Optimization of linear pre/decoders for multi-user closed-loop MIMO-OFDM

Jonathan Duplicy[1], Jérôme Louveaux[2], and Luc Vandendorpe[1]

[1]Communications and Remote Sensing Laboratory
Université catholique de Louvain, Belgium
{duplicy,vandendorpe}@tele.ucl.ac.be.

[2]Circuit & Systems, Dept. of Electrical Eng.
Delft University of Technology, The Netherlands
louveaux@cas.et.tudelft.nl

***Abstract*— This paper deals with a multi-user downlink MIMO-OFDM scheme with perfect channel knowledge at both the transmit and receive sides. The objective is to optimize the average BER while fulfilling each user's rate and satisfying a global transmit power constraint. Space Division Multiple Access (SDMA) is investigated and three new iterative schemes are proposed to compute the linear pre/decoders. The results exhibit, inter alia, that orthogonalizing the channels does not lead to the best results.**

***Index Terms*— Multi-user, Downlink, SDMA, MIMO, OFDM.**

I. Introduction

Recently, MIMO (Multi-Input Multi-Output) systems arising from the use of spatial diversity at both the transmitter and the receiver have drawn considerable attention since they offer a large potential capacity increase compared with single antenna systems [1][2]. Concurrently, OFDM (Orthogonal Frequency Division Multiplexing) has also encountered an increasing popularity since it provides a low complexity solution to the intersymbol interference (ISI) induced by frequency selective channels.

Various schemes have been recently proposed to combine the advantages of both the MIMO system and the OFDM modulation. Most of these schemes assume perfect channel state information (CSI) at the receiver but no channel knowledge at the transmitter. In a number of applications, CSI is also available at the transmitter. It can be achieved either by means of a feedback channel or by estimating the received channel and exploiting reciprocity property when applicable. Assuming channel state information at the transmitter (CSIT) allows to improve performance significantly by optimally allocating resources [3][4].

Multi-user OFDM has been well-studied for the SISO (Single-Input Single-Output) case [5] and for the MISO (Multiple-Input Single-Output) case [6], but remains largely unexplored for the MIMO case. In [7], Doostnejad et al. introduced space-frequency spreading codes for the downlink with no or second-order CSIT. [8] proposed a no-CSIT Multiple access scheme which allows to gradually vary the amount of user collision in signal space by assigning different subsets of the available OFDM tones to different users. Finally, assuming perfect CSIT, [10] presented an adaptive scheme for the QoS-constrained uplink scenario.

In this paper, we consider a multi-user downlink MIMO-OFDM scheme with perfect CSI at both the base station (BS) and the mobile terminals. Our objective is to optimize the mean BER while fulfilling each user's rate and satisfying a global transmit power constraint. To achieve high spectral efficiency, SDMA, in which each OFDM tone is shared simultaneously by the whole set of users, is investigated. A linear process is used at both sides of the link and three new iterative schemes are proposed to compute the pre/decoders.

II. System model

A. Notations

Throughout this paper, we will use the following notations. $A^\dagger$ denotes the conjugate transpose (Hermitian) of a matrix A, $\mathrm{tr}\{A\}$ denotes its trace ; $\mathrm{diag}\,(A_1\ A_2\ \ldots\ A_n)$ builds a block diagonal matrix made of matrices A_i. I is the identity matrix and $E\{.\}$ is the mathematical expectation operator.

B. Channel model

The frequency selective MIMO channels are modeled as tap delay lines :

$$h_{ij}(t) = \sum_{l=0}^{L_{ij}-1} \beta_{ij}(l)\,\delta(t-\tau_{ij}(l)) \qquad \begin{cases} 1 \le i \le N_r \\ 1 \le j \le N_t \end{cases}, \quad (1)$$

where N_r and N_t are the number of receive and transmit antennas. $\tau_{ij}(l)$ and $\beta_{ij}(l)$ are respectively the delay and the complex amplitude associated with the l^{th} path. The $\beta_{ij}(l)$'s are modeled as zero-mean, complex Gaussian random variables with variances σ^2_{lij}. These variances are normalized so that $\sum_{l=0}^{L_{ij}-1}\sigma^2_{lij}=1\ \ \forall i,j$. Finally, we suppose that the channels are invariant during each OFDM block, but are allowed to vary from block to block.

