily decrease with the frequency, even if the propagation factor in (2) is monotically decreasing.

Let us consider a simple example of a regular-pattern wireline access network with $\overline{K} = 20$ derivations, identical cable segments of length $d_k = d'_k = 15$ m, and ideal matched terminations (see Fig. 1). The lossy cables parameters are $\kappa = 2.3$, $\delta_d = 0.02$, and $\delta_{c1} = 85\sqrt{\text{Hz}}$. Fig. 2 gives the associated channel frequency responses $|H_k(\omega)|^2$ in a frequency range of 0–10 MHz, with $H_k(\omega)$ given by (1).[1] This example illustrates the rather extreme attenuation characteristic of such channels. Through the combined effect of cable losses and multiple reflections on the cable derivations, the channel gains for the remote

[1]Actually, the propagation model in (2) is only valid at high frequencies. However, this simple model was used to generate the channel responses of Fig. 2, even at the lower frequencies. The impact of this approximation on the resulting capacity computations is negligible.

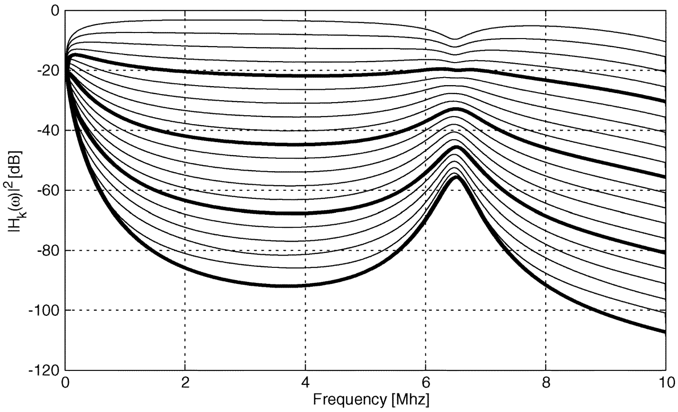

Fig. 2. Channel frequency responses associated with a regular-pattern wireline network with $\overline{K} = 20$ users.

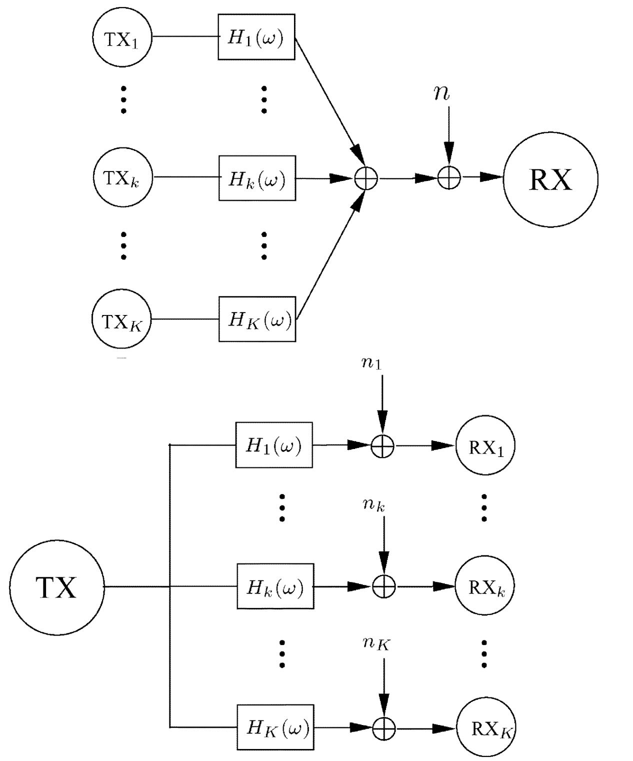

Fig. 3. MA and BC channels.

users can go below -100 dB at some frequencies. Furthermore, the mean attenuation level increases dramatically with the position of the considered user along the main cable. This is mainly the effect of the network topology, as a significant part of the transmitted power is dissipated in the intermediate terminations. As distant users experience much poorer channel characteristics than nearby users, it is crucial to introduce specific fairness constraints in the system design, in order to make sure that every user gets a minimal quality of service.

The *uplink* (data transmission from the user modems to the cable head-end) corresponds to an MA channel, while the *downlink* (data transmission from the cable head-end to the user modems) corresponds to a BC. The schemes of these two types of multiuser channels are given in Fig. 3. The additive noises at all receivers (n and $\{n_1, \ldots, n_K\}$) are supposed to be

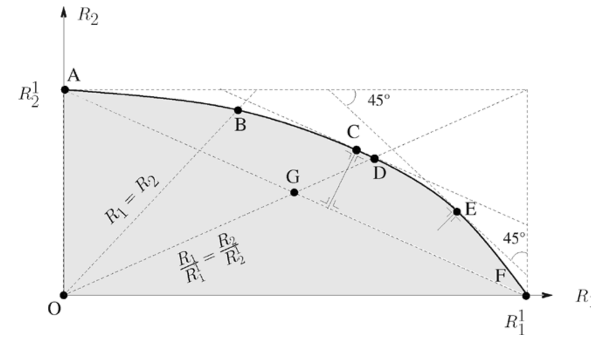

Fig. 4. Two-user capacity region and specific points of the boundary.

Gaussian and white, with a double-sided power spectral density (PSD) $N_0/2$. The downlink analysis with different noise levels can be easily performed in this context by an appropriate scaling of the respective channel frequency responses, as only the channel-gain-to-noise-variance ratio has an impact on the capacity.

In a practical wireline communications system, constraints are put on the *bandwidth*, on the *powers*, and on the *PSDs* of the transmitted signals. We assume, in the following, a transmission bandwidth $[0, B]$. The power constraint $\overline{P}$ and the PSD constraint $\overline{\gamma}$ will be analyzed separately. While in the downlink, the power constraint actually refers to the sum of all the user signals (located in the unique downlink transmitter); in the uplink, individual power constraints are generally associated with each transmitter. In this paper, however, the individual power constraint is relaxed, and a single power-sum constraint is considered both for the downlink and the uplink. Some results referring to the (more complex) individual power-constraint scenario in the uplink (see [20] and [21]) will be given for the purpose of comparison, but the associated analysis and algorithms will be reported in a later paper.

III. Multiuser Capacity Region and Balanced Capacity

A multiuser capacity region is defined as the set of achievable rates $\{R_k\}$ at which the receiver(s) may decode information from the transmitter(s) without error for a given set of (power) constraints. For the Gaussian K-user channel, it is a convex region in the K-dimensional space. This region reflects the tradeoff among the individual data rates of the different users competing for the limited resources.

The capacity regions investigated in this paper have the additional property to be *strictly convex* (see Fig. 4), under the assumption that the respective user channel responses are different. Consequently, the boundary of the capacity region can be traced out by means of a set of relative priority coefficients α_k with $\sum_k \alpha_k = 1$. Each boundary point of the capacity region maximizes the linear combination of the user rates $R_\alpha = \sum_k \alpha_k R_k$. Let $\mathbf{R}^* = [R_1^*, \ldots, R_k^*]^T$ be the vector of maximum user rates associated with a given vector of relative priorities $\boldsymbol{\alpha} = [\alpha_1, \ldots, \alpha_k]^T$. Actually, the coefficients α_k do not bring any information about the distribution of the maximum user rates R_k^*. Even with a higher relative priority $\alpha_k > 1/K$, a given user k with a poor channel quality could obtain a lower data rate R_k^* than the other users, or even a zero data rate. The only interpretation that can be associated with the relative priorities α_k is that $\mathbf{dR}^* \perp \boldsymbol{\alpha}$, which means that the boundary of the capacity region is normal to the vector $\boldsymbol{\alpha}$ in the neighborhood of the point $\mathbf{R}^*$. To obtain a practical characterization of the access network capacity, it is useful to put forward some specific points of the boundary.

- The *single-user rates* R_k^1 are the maximum achievable rates in the single-user communication scenario. They correspond to having only one alpha different from zero.
- The *maximum sum rate* corresponds to the setting $\alpha_k = 1/K$. It generally results in unfair situations where the users with the best channels have a much higher rate than the others, which is not desirable in practical applications. It may even happen that some subscribers do not have any bit rate at all.
- The *maximum common rate* [16] or *symmetric capacity* [22] is such that $R_1 = \cdots = R_K$. When the single-user rates are very different, the common rate constraint is generally a waste of resources, as it forces the users with the best channels to lower their rate dramatically to reach the level of the weakest channels.
- The *maximum balanced rate* is such that $R_1/R_1^1 = \cdots = R_K/R_K^1$. In other words, all users transmit at a rate R_k, which is in the same proportion with respect to (w.r.t.) the potential rate R_k^1 offered by their own channel. The relative cost implied by the coexistence with the other users is the same for all the users.

Fig. 4 gives an example of a convex capacity region and the corresponding boundary points for $K = 2$. A and F give the single-user rates. E gives the maximum sum rate; it corresponds to the point where the tangent has a -45° slope. B gives the maximum common rate, and D gives the maximum balanced rates.

As a lower bound on the balanced rates, TDM or TDMA can be considered, with time slots of equal duration for each user. This strategy gives a set of rates $R_k = R_k^1/K$ (line AGF on Fig. 4). Allowing simultaneous transmission of all the users, with a smart power and spectrum management, offers higher balanced rates, of course. In any case, the maximum balanced rates can be written

$$R_k = g\frac{R_k^1}{K} \tag{3}$$

where g is the rate gain with respect to the TDM(A) strategy (OD/OG on Fig. 4). Thanks to the convexity property of the capacity region, we know that $g \geq 1$. This gain will be referred to as the "multiuser gain" in the following. The multiuser gain increases when the user channel frequency responses are very different. An equivalent concept in the context of multiuser fading channels is the concept of multiuser diversity (e.g., [23]).

If the constraint is put on the power sum, the multiuser gain g is equal to one in the special case where all the users have identical channel responses $(H_k(\omega) = H_{k'}(\omega)\ \forall k, k')$. In other words, allowing simultaneous transmission by the different users does not increase the size of the capacity region if the channels of all users are identical. On the opposite, the maximum gain g is obtained when the user channels are sufficiently different to provide single-user optimal power allocations in K mutually exclusive sets of frequency bins. In that case, the users are not in competition for the best frequencies, and they can all transmit at their single-user maximum rate $R_k = R_k^1\ \forall k$. An upper bound for the multiuser gain is thus $g \leq K$.

The aim of this paper is to propose practical algorithms allowing the *computation of the maximum balanced rates for an arbitrary number of users*, and the corresponding (downlink and uplink) power allocations. The derivation of these algorithms basically involves three steps.

- The computation of the maximum aggregate rate R_α for a fixed power allocation and a fixed sequence of priority coefficients $\{\alpha_k\}$.
- The computation of the optimal power allocation (i.e., maximizing R_α) for a fixed set of priority coefficients $\{\alpha_k\}$.
- The computation of the appropriate sequence $\{\alpha_k\}$ providing balanced user rates.

These three steps will first be analyzed in detail for the case of a multiuser memoryless channel (Section IV). Extensions will then be provided for a multiuser frequency-selective channel with various kinds of power constraints (Section V).

IV. Memoryless Channel

In the memoryless scenario, the channels are fully defined by the set of attenuation factors $\{h_k^2 \triangleq |H_k(\omega)|^2\}$ and the additive white Gaussian noise (AWGN) variance $\sigma^2 = N_0 B$. A total power $\overline{P}$ should be allocated among the different users. The maximum balanced rates $\{R_k^*\}$ are here associated with a specific power allocation $\{P_k\}$, and a specific decoding order.

A. Maximum Aggregate Rate for a Given Power Allocation

This section provides explicit expressions for the MA and BC maximum aggregate rates R_α associated with a fixed power allocation $\{P_k\}$ and a fixed set $\{\alpha_k\}$. This allows formulating the power-allocation problem as a constrained optimization problem.

1) MA Channel: The capacity region for the MA channel (uplink) is known to be [15]

$$\left\{\{R_k\} : \sum_{k\in\mathcal{S}} R_k \leq \mathcal{C}\left(\frac{\sum_{k\in\mathcal{S}} P_k h_k^2}{\sigma^2}\right)\ \forall\mathcal{S}\subset\{1,\ldots,K\}\right\} \tag{4}$$

where $\mathcal{C}(x) \triangleq B\log(1+x)$.[2] This region is defined by $2^K - 1$ constraints, each one corresponding to a nonempty subset $\mathcal{S}$ of users. It has $K!$ vertices in the positive quadrant. Each vertex is achievable by a successive decoding using one of the $K!$ possible orderings of the users. In the special case $K = 2$, the capacity region is a pentagon whose two useful vertices correspond to the decoding orders (1,2) and (2,1), respectively (see Fig. 5). The global capacity region, corresponding to a constraint on the power sum, is generated by the union of such polyhedrons, each one corresponding to a specific power allocation that satisfies the global power constraint. The resulting boundary of the global capacity region is curved, and it is

[2]The logarithm used in the analytical derivations has a natural base, but a base of two is used in Section VI in order to obtain numerical results expressed in Mb/s.

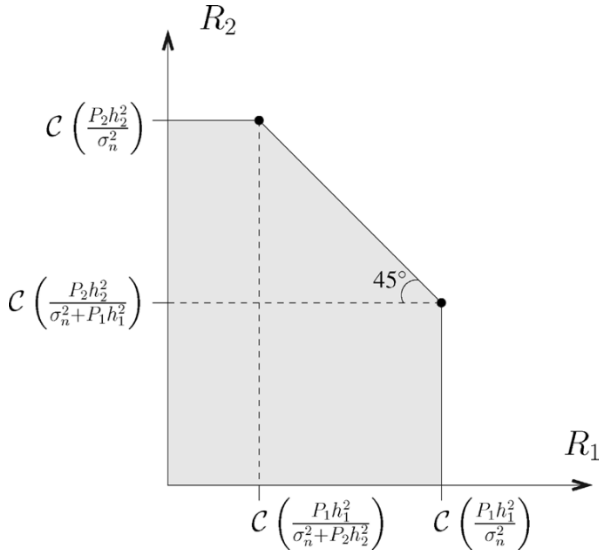

Fig. 5. Two-user capacity regions for a fixed power distribution (MA channel).

generated by the union of well-selected vertices. Consequently, each point on the boundary of the global capacity region can be obtained by a simple successive decoding with an appropriate power allocation and an appropriate decoding order, and special techniques like rate splitting [24] are not required.

From the structure of the capacity region in (4), it is obvious that the maximum aggregate rate $R_\alpha = \sum_k \alpha_k R_k$ is obtained at a single vertex of the polyhedron, corresponding to a successive decoding of the received signals in ascending order of the relative priorities $\{\alpha_k\}$. In other words, the messages with the lower priorities are decoded first and subtracted from the received signal before decoding the messages with higher priorities. In this paper, in order to simplify notations, we assume that $\alpha_1 \leq \cdots \leq \alpha_K$. Under this assumption, the optimal decoding order in the receiver is known to be $\{1,\ldots,K\}$. The maximum weighted rate is then given by

$$R_{\alpha,\mathbf{MA}} = \sum_{k=1}^{K} \alpha_k \mathcal{C}\left(\frac{P_k h_k^2}{\sigma^2 + \sum_{l=k+1}^{K} P_l h_l^2}\right) \quad (5)$$

$$= \sum_{k=1}^{K} (\alpha_k - \alpha_{k-1}) \mathcal{C}\left(\frac{\sum_{l=k}^{K} P_l h_l^2}{\sigma^2}\right) \quad (6)$$

with $\alpha_0 \triangleq 0$. The expression in (6) is obtained by using the following chain rule:

$$\mathcal{C}\left(\frac{x+y}{\sigma^2}\right) = \mathcal{C}\left(\frac{x}{\sigma^2+y}\right) + \mathcal{C}\left(\frac{y}{\sigma^2}\right) \quad (7)$$

for arbitrary positive values of x, y, and σ^2.

2) BC Channel: The scalar Gaussian BC is known to be a *degraded* BC, which means that the K users can be absolutely ranked by their channel gains h_k^2. From this observation, Cover [14], [15] computed the capacity region of the Gaussian BC as follows:

$$\left\{\{R_k\} : R_k \leq \mathcal{C}\left(\frac{P_k h_k^2}{\sigma^2 + \left(\sum_{h_k^2 < h_l^2} P_l\right) h_k^2}\right)\right\}. \quad (8)$$

Every point on the boundary of the Gaussian BC capacity region in (8) corresponds to a given power allocation $\{P_k\}$ that satisfies the global power constraint, and can be achieved by successive decoding in the different receivers, with an appropriate decoding order. Here the decoding order is not dictated by the user relative priorities, but by the relative channel gains. A given receiver is able to decode the messages intended for the users with lower channel gains (and thus, transmitted at a lower rate) before decoding its own message. The remaining messages are considered as noise.