C. Single-user case

To deal with the frequency-selectivity of the channel, an N-tone OFDM modulation is used. Assuming convenient cyclic prefix and perfect synchronization to avoid inter-carrier/symbol interference, the system model for tone n can be written as :

$$r_n = H_n x_n + v_n \qquad 1 \le n \le N, \quad (2)$$

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where $x_n \in \mathbb{C}^{N_t \times 1}$ is the transmitted signal vector. $r_n, v_n \in \mathbb{C}^{N_r \times 1}$ are respectively the received signal vector and the additive white Gaussian noise with correlation matrix R_n. Moreover, we assume $E\{v_n v_m^\dagger\} = 0_{(N_r \times N_r)}$ for $n \neq m$. Finally, $H_n \in \mathbb{C}^{N_r \times N_t}$ is the frequency response of the n^{th} subcarrier whose elements are given by :

$$(H_n)_{ij} = \sum_{l=0}^{L_{ij}-1} \beta_{ij}(l) e^{-j2\pi n \tau_{ij}(l)/N}. \tag{3}$$

We assume perfect channel knowledge at both sides of the communication link and we use a linear processing illustrated by Fig. 1. For each subcarrier, the N_t antennas transmit a linearly precoded version of the *same* N_{sn} symbols. Noisy received symbols are then decoded to give an estimate of the transmitted symbols. Hence, the estimated symbols for subcarrier n are :

$$\hat{s}_n = G_n(H_n F_n s_n + v_n) \ , \tag{4}$$

where $s_n \in \mathbb{C}^{N_{sn} \times 1}$ is the vector the of transmitted symbols, $F_n \in \mathbb{C}^{N_t \times N_{sn}}$ is the precoding matrix and $G_n \in \mathbb{C}^{N_{sn} \times N_r}$ is the decoding matrix. Additionally, we impose the following transmit power constraint :

$$\sum_{n=1}^{N} \mathrm{tr}\left\{F_n {F_n}^\dagger\right\} \leq P_t \ . \tag{5}$$

Optimizing F_n and G_n subject to the power constraint is a well-studied problem and it will not be discussed in this paper. We refer the reader to [3] for a complete overview. We restrict to non-adaptive schemes : $N_{sn}{=}N_s \ \forall n$. Furthermore, we do not consider coding across subcarriers (frequency coding) since it implies a much higher computational burden.

D. Multi-user case

This paper focuses on a multi-user downlink SDMA scheme. The base station is simultaneously transmitting to the U remote stations on the whole set of subcarriers. The estimated symbol vector for subcarrier n of user u is therefore given by :

$$\hat{s}_n^u = G_n^u \left(H_n^u \sum_{j=1}^{U} F_n^j s_n^j + v_n^u \right) \ . \tag{6}$$

The challenge is to compute F_n^u and $G_n^u \ \forall n, u$ to optimize the mean BER while satisfying the following global power constraint :

$$\sum_{u=1}^{U} \sum_{n=1}^{N} \mathrm{tr}\left\{F_n^u {F_n^u}^\dagger\right\} \leq P_t \ . \tag{7}$$

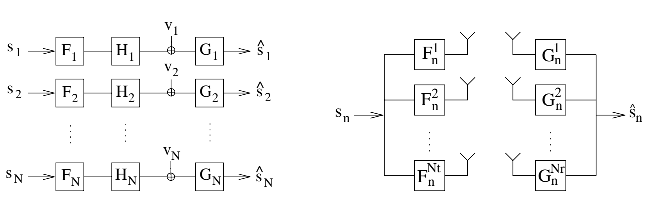

Fig. 1. Single-user case : global system (left) and detail for subcarrier n (right).

Weight optimization is centralized at the base station where perfect CSI is assumed and the rx-weights are transmitted to the users via control channels. In this way, the remote stations complexity can be made very low.

III. Proposed schemes

A. Orthogonalization

An easy way to deal with the multi-user interference (MUI) induced by SDMA is to annihilate it by orthogonalizing the channels as proposed in [13] for the single carrier case. For each subcarrier, we rewrite the system model in the following concatenated way (with an aim of conciseness, we leave out the subcarrier index n) : $\hat{s} = G\left[HFs + v\right]$:

$$\begin{pmatrix} \hat{s}^1 \\ \hat{s}^2 \\ \vdots \\ \hat{s}^U \end{pmatrix} = \mathrm{diag}\begin{pmatrix} G^1 \\ G^2 \\ \vdots \\ G^U \end{pmatrix} \cdot \left[\begin{pmatrix} H^1 \\ H^2 \\ \vdots \\ H^U \end{pmatrix} \begin{pmatrix} F^1 \\ F^2 \\ \vdots \\ F^U \end{pmatrix}^T \begin{pmatrix} s^1 \\ s^2 \\ \vdots \\ s^U \end{pmatrix} + \begin{pmatrix} v^1 \\ v^2 \\ \vdots \\ v^U \end{pmatrix} \right] \tag{8}$$