Let π be a permutation on the set $\{1,\ldots,K\}$, such that $h_{\pi(i)}^2 \geq h_{\pi(j)}^2$ for $i < j$. In other words, this permutation is used to sort the users in decreasing order of their channel gains. The weighted rate R_α associated with a given power allocation $\{P_k\}$ for the BC channel can finally be written as

$$R_{\alpha,\mathbf{BC}} = \sum_{k=1}^{K} \alpha_{\pi(k)} \mathcal{C}\left(\frac{P_{\pi(k)} h_{\pi(k)}^2}{\sigma^2 + \left[\sum_{l=1}^{k-1} P_{\pi(l)}\right] h_{\pi(k)}^2}\right). \quad (9)$$

3) Formulation of the Optimal Power-Allocation Problem: For a given constraint $\overline{P}$ on the total transmitted power, our objective is now to find the optimal power distributions $\{P_k^\circ\}$ and $\{P_k^*\}$ that maximize R_α for the MA and BC channels, respectively

$$\{P_k^\circ\} = \arg\max\left[R_{\alpha,\mathbf{MA}}\left(\{P_k\}\right)\right] \quad (10)$$

$$\{P_k^*\} = \arg\max\left[R_{\alpha,\mathbf{BC}}\left(\{P_k\}\right)\right] \quad (11)$$

with constraints $\sum_k P_k \leq \overline{P}$ and $P_k \geq 0\ \forall k$. In each case, the solution satisfies the Kuhn–Tucker conditions

$$\frac{1}{B}\frac{\partial R_\alpha}{\partial P_k} \begin{cases} = \mu, & P_k > 0 \\ < \mu, & P_k = 0 \end{cases} \quad (12)$$

where μ is a positive Lagrange multiplier.

B. Optimal Power Allocation for the BC Memoryless Channel (Given $\{\alpha_k\}$)

The users are supposed to be sorted in decreasing order of their channel gains through the permutation π. Starting the allocation process with $P_{\pi(k)} = 0\ \forall k$, the marginal rate gain for each user is given by $(1/B)(\partial R_\alpha/\partial P_{\pi(k)}) = D_{\pi(k)}(0)$, where the functions $D_k(x)$ are defined by

$$D_k(x) \triangleq \frac{\alpha_k h_k^2/\sigma^2}{1 + x h_k^2/\sigma^2}. \quad (13)$$

Some power $P_{\pi(k_1)}$ is first allocated to the user $\pi(k_1)$ with the maximum $\alpha_{\pi(k)} h_{\pi(k)}^2$.

The users with a higher $h_{\pi(k)}^2$ but a lower $\alpha_{\pi(k)} h_{\pi(k)}^2$ can be discarded. Indeed, (9) ensures that for $k < k_1$ (i.e., for users with a higher channel gain)

$$\frac{1}{B}\frac{\partial R_\alpha}{\partial P_{\pi(k)}} = \frac{1}{B}\frac{\partial R_\alpha}{\partial P_{\pi(k_1)}} + \frac{1}{\sigma^2}\left(\alpha_{\pi(k)} h_{\pi(k)}^2 - \alpha_{\pi(k_1)} h_{\pi(k_1)}^2\right)$$

$$< \frac{1}{B}\frac{\partial R_\alpha}{\partial P_{\pi(k_1)}} \quad (14)$$

$$\text{if } P_{\pi(k)}^* = 0 \quad (15)$$

which is consistent with the Kuhn–Tucker conditions (12).

The marginal gain for the remaining users becomes $D_{\pi(k)}(P_{\pi(k_1)})$ with $k \in \{k_1, \ldots, K\}$. The power $P_{\pi(k_1)}$ is then increased until a second user $\pi(k_2)$ (with $k_2 > k_1$) reaches a marginal gain equal to that of user $\pi(k_1)$ (unless $\overline{P}$ is reached before). As $h^2_{\pi(k_2)} < h^2_{\pi(k_1)}$, this is only possible if $\alpha_{\pi(k_2)} > \alpha_{\pi(k_1)}$. When this happens, the common marginal gain can be shown to be

$$D_{\pi(k_1)}\left(P^*_{\pi(k_1)}\right) = D_{\pi(k_2)}\left(P^*_{\pi(k_1)}\right) = \left(\frac{\alpha_{\pi(k_2)} h^2_{\pi(k_2)}}{\sigma^2}\right) F_{\pi(k_1),\pi(k_2)} \quad (16)$$

where the modification factor $F_{i,j} < 1$ is defined as

$$F_{i,j} = \frac{1 - \alpha_i/\alpha_j}{1 - h_i^{-2}/h_j^{-2}}. \quad (17)$$

User $\pi(k_2)$ is thus selected as the one with the largest $(\alpha_{\pi(k)} h^2_{\pi(k)}) F_{\pi(k_1),\pi(k)}$ on the set $k \in \{k_1 + 1, \ldots, K\}$. The power allocated to user $\pi(k_1)$ is then fixed to

$$P^*_{\pi(k_1)} = \sigma^2 G_{\pi(k_1),\pi(k_2)} \quad (18)$$

where

$$G_{i,j} \triangleq \frac{\alpha_i h_i^2 - \alpha_j h_j^2}{(\alpha_j - \alpha_i) h_i^2 h_j^2} = h_j^{-2}\left(F_{i,j}^{-1} - 1\right). \quad (19)$$

The users with a higher $h^2_{\pi(k)}$, but a lower $(\alpha_{\pi(k)} h^2_{\pi(k)}) F_{\pi(k_1),\pi(k)}$ can be discarded. From (9), it can be checked that

$$\frac{1}{B}\frac{\partial R_\alpha}{\partial P_{\pi(k)}} = \frac{1}{B}\frac{\partial R_\alpha}{\partial P_{\pi(k_2)}} + \frac{1}{\sigma^2}\left(\alpha_{\pi(k)} h^2_{\pi(k)} F_{\pi(k_1),\pi(k)} - \alpha_{\pi(k_2)} h^2_{\pi(k_2)} F_{\pi(k_1),\pi(k_2)}\right) < \frac{1}{B}\frac{\partial R_\alpha}{\partial P_{\pi(k_2)}} \quad (20)$$

$$\text{if } P^*_{\pi(k)} = 0 \quad \forall k \in \{k_1 + 1, \ldots, k_2 - 1\} \quad (21)$$

which is again consistent with the Kuhn–Tucker conditions (12).

The rate $R^*_{\pi(k_1)}$ for user $\pi(k_1)$ is given by

$$R^*_{\pi(k_1)} = B \log\left(\frac{\alpha_{\pi(k_1)} h^2_{\pi_{k_1}}}{\alpha_{\pi(k_2)} h^2_{\pi(k_2)} F_{\pi(k_1),\pi(k_2)}}\right). \quad (22)$$

This rate is left unaffected by the allocation of power to the other users (with $k > k_1$), as those users have a lower channel gain, and their signals can be perfectly decoded and subtracted by receiver k_1. If there is still some power to allocate, then the allocation goes on with $P_{\pi(k_2)}$, which increases the rate $R_{\pi(k_2)}$ while leaving $R_{\pi(k_1)}$ unchanged, until a third user $\pi(k_3)$ (with a smaller $h^2_{\pi(k_3)}$ but a higher $\alpha_{\pi(k_3)}$) reaches the same marginal gain, and so on.

At the end of the allocation process, a subset $S_J = \{\pi(k_1), \ldots, \pi(k_J)\} \subset \{1, \ldots, K\}$ of $J \leq K$ users is obtained, who get a nonzero fraction of the total power $\overline{P}$. The users in this subset are sorted in *decreasing order of the channel gains* h_k^2, and in *increasing order of the priority coefficients* α_k. As a consequence, the allocation algorithm can work as well on the initial sequence $\{1, \ldots, K\}$ as on the ordered sequence $\{\pi(1), \ldots, \pi(K)\}$. The allocation rule can be stated in very simple terms: *always add power to the user with the maximum* $D_k(P)$ where P is the power already allocated. Mathematically, we get

$$R_\alpha = B \sum_{i=1}^{J} \int_{P^*_{k_1}+\cdots+P^*_{k_{i-1}}}^{P^*_{k_1}+\cdots+P^*_{k_i}} D_{k_i}(x)dx = B \int_0^{\overline{P}} \max_{k \in \{1,\cdots,K\}} D_k(x)dx \quad (23)$$

which provides an intuitive graphical interpretation.

Fig. 6 illustrates the evolution of D_k and the individual powers P_k^* as a function of the total allocated power P for $K = 4$. In this example, $h_1^2 > h_2^2 > h_3^2 > h_4^2$ and $\alpha_1 < \alpha_2 < \alpha_3 < \alpha_4$. At the beginning of the allocation process ($P = 0$), user 1 has the maximum marginal rate $D_1(0)$ (see upper figure). The power allocated to user 1, i.e., P_1^*,

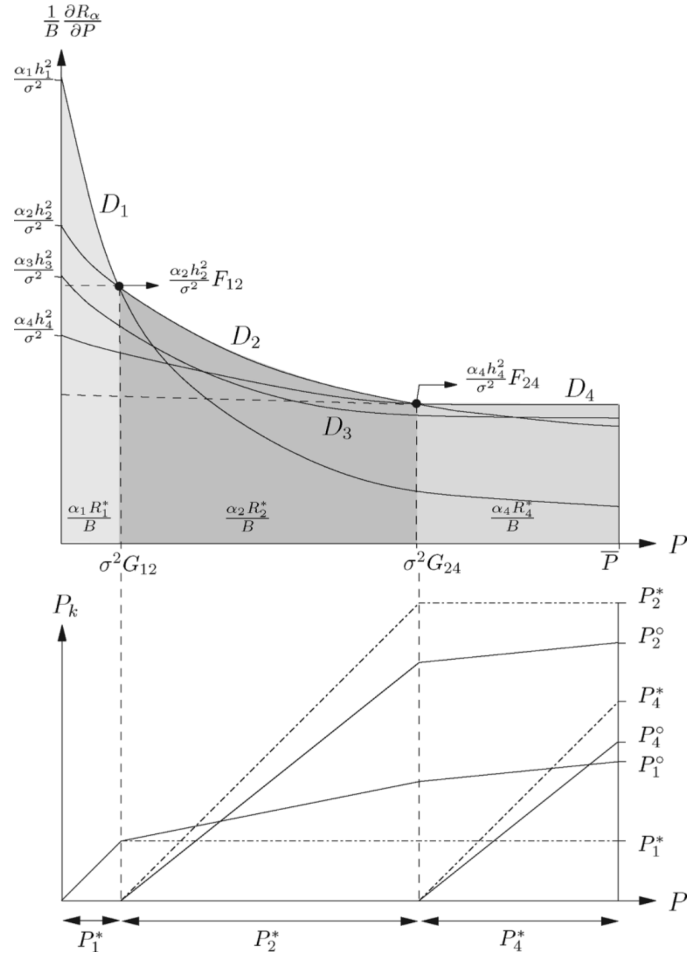

Fig. 6. Marginal gain and power distribution during the allocation process ($K = 4$).

is then equal to P (see lower figure), while the other power allocations (P_2^*, P_3^*, P_4^*) are left to zero. This continues until $D_1(P) = D_2(P)$. When this happens, $P = P_1^* = \sigma^2 G_{12}$. From that point, no extra power will be allocated to user 1, and P_1^* has reached its maximal value. The resulting rate R_1^* is proportional to the area below the curve $D_1(P)$ in the interval $[0, \sigma^2 G_{12}]$. Next, power is allocated to user 2, which has the maximum marginal rate $D_2(P)$ in the interval $[\sigma^2 G_{12}, \sigma^2 G_{24}]$ (see upper figure). In this interval, power is just fully allocated to user 2, and P_2^* goes from zero to its ultimate value $\sigma^2(G_{24} - G_{12})$. Finally, the rest of the available power is allocated to user 4, which has the maximum marginal rate $D_4(P)$ in the interval $[\sigma^2 G_{24}, \overline{P}]$. User 3 does not receive any power. The optimal allocation gives $J = 3$ and $S_J = \{1, 2, 4\}$. For decreasing values of $\overline{P}$, the subset S_J can be reduced to {1,2} or {1}. Obviously, when the power to be allocated is large enough, the last user in the subset S_J is always the user with the highest priority α_k.

Once the subset S_J is known, the set of optimal powers $\{P_{k_i}^*\}$ can be computed by solving the following system:

$$D_{k_i}\left(\sum_{l=1}^{i} P_{k_l}^*\right) = D_{k_{i+1}}\left(\sum_{l=1}^{i} P_{k_l}^*\right), \quad i \le J-1$$
$$\sum_{i=1}^{J} P_{k_i}^* = \overline{P}. \tag{24}$$

Finally, the complete allocation procedure and the corresponding user rates are explicitly given as follows.

Algorithm 1: Algorithm for the selection of the user sequence receiving a fraction of the total power $\overline{P}$ (Subset S_J)

- Initialization $(J = 1)$:

$$k_1 = \arg\max_{k\in[1,K]} \left(\alpha_k h_k^2\right). \tag{25}$$

- While $k_J < K$:

— Compute the next candidate:

$$k^* = \arg\max_{k\in[k_J+1,K]} \left(\alpha_k h_k^2 F_{k_J,k}\right). \tag{26}$$

— If there is enough power:

$$\overline{P} \ge \sigma^2 G_{k_J,k^*} \tag{27}$$

then: $J = J + 1$, $k_J = k^*$.

— Else: stop.

Optimal BC allocated powers P_k^* (with $k \in S_J$)

$$\begin{pmatrix} P_{k_1}^* \\ P_{k_2}^* \\ \cdots \\ P_{k_j}^* \\ \cdots \\ P_{k_{J-1}}^* \\ P_{k_J}^* \end{pmatrix} = \sigma^2 \begin{pmatrix} G_{k_1,k_2} \\ G_{k_2,k_3} - G_{k_1,k_2} \\ \cdots \\ G_{k_j,k_{j+1}} - G_{k_{j-1},k_j} \\ \cdots \\ G_{k_{J-1},k_J} - G_{k_{J-2},k_{J-1}} \\ -G_{k_{J-1},k_J} \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \cdots \\ 0 \\ \cdots \\ 0 \\ \overline{P} \end{pmatrix}. \tag{28}$$

Optimal user rates R_k^* (with $k \in S_J$)

$$\begin{pmatrix} R_{k_1}^* \\ R_{k_2}^* \\ \cdots \\ R_{k_j}^* \\ \cdots \\ R_{k_{J-1}}^* \\ R_{k_J}^* \end{pmatrix} = B \log \begin{pmatrix} \frac{\alpha_{k_1} h_{k_1}^2}{\alpha_{k_2} h_{k_2}^2 F_{k_1,k_2}} \\ \frac{\alpha_{k_2} h_{k_2}^2 F_{k_1,k_2}}{\alpha_{k_3} h_{k_3}^2 F_{k_2,k_3}} \\ \cdots \\ \frac{\alpha_{k_j} h_{k_j}^2 F_{k_{j-1},k_j}}{\alpha_{k_{j+1}} h_{k_{j+1}}^2 F_{k_j,k_{j+1}}} \\ \cdots \\ \frac{\alpha_{k_{J-1}} h_{k_{J-1}}^2 F_{k_{J-2},k_{J-1}}}{\alpha_{k_J} h_{k_J}^2 F_{k_{J-1},k_J}} \\ \left[1 + \frac{\overline{P} h_{k_J}^2}{\sigma^2}\right] F_{k_{J-1},k_J} \end{pmatrix}. \tag{29}$$

Algorithm 1 ensures that the obtained powers and rates are positive.

C. Optimal Power Allocation for the MA Memoryless Channel (Given $\{\alpha_k\}$)

The computation of the optimal allocation is very similar in the MA and BC channels. These two channels are known to have the same capacity region when the constraint is put on the transmitted power sum [12], [13]. However, the power distributions corresponding to a given boundary point of the capacity region are different. In this section, the expression of the optimal user rates is derived again in the MA context, allowing the computation of an explicit expression for the optimal MA power allocation $\{P_k^\circ\}$. The MA optimal rates turn out to have the same form as the BC optimal rates $\{R_k^*\}$ given by (29).

The users are now sorted in increasing order of their priority coefficients α_k. For a given power allocation $\{P_k\}$, we define the sequence $\{x_k\} \le 1$ as

$$x_k = \left(1 + \frac{\sum_{i=k}^{K} P_i h_i^2}{\sigma^2}\right)^{-1}. \tag{30}$$

From (5), the marginal rate gains can be obtained iteratively as follows:

$$\frac{1}{B}\frac{\partial R_\alpha}{\partial P_1} = \frac{\alpha_1 h_1^2}{\sigma^2} x_1$$
$$\frac{1}{B}\frac{\partial R_\alpha}{\partial P_{k+1}} = \frac{h_{k+1}^2}{h_k^2}\left(\frac{1}{B}\frac{\partial R_\alpha}{\partial P_k}\right) + (\alpha_{k+1} - \alpha_k)\frac{h_{k+1}^2}{\sigma^2} x_{k+1}. \tag{31}$$

The initial marginal rate gain for each user $(x_k = 1\ \forall k)$ is again proportional to $\alpha_k h_k^2$. The power is thus first allocated to user k_1 with the maximum $\alpha_k h_k^2$.

The lower indexed users (i.e., the users with a lower priority) are discarded; $P_k^\circ = 0$ for $k < k_1$. From (31), it follows that:

$$x_k = x_{k_1} \tag{32}$$
$$\frac{\partial R_\alpha}{\partial P_k} = \frac{\alpha_k h_k^2}{\alpha_{k_1} h_{k_1}^2}\frac{\partial R_\alpha}{\partial P_{k_1}} < \frac{\partial R_\alpha}{\partial P_{k_1}} \tag{33}$$

for $k < k_1$, which is consistent with the Kuhn–Tucker conditions (12).

The marginal gains for the remaining users is again $D_k(P_{k_1})$. Power is thus allocated to user k_1 until $P_{k_1} = \sigma^2 G_{k_1,k_2}$ for a given user k_2 with

$$k_2 = \arg \max_{k \in \{k_1+1,\ldots,K\}} \left(\alpha_k h_k^2 F_{k_1,k}\right) \tag{34}$$

and G_{k_1,k_2} is defined in (19). At that time, the power allocated to user k_1 is equal to $P^*_{k_1}$, that is to say, the optimal power that would be allocated in a BC scenario (see Fig. 6).