As shown by the left side of Fig. 2, we require that symbols from user u generate zeros on each antenna of the other $U-1$ users. Mathematically, it means that F^u is in the null space of $\underline{H}^u$, the concatenated channel matrix where lines related to user u have been removed :

$$\underline{H}^u F^u = 0_{\left(\left(\sum_{j\neq u} N_r^j\right) \times N_s^u\right)} \quad \equiv \quad F^u \in \mathrm{null}\{\underline{H}^u\} \qquad \forall u \ . \tag{9}$$

To achieve this, we split the precoding matrix F, in two matrices F_A and F_B :

$$F = F_A F_B = \left(F_A^1 \ F_A^2 \ \ldots \ F_A^U\right) \mathrm{diag}\left(F_B^1 \ F_B^2 \ \ldots \ F_B^U\right). \tag{10}$$

F_B^u is a $\left(\left(N_t - \sum_{j\neq u} N_r^j\right) \times N_s^u\right)$ matrix whereas F_A^u is a $\left(N_t \times \left(N_t - \sum_{j\neq u} N_r^j\right)\right)$ matrix, basis of the null space of $\underline{H}^u$. Once F_A^u matrices are calculated, we revert to single-user systems with enhanced channels ($H^u F_A^u$) and we can apply classical solutions [3] to compute G^u and F_B^u matrices.

As can be deduced from (9) and (10), this technique imposes very restrictive feasibility conditions given by (11).

$$\left\{ \begin{array}{l} N_s^u \leq N_r^u \\ N_s^u \leq N_t - \sum_{j\neq u} N_r^j \qquad \forall u \end{array} \right. \tag{11}$$

For example, a system with $N_t{=}4$ and $N_r^u{=}3$, $N_s^u{=}1 \ \forall u$ cannot accomodate more than 2 users. However, capacity can be greatly improved by the method we propose hereafter.

Improvement: Indeed, imposing conditions (9) is over-designed. The right side of Fig. 2 shows the principle of our new method : zeros are now required after linear decoding of the other $U-1$ users and not before. Hence the availability conditions turn into :

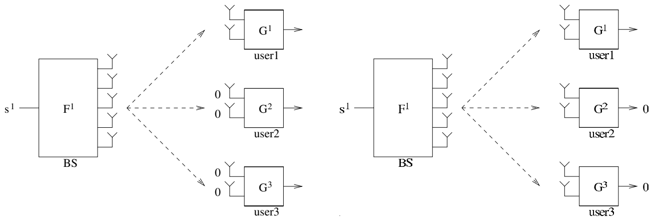

Fig. 2. Orthogonalization principles : simple scheme (left) and improved scheme (right).

$$\left\{ \begin{array}{l} N_s^u \leq N_r^u \\ N_s^u \leq N_t - \sum_{j \neq u} N_s^j \qquad \forall u \; . \end{array} \right. \tag{12}$$

Applying this scheme to our example (N_t=4 and N_r^u=3, N_s^u=1 $\forall u$) doubles the capacity of the system : 4 users can now be accommodated simultaneously compared with only 2 users for the simple scheme. Unfortunately, F_A is needed to compute F_B and G and vice versa, hence we have to resort to an iterative processing given by table I. Note that G_n^u weights initialization is made with the single-user case.

Interference-free transmission constraints strictly the maximum number of accommodated users. If we want to remove this capacity constraint, we have to allow MUI. Direct optimization in terms of BER is not tractable, we propose therefore two suboptimal schemes in the two following sections.