Now the allocation process becomes different. Whereas in the BC channel, any power increase dP would be fully allocated to user k_2 (which increases R_{k_2} and leaves R_{k_1} unaffected), in the MA channel, dP has to be shared by users k_1 and k_2. The fraction of power dP_{k_2} allocated to user k_2 directly increases the rate R_{k_2}. But this power is seen as noise by user k_1 (which is decoded *before* user k_2). Some power dP_{k_1} is then allocated to user k to compensate for the noise enhancement and leave R_{k_1} unchanged. Some power is thus allocated simultaneously to users k_1 and k_2.

Users with an intermediate priority are discarded; $P^\circ_k = 0$ with $k_1 < k < k_2$. From (31), we obtain the system

$$\frac{1}{B}\frac{\partial R_\alpha}{\partial P_{k_1}} = \mu \tag{35}$$

$$\frac{1}{B}\frac{\partial R_\alpha}{\partial P_k} = \frac{h_k^2}{h_{k_1}^2}\mu + (\alpha_k - \alpha_{k_1})\frac{h_k^2}{\sigma^2}x_{k_2} \tag{36}$$

$$\frac{1}{B}\frac{\partial R_\alpha}{\partial P_{k_2}} = \frac{h_{k_2}^2}{h_{k_1}^2}\mu + (\alpha_{k_2} - \alpha_{k_1})\frac{h_{k_2}^2}{\sigma^2}x_{k_2} = \mu. \tag{37}$$

Solving this system, we get

$$\begin{aligned}\frac{\partial R_\alpha}{\partial P_k} &= \frac{\partial R_\alpha}{\partial P_{k_1}}\Big(1 - h_k^2(\alpha_k - \alpha_{k_1}) \\ &\qquad \times \left[\left(\alpha_k h_k^2 F_{k_1,k}\right)^{-1} - \left(\alpha_{k_2} h_{k_2}^2 F_{k_1,k_2}\right)^{-1}\right]\Big) \\ &< \frac{\partial R_\alpha}{\partial P_{k_1}}\end{aligned} \tag{38}$$

for $k_1 < k < k_2$, which is again consistent with the Kuhn–Tucker conditions (12).

The optimal rate $R^*_{k_1}$ for user k_1 is found to be the same as (22), and the common marginal gain can be shown to be

$$\mu = D_{k_2}\left(P_{k_1} + P_{k_2}\right). \tag{39}$$

The two-user allocation continues until a third user $k_3 \in \{k_2 + 1,\ldots,K\}$ reaches the same marginal gain. When this happens, we have $P_{k_1} + P_{k_2} = P^*_{k_1} + P^*_{k_2}$, that is to say, the total power allocated to users k_1 and k_2 is the same as the optimal power sum that would be allocated in a BC scenario (see Fig. 6). Power is then allocated simultaneously to users (k_1, k_2, k_3), and so on.

The relation (23) given for the BC scenario remains valid for the MA channel. The allocation rule can be restated as *always increase the rate of the user k_i with the maximum $D_k(P)$ while leaving the other rates unchanged*, where P is the power already allocated. This can be obtained by allocating power simultaneously to all the users in the set $\{k_1,\ldots,k_i\}$. The user subset S_J and the optimal user rates are found to be the same as in the BC [see *Algorithm 1* and (29)].

Once the subset S_J is known, the set of optimal powers $\{P^\circ_{k_i}\}$ can be computed from (31) and (12). The sequence x_{k_j} is first computed as

$$\begin{aligned}x_{k_1} &= \mu\left(\frac{\alpha_{k_1} h_{k_1}^2}{\sigma^2}\right)^{-1} \\ x_{k_{j+1}} &= \mu\left(\frac{\alpha_{k_{j+1}} h_{k_{j+1}}^2}{\sigma^2}F_{k_j,k_{j+1}}\right)^{-1}, \quad j \in \{1,\ldots,J-1\}.\end{aligned} \tag{40}$$

The optimal powers $P^\circ_{k_j}$ are then given by

$$\begin{aligned}P^\circ_{k_j} &= \sigma^2 h_{k_j}^{-2}\left(x_{k_j}^{-1} - x_{k_{j+1}}^{-1}\right), \quad j \in \{1,\ldots,J-1\} \\ P^\circ_{k_J} &= \sigma^2 h_{k_J}^{-2}\left(x_{k_J}^{-1} - 1\right).\end{aligned} \tag{41}$$

Using the power constraint, the marginal rate is computed as

$$\mu = \frac{\alpha_{k_J} h_{k_J}^2/\sigma^2}{1 + \overline{P} h_{k_J}^2/\sigma^2}. \tag{42}$$

Finally, the *optimal MA allocated powers* P°_k (with $k \in S_J$) are given explicitly, as follows:

$$\begin{pmatrix} P^\circ_{k_1} \\ P^\circ_{k_2} \\ \cdots \\ P^\circ_{k_j} \\ \cdots \\ P^\circ_{k_{J-1}} \\ P^\circ_{k_J}\end{pmatrix} = \frac{1}{\alpha_{k_J}}\left(\overline{P} + \frac{\sigma^2}{h_{k_J}^2}\right)\mathbf{\Pi}(\boldsymbol{\alpha},\mathbf{h}) - \begin{bmatrix} 0 \\ 0 \\ \cdots \\ 0 \\ \cdots \\ 0 \\ \frac{\sigma^2}{h_{k_J}^2}\end{bmatrix} \tag{43}$$

with vector $\mathbf{\Pi}(\boldsymbol{\alpha},\mathbf{h})$ defined as

$$\begin{bmatrix} h_{k_1}^{-2}\left(\alpha_{k_1} h_{k_1}^2 - \alpha_{k_2} h_{k_2}^2 F_{k_1,k_2}\right) \\ h_{k_2}^{-2}\left(\alpha_{k_2} h_{k_2}^2 F_{k_1,k_2} - \alpha_{k_3} h_{k_3}^2 F_{k_2,k_3}\right) \\ \cdots \\ h_{k_j}^{-2}\left(\alpha_{k_j} h_{k_j}^2 F_{k_{j-1},k_j} - \alpha_{k_{j+1}} h_{k_{j+1}}^2 F_{k_j,k_{j+1}}\right) \\ \cdots \\ h_{k_{J-1}}^{-2}\left(\alpha_{k_{J-1}} h_{k_{J-1}}^2 F_{k_{J-2},k_{J-1}} - \alpha_{k_J} h_{k_J}^2 F_{k_{J-1},k_J}\right) \\ h_{k_J}^{-2}\left(\alpha_{k_J} h_{k_J}^2 F_{k_{J-1},k_J}\right)\end{bmatrix}. \tag{44}$$

Algorithm 1 also ensures that the obtained powers are positive.

Fig. 6 illustrates the evolution of the MA power allocation P°_k as a function of the total allocated power P, for the four-user example introduced in Section IV-B. In the first interval, $[0, \sigma^2 G_{12}]$, the power is again fully allocated to user 1, which provides the full rate R^*_1. In the second interval, $[\sigma^2 G_{12}, \sigma^2 G_{24}]$, the power allocation is shared between P°_2 (which raises the associated data rate from zero to R^*_2) and P°_1 (which keeps R^*_1 to a constant level). In the last interval, the rest of the available power is finally shared between user

4 (which raises the associated data rate from zero to R_4^*), and users 1 and 2 (which keeps R_1^* and R_2^* to a constant level).

D. Maximum Balanced Rates

The previous section explained how to compute the optimal power allocation $\{P_k\}$ for a given set of priority coefficients $\{\alpha_k\}$ and a total power $\overline{P}$. With this result, the whole boundary of the capacity region can be numerically determined, just by ranging over all possible combinations of relative priorities. For a large number of users $K \gg 2$, however, the full characterization of the capacity region becomes prohibitively complex. It can be useful to characterize the K-user capacity region by computing just a few points corresponding to desirable working points. Remarkable points on the boundary of the capacity region imply specific relations between the individual data rates. Unfortunately, the selected set of relative priorities $\{\alpha_k\}$ does not bring any information about the associated distribution of individual data rates.

To obtain a specific boundary point of the capacity region such that the corresponding user rates satisfy

$$\frac{R_1^*}{\beta_1} = \cdots = \frac{R_K^*}{\beta_K} = \overline{R} \tag{45}$$

for a predefined set of normalization coefficients $\{\beta_k\}$, the proper set of priority coefficients $\{\alpha_k\}$ has to be found iteratively. Computation of the maximum balanced rates corresponds to $\beta_k = R_k^1$, but the idea is the same to get any other point of the capacity region. Equation (45) implies that every user gets a nonzero fraction of the power. Sorting the users in decreasing order of their channel gain h_k^2, the solution of our problem requires the computation of an appropriate set of priority coefficients $\alpha_1 < \cdots < \alpha_K$, such that *Algorithm 1* selects the whole set of users ($J = K$ and $S_K = \{1, \ldots, K\}$), and that the associated user rates in (29) satisfy the constraint in (45).

Actually, the problem of maximum balanced-rates computation can be modeled a nonlinear system with K unknowns α_k as follows:

$$\mathbf{F}(\boldsymbol{\alpha}) = \begin{pmatrix} \frac{R_2^*}{\beta_2} - \frac{R_1^*}{\beta_1} \\ \vdots \\ \frac{R_K^*}{\beta_K} - \frac{R_1^*}{\beta_1} \\ \sum_k \alpha_k - 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix}. \tag{46}$$

The shape of the capacity region (a convex set in the positive quadrant) guarantees the existence and uniqueness of the solution.

A Newton–Raphson iterative search is proposed to find the solution.

Algorithm 2: Maximum balanced rates for a Gaussian memoryless multiuser channel

- Start with $\boldsymbol{\alpha}^{(0)} = [1/R_1^1, \ldots, 1/R_K^1]^T / \sum_k (1/R_k^1)$, set $i = 0$.
- Compute the optimal power allocation with *Algorithm 1*. Compute the corresponding user rates $\{R_k^*\}$ and the normalized rates $\{R_k^*/\beta_k\}$.
- Compute the average normalized rate μ_{R_β} and the standard deviation σ_{R_β}, defined by

$$\mu_{R_\beta} \triangleq \frac{1}{K} \sum_k \left(\frac{R_k^*}{\beta_k} \right) \tag{47}$$

$$\sigma_{R_\beta} \triangleq \sqrt{\frac{1}{K} \sum_k \left(\frac{R_k^*}{\beta_k} - \mu_{R_\beta} \right)^2}. \tag{48}$$

- While $\sigma_{R_\beta}/\mu_{R_\beta} > \varepsilon$

— Update the set of priority coefficients as follows:

$$\boldsymbol{\Delta}_\alpha^{(i)} = -\left(\frac{\partial \mathbf{F}}{\partial \boldsymbol{\alpha}} \right)^{-1}_{\boldsymbol{\alpha} = \boldsymbol{\alpha}^{(i)}} \mathbf{F}\left(\boldsymbol{\alpha}^{(i)} \right) \tag{49}$$

$$\boldsymbol{\alpha}^{(i+1)} = \boldsymbol{\alpha}^{(i)} + \lambda_i \boldsymbol{\Delta}_\alpha^{(i)}. \tag{50}$$

— Update $\{R_k^*\}$, $\{R_k^*/\beta_k\}$, μ_{R_β}, and σ_{R_β}, set $i = i + 1$ where ε is a tolerance parameter and the sequence $\lambda_i \leq 1$ is used to ensure convergence in the first few iterations.

The computation of the update direction $\boldsymbol{\Delta}_\alpha^{(i)}$ requires the knowledge of the Jacobian matrix $(1/B)(\partial \mathbf{R}/\partial \boldsymbol{\alpha})$. From (29), it is a symmetric tridiagonal matrix $\mathbf{T}(\boldsymbol{\alpha})$ with the following entries on the three main diagonals:

$$\begin{aligned} \mathbf{T}_{k_j,k_j} &= \frac{\alpha_{k_{j+1}} - \alpha_{k_{j-1}}}{\left(\alpha_{k_j} - \alpha_{k_{j-1}}\right)\left(\alpha_{k_{j+1}} - \alpha_{k_j}\right)} \\ \mathbf{T}_{k_j,k_{j+1}} &= \frac{-1}{\alpha_{k_{j+1}} - \alpha_{k_j}} = \mathbf{T}_{k_{j+1},k_j} \end{aligned} \tag{51}$$

where $\alpha_{k_0} = \alpha_{k_{J+1}} \triangleq 0$. In particular, the Jacobian matrix is zero on the rows and columns corresponding to users who are not in the subset S_J. As this matrix should be invertible to allow the computation of the update direction, it is important to choose a convenient starting vector $\boldsymbol{\alpha}^{(0)}$, and to control the size of the updating step λ_i in order to keep $\boldsymbol{\alpha}^{(i+1)}$ close to the solution and keep the matrix inversion possible. The convexity property of the capacity region guarantees that every user is included in the allocation subset for the proposed $\boldsymbol{\alpha}^{(0)}$. Fig. 4 illustrates the initial rates obtained with $\boldsymbol{\alpha}^{(0)}$ (point C) in a two-user problem. After some iterations, the solution converges to the maximum balanced rates (point D).

To conclude this section, it should be noted that the computation method proposed here is probably not the most efficient one in the case of a multiuser memoryless channel. However, it can be easily extended to the case of a multiuser frequency-selective channel, which is the purpose of the next section.

V. Frequency-Selective Channel

In the case of frequency-selective channels with frequency responses $H_k(\omega)$, the power-allocation problem can be discretized by dividing the frequency spectrum into a large number N of frequency bins of width df. As N increases to infinity, a piecewise-constant channel model converges to the actual channels. Each frequency bin $n \in [1, N]$ corresponds to a multiuser memoryless Gaussian channel with channel gains $h_{kn}^2 \triangleq |H_k(\omega_n)|^2$ and a bandwidth B/N. The problem is now

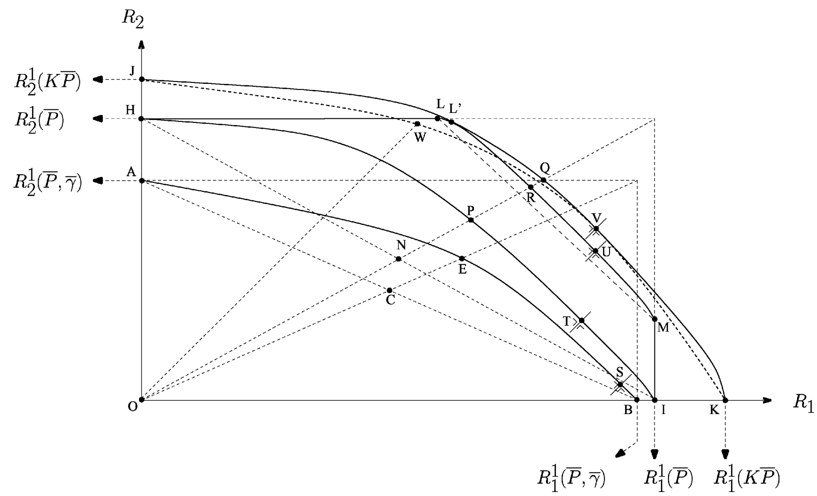

Fig. 7. Various capacity regions for ISI channels.

to find the K optimal power spectra P_{kn} corresponding to a given boundary point of the capacity region. The user rates R_k are computed by summing the partial rates R_{kn} corresponding to the N parallel subchannels.

A. *Types of Constraints*

Talking about the capacity region of a multiuser channel may be confusing, as this region actually depends on the nature of the power constraints. Various types of constraints can be considered, leading to distinct capacity regions. The next items should be taken in consideration in the definition of the capacity region.

- Considering a given user k, there can be one constraint on the total transmitted power $\sum_n(P_{kn})$, or N constraints on the power in each frequency bin P_{kn}. In a wired system, it is usual to impose a transmission mask on the PSD. This makes sense, for example, to solve the problem of electromagnetic compatibility with existing narrowband services (egress problem). Both constraints are also commonly combined. In that case, a total power $\overline{P}$ has to be distributed on the frequency axis, and the power in each frequency bin can not go beyond a given limit $\overline{\gamma}$ with $\overline{\gamma}df > \overline{P}/N$. The two types of constraints will be analyzed separately in the sequel.
- In the uplink, the K different transmitters can be constrained separately or considered as a single "distributed" entity, with a single constraint on the power sum or PSD sum. This makes sense again when the egress problem is considered, as an external receiver gets interference from the K transmitters simultaneously. For example, in a time-division duplex scheme, the signals are produced alternately by the single head-end (during the downstream time slots) and by the K user modems (during the upstream time slots). In the downlink, the sum of the signals intended to the different receivers will be considered as a unique power-constrained signal.

Let us illustrate these considerations on a qualitative example with $K = 2$. Fig. 7 gives the boundaries of various capacity regions. All these regions are valid for both the MA and BC channels, except when explicitly noted.

1) Flat PSD-Constrained Capacity Regions: Points A and B give the single-user rates $R_2^1(\overline{P},\overline{\gamma})$ and $R_1^1(\overline{P},\overline{\gamma})$ associated with a flat PSD constraint $(P_{1n} = 0, P_{2n} = \overline{\gamma} = \overline{P}/N)$ and $(P_{1n} = \overline{\gamma} = \overline{P}/N, P_{2n} = 0)$, respectively. The line ACB bounds the flat-PSD TDMA capacity region; every point on this line can be obtained by time-sharing between the single-user solutions. The line OCE is the locus of balanced-rates solutions. The curve AEB gives the capacity region for a constraint on the PSD sum, $P_{1n}+P_{2n} = \overline{\gamma} = \overline{P}/N$. Point S gives the maximum sum rate, while point E gives the maximum balanced rate. The multiuser gain is obtained here by the ratio OE/OC.

2) Power-Constrained Capacity Regions: Points H and I give the single-user rates $R_2^1(\overline{P})$ and $R_1^1(\overline{P})$ associated with a total power constraint $(\sum_{n=1}^N P_{1n} = \overline{P}, P_{2n} = 0)$ and $(P_{1n} = 0, \sum_{n=1}^N P_{2n} = \overline{P})$, respectively. These rates are, of course, higher than rates A and B as the flat PSD constraint is relaxed. The line HNI bounds the unconstrained-PSD TDMA capacity region. The locus of balanced-rates solutions is here given by the line ONQ. The curve HPI gives the capacity region with a single constraint on the transmitted power sum, $\sum_n(P_{1n} + P_{2n}) = \overline{P}$. The multiuser gain is given here by OP/ON. The curve HLMI bounds the MA capacity region with constraints on the individual powers, $(\sum_{n=1}^N P_{1n} = \overline{P}, \sum_{n=1}^N P_{2n} = \overline{P})$. Only the LM portion of the curve is useful. The multiuser gain is here OR/ON. JQVK gives the capacity region obtained when the total power $2\overline{P}$ can be redistributed arbitrarily to the users. In other words, the constraint on the power sum is here $\sum_n(P_{1n} + P_{2n}) = 2\overline{P}$. This curve is tangent to the HLMI capacity curve at some point L', where the optimal power allocation happens to share the available power $2\overline{P}$ in two equal parts. On the JL$'$ segment, the power allocated to user 1 is lower, while on the L$'K$ segment, it is higher. Finally, the dashed line JVK represents the same MA capacity region, but with an additional FDMA constraint (i.e., each frequency bin is exclusively allocated to one user). Both curves have a common point V with a tangent at 45°. It is well known, indeed, that the maximum sum-rate solution has the FDMA property [3]. The authors of [16] proposed a method to compute the maximum common rate of MA channels with that FDMA constraint, i.e., point W in the figure.

The object of the following sections is to provide efficient algorithms to compute the balanced-rates solutions E, P (or Q), and R, as well as the associated power allocations, for an arbitrary number of users K. Note that these algorithms could be used to compute other boundary points of the capacity regions.

B. Maximum Aggregate Rate for a Given Power Allocation

As the total rates are simply obtained by summing the partial rates in each frequency bin, the maximum aggregate rate R_α for a given power allocation $\{P_{kn}\}$ is given by

$$R_\alpha = \sum_{k=1}^{K} \alpha_k \left(\sum_{n=1}^{N} R_{kn} \right) \tag{52}$$

with

$$(R_{kn})_{\mathbf{MA}} = \frac{1}{N} \mathcal{C} \left(\frac{P_{kn} h_{kn}^2}{\sigma_n^2 + \sum_{l=k+1}^{K} P_{ln} h_{ln}^2} \right) \tag{53}$$

$$(R_{kn})_{\mathbf{BC}} = \frac{1}{N} \mathcal{C} \left(\frac{P_{kn} h_{kn}^2}{\sigma_n^2 + \left(\sum_{h_k^2 < h_l^2} P_{ln} \right) h_{kn}^2} \right). \tag{54}$$

C. Optimal Power Allocation (Given $\{\alpha_k\}$)

We wish now to compute the power allocations $\{P_{kn}^*\}$ and $\{P_{kn}^\circ\}$ maximizing the aggregate rate R_α for the MA and BC frequency-selective channels, respectively. The general strategy used to solve this problem is made of two steps.

1) Find the solution along the frequency axis, i.e., compute the optimal spectral allocation of the power sum $\overline{P}_n = \sum_k P_{kn}$.
2) Find the solution along the multiuser axis, i.e., allocate the power $\overline{P}_n$ to the K users separately in each frequency bin n by using the results of Section IV.

Let us analyze in more detail the three different scenarios.

PSD-sum constraint: $\sum_k \gamma_k(\omega) = \overline{\gamma}(\omega)$.

The spectrum allocation is trivially given by $\overline{P}_n = \sum_k P_{kn} = \overline{\gamma}(\omega_n) df$. The Kuhn–Tucker conditions are

$$\frac{N}{B} \frac{\partial R_\alpha}{\partial P_{kn}} \begin{cases} = \mu_n, & P_{kn} > 0 \\ < \mu_n, & P_{kn} = 0. \end{cases} \tag{55}$$

The power-allocation algorithm is then applied separately in the N frequency bins. A constraint on the PSD sum actually generates independent constraints on the power sum inside every frequency bin.

Power-sum constraint: $\sum_k \sum_n P_{kn} = \overline{P}$.

The Kuhn–Tucker conditions are now

$$\frac{N}{B} \frac{\partial R_\alpha}{\partial P_{kn}} \begin{cases} = \mu, & P_{kn} > 0 \\ < \mu, & P_{kn} = 0. \end{cases} \tag{56}$$

Using (23), the partial derivative for each frequency bin is $\max_k \{D_{kn}(\overline{P}_n)\}$ [the KN functions $D_{kn}(x)$ are defined in the same way as (13)], which provides the solution

$$\mu \overline{P}_n = \max_{k \in [1,K]} \left(1 - \left[\frac{\mu}{\frac{h_{kn}^2}{\sigma^2}} + (1 - \alpha_k) \right] \right)_+ \tag{57}$$

where $(x)_+$ denotes $\max(x, 0)$. The spectral allocation is the multiuser extension of the well-known waterfilling algorithm ([3]–[15]). The solution can be represented in a single waterfilling diagram, with a common water level fixed to 1 and K containers $\mu\sigma^2/h_{kn}^2$ modified by the correction terms $(1 - \alpha_k)$, reflecting the effect of the user relative priorities. The bottom of the resulting multiuser container is the minimum of the K modified individual containers. A simple binary search must be performed to find the right coefficient μ that satisfies the total power constraint. The power-allocation algorithm is then applied separately for the N frequency bins.

Individual power constraint: $\sum_n P_{kn} = \overline{P}_k$ (MA channel only).

The Kuhn–Tucker conditions are

$$\frac{N}{B} \frac{\partial R_\alpha}{\partial P_{kn}} \begin{cases} = \mu_k, & P_{kn} > 0 \\ < \mu_k, & P_{kn} = 0 \end{cases} \tag{58}$$

which is not directly compatible with (12), as the marginal gain for each user is now different. The solution uses the concept of *equivalent channels* [3] or *power prices* [4]. K distinct parameters μ_k have to be found to satisfy the K individual power constraints simultaneously. Once these parameters are known, (29) and (43) can be extended to provide explicit expressions for the optimal power allocation and user rates in each frequency bin. This extension is not given here, due to lack of space.

D. Maximum Balanced Rates

Algorithm 2 can be extended to compute the maximum balanced rates of frequency-selective multiuser channels. Let us analyze in more detail the three different scenarios.

PSD-sum constraint: The Jacobian matrix $\partial \boldsymbol{R}/\partial \boldsymbol{\alpha}$ is now the sum of Jacobian matrices corresponding to each frequency bin. For each term, the $J(n) \times J(n)$ submatrix corresponding to the users active on the frequency bin n is a symmetric tridiagonal matrix $\mathbf{T}_n(\boldsymbol{\alpha})$ with nonzero entries defined in (51)

$$\frac{N}{B} \frac{\partial \boldsymbol{R}}{\partial \boldsymbol{\alpha}} = \sum_{n=1}^{N} \mathbf{T}_n(\boldsymbol{\alpha}). \tag{59}$$

The resulting Jacobian matrix is generally not tridiagonal. The sum should be invertible to allow the computation of the update direction, which requires that each user gets some power in at least one frequency bin.

Power-sum constraint: The total power to be allocated in each frequency bin $\overline{P}_n$ is now dependent on the user relative priorities. From (29), it follows that an additional term $\mathbf{Q}_n(\boldsymbol{\alpha})$ should be included in the expression of $\partial \mathbf{R}_n / \partial \boldsymbol{\alpha}$

$$\frac{N}{B} \frac{\partial \boldsymbol{R}}{\partial \boldsymbol{\alpha}} = \sum_{n=1}^{N} \left[\mathbf{T}_n(\boldsymbol{\alpha}) + \mathbf{Q}_n(\boldsymbol{\alpha}) \right]. \tag{60}$$

Let us define N_k as the number of frequency bins n, such that $k_J(n) = k$ (i.e., the last user in the allocation process (*Algorithm 1*) is user k), with $\sum_k N_k = N$. Combining the waterfilling solution (57) with the power-sum constraint, the marginal rate μ and the total power allocation $\overline{P}_n$ can be expressed as

$$\mu = \frac{\sum_{k=1}^{K} \alpha_k N_k}{\overline{P} + \sigma^2 \sum_{n=1}^{N} h_{k_J(n)}^{-2}} \tag{61}$$

$$\overline{P}_n = \frac{\alpha_{k_J(n)}}{\sum_{k=1}^{K} \alpha_k N_k} \left(\overline{P} + \sigma^2 \sum_{l=1}^{N} h_{k_J(l)}^{-2} \right) - \sigma^2 h_{k_J(n)}^{-2}. \tag{62}$$

Using (29) and (62), the additional term $\mathbf{Q}_n(\boldsymbol{\alpha})$ has the following entries on its k_Jth row:

$$[\mathbf{Q}_n(\boldsymbol{\alpha})]_{k_J(n),k} = \left(\frac{h_{k_J n}^2/\sigma^2}{1 + P_n h_{k_J n}^2/\sigma^2} \right) \frac{\partial P_n}{\partial \alpha_k} = \frac{1}{\alpha_{k_J(n)}} \left(\delta_{k_J(n),k} - \frac{\alpha_{k_J(n)} N_k}{\sum_{l=1}^{K} \alpha_l N_l} \right). \tag{63}$$

The $K-1$ other rows of $\mathbf{Q}_n(\boldsymbol{\alpha})$ are all zero. The algorithm for the computation of the maximum balanced rates is then identical to the original algorithm, except that a new water level μ and a new spectral allocation $\{\overline{P}_n\}$ have to be computed by (57) for each new set of priority coefficients $\boldsymbol{\alpha}^{(i+1)}$.

Individual power constraint: The computation of maximum balanced rates is much more complex in this case, as the optimal vectors $\boldsymbol{\alpha}$ and $\boldsymbol{\mu}$ have to be found that simultaneously satisfy the K balanced-rates equations and the K individual power constraints. Practical algorithms will be given in a later paper (see [20] and [21]), but simulation results corresponding to this scenario are already included in Section VI of this paper for the purpose of comparison.

VI. Results

To illustrate the important multiuser gain that can be expected in a practical scenario, we consider the 20-user network example introduced in Section II. The channel frequency responses $|H_k(\omega)|^2$ are given by Fig. 2. The noise level $N_0/2$ is chosen as -120 dBm/Hz.

Two kinds of power constraints are put on the single-user transmission power: a maximum transmission PSD of $\overline{\gamma} = -60$ dBm/Hz (flat PSD), or a maximum transmission power of $\overline{P} = \overline{\gamma} B = 10$ mW (waterfilling PSD). Fig. 8 gives the evolution of the single-user capacities R_k^1 for the two kinds of power constraints. The single-user capacity decreases rapidly for the remote users. As expected, the waterfilling solution provides a higher rate with respect to the flat PSD solution only for the users with a low received signal-to-noise ratio (SNR), i.e., the most remote users.

In a multiuser scenario, a subset $\mathcal{C}_K \subset \{1, \ldots, \overline{K}\}$ of $K \leq \overline{K}$ active users transmit their signals simultaneously. Table I gives the maximum balanced rates for $\mathcal{C}_4 = \{5, 10, 15, 20\}$ and different types of constraints, and compares them with the Single User (SU) and TDMA rates. The sixth column gives the ratio of the rates obtained in the multiuser configuration with respect to the corresponding single-user rates, expressed in percentage: this ratio is the same for all users as the balanced-rate condition is satisfied. The last column gives the corresponding multiuser gain, as defined in (3).

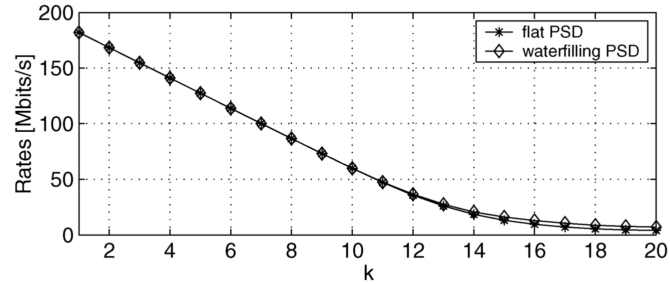

Fig. 8. Single-user rates versus user index k.

TABLE I
MAXIMUM BALANCED RATES (IN Mb/s) FOR $\mathcal{C}_4 = \{5, 10, 15, 20\}$

Constraint	User 5	User 10	User 15	User 20	%	g
Flat SU	127.41	59.82	13.12	4.05	100	
PSD-sum	71.52	33.58	7.36	2.27	56	2.24
Flat TDMA	31.85	14.96	3.28	1.01	25	1
SU	127.41	59.85	16.04	7.06	100	
Power-sum ($4\overline{P}$)	84.47	39.68	10.63	4.68	66	2.65
Individual power	77.94	36.61	9.81	4.32	61	2.45
Power-sum ($\overline{P}$)	65.99	31.00	8.31	3.66	52	2.07
TDMA	31.85	14.96	4.01	1.77	25	1

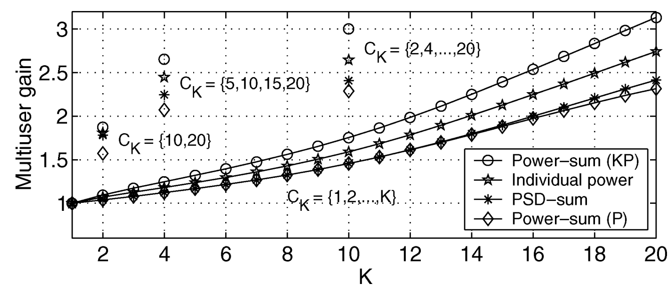

Fig. 9. Multiuser gain versus number of active users K.

Fig. 9 gives the multiuser gain g versus K for various sets of active users $\mathcal{C}_K$. The main curves correspond to $\mathcal{C}_K = \{1, 2, \ldots, K\}$, i.e., the $K \leq \overline{K}$ closest modems are supposed to be active. The multiuser gain increases with K, and is much larger when the user channels in $\mathcal{C}_K$ are very different. Comparing these different curves, we get some insight in the respective gains that can be expected from the following.

- The simultaneous transmission by all users (with the same total power $\overline{P}$) instead of transmission in separate time slots.
- The allocation of $K\overline{P}$ to the users ($\overline{P}$ for each) instead of $\overline{P}$.
- The distribution of the total power $K\overline{P}$ in unequal parts.

Fig. 10 gives the results of the optimal MA and BC power allocations (normalized with respect to $\overline{\gamma}$) for $\mathcal{C}_4 = \{5, 10, 15, 20\}$ and a constraint on the transmitted power sum $\overline{P}$. The normalized optimal PSD sum is also given (upper lines). As expected, the users with the weakest channels concentrate their transmission power in the best parts of the frequency spectrum, while the users with the best channels (and lowest priorities) can make use of the remaining frequencies. The remote users require more power in the BC (where they

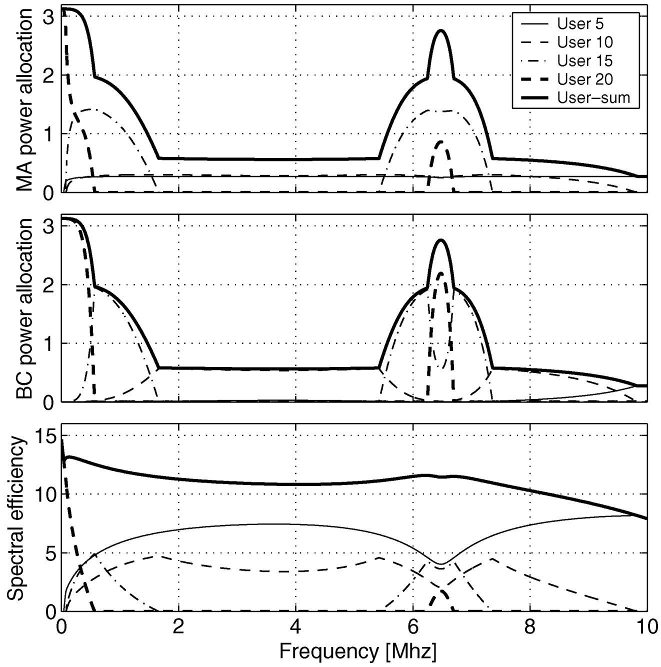

Fig. 10. Optimal power allocations and spectral efficiency for $\mathcal{C}_4 = \{5, 10, 15, 20\}$ and a power-sum constraint.

are decoded first) than in the MA channel (where they are decoded last). The lower part of the figure illustrates the spectral efficiency in b/s/Hz.

VII. Conclusion

This paper introduces the concept of balanced capacity to characterize the performance of multiuser channels. The uplink (MA channel) and downlink (BC) balanced capacities of wireline MA networks are computed and compared for an arbitrary number of users. A detailed analysis of the optimal power allocation in multiuser memoryless channels is first given, allowing the derivation of an iterative algorithm for the computation of maximum balanced rates. This algorithm is then extended to the case of frequency-selective multiuser channels. Two kinds of power constraints (with increasing complexity) are considered: a constraint on the total transmitted PSD, and a constraint on the total transmitted power. Results are provided for a wireline access network with $K = 20$ users.

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