B. min-MSE

In this section, we aim at minimizing the overall Mean Square Error (MSE) of the system subject to the transmit power constraint, i.e. :

$$\min_{\substack{F,G \\ s.t.(7)}} \quad MSE = \sum_{u=1}^{U} \sum_{n=1}^{N} MSE_n^u \tag{13}$$

The MSE for subcarrier n of user u is given by

$$MSE_n^u = E\left\{ \mathrm{tr}\left\{ E_n^u E_n^{u\dagger} \right\} \right\}, \tag{14}$$

where E_n^u is the error vector :

$$E_n^u = G_n^u \left(H_n^u \sum_{j=1}^{U} F_n^j s_n^j + v_n^u \right) - s_n^u \; . \tag{15}$$

Simple algebra leads to :

$$MSE_n^u = \mathrm{tr}\left\{ G_n^u H_n^u \left(\sum_{j=1}^{U} F_n^j F_n^{j\dagger} \right) H_n^{u\dagger} G_n^{u\dagger} - F_n^{u\dagger} H_n^{u\dagger} G_n^{u\dagger} - G_n^u H_n^u F_n^u + I + G_n^u R_n^u G_n^{u\dagger} \right\}. \tag{16}$$

We resort to the Lagrange multipliers technique to solve problem (13). The corresponding Lagrangian is given by

$$L = \sum_{u=1}^{U} \sum_{n=1}^{N} MSE_n^u + \mu \left(\sum_{u=1}^{U} \sum_{n=1}^{N} \mathrm{tr}\left\{ F_n^u F_n^{u\dagger} \right\} - P_t \right). \tag{17}$$

Forcing to zero the partial derivatives relatives to F_n^u and G_n^u with all other variables fixed, we easily get :

$$G_n^u = F_n^{u\dagger} H_n^{u\dagger} \left[R_n^u + \sum_{j=1}^{U} H_n^u F_n^j F_n^{j\dagger} H_n^{u\dagger} \right]^{-1} \tag{18}$$

$$F_n^u = \left(\mu I + \sum_{j=1}^{U} H_n^{j\dagger} G_n^{j\dagger} G_n^j H_n^j \right)^{-1} H_n^{u\dagger} G_n^{u\dagger} \; , \tag{19}$$

where μ is the waterlevel chosen to satisfy the power constraint with equality.

Naturally, transmit weights rely on receive weights and vice versa, hence an iterative processing is required and given in table II. This algorithm decreases the MSE at each step. Nevertheless, convexity is not assured and convergence to a local minimum is possible. Note that a similar algorithm has been derived for the single-carrier case in [9], but convergence to local minima was also encountered.

C. max-min-SINR

The average BER is mainly dominated by substreams with the lowest SINRs, so it makes sense to maximize the minimum of SINRs among all substreams of all subcarriers :

$$\max_{\substack{F,G \\ s.t.(7)}} \quad \min_{k,n,u} SINR_{kn}^u \; , \tag{20}$$

where $k \in [1, N_s^u]$ designs the substream index. Obviously, choosing this criterion leads to achieving equal SINRs, i.e.

$$SINR_{kn}^u = SINR_0 \qquad \forall k, n, u \; . \tag{21}$$

TABLE I

IMPROVED ORTHOGONALIZATION ALGORITHM.

$\forall n, u$

1. initialize G_n^u with the single-user case
2. compute ${F_A}_n^u$ for enhanced channel $G_n^u H_n^u$
3. compute G_n^u and ${F_B}_n^u$ satisfying (7) for enhanced channel $H_n^u {F_A}_n^u$
4. go to step 2 until convergence

TABLE II

min-MSE ALGORITHM.

$\forall n, u$

1. initialize G_n^u with the single-user case
2. compute F_u^n like (19) with μ satisfying (7)
3. compute G_n^u like (18)
4. go to step 2 until convergence

Problem (20) has recently been solved for the MISO single-carrier case [12]. However, it can be extended to our multi-carrier MIMO case. Passing from the single-carrier to the multi-carrier case is simply done by regarding the N sub-carriers of the U users like $N \times U$ users with no interference between tones with different indexes. Extension to the MIMO case is tougher and requires an iterative process given by table III and which is similar to previous ones. Thanks to [12], the algorithm converges to the global optimum.

IV. SIMULATION RESULTS

In this section, we provide some simulation results illustrating the schemes described above. We choose the HiperLAN2 channel model A (τ_{rms}=50ns) which corresponds typically to a building of offices without line of sight [11]. For purpose of simplicity, we assume no correlation, i.e. $\beta_{ij}^u(l)$ and $\tau_{ij}^u(l) \perp i, j, l, u$ in (1). White noise is also assumed to be uncorrelated : $R_n^u = \sigma^2 I \; \forall n, u$. Moreover, the 20MHz channel is turned into N=64 subchannels by OFDM modulation and 4-QAM Gray-mapped constellations are used.

We restrict to the single-symbol case, i.e. N_s^u=1 $\forall u$. For the orthogonalizing schemes we use the single-user *max-MSE* criterion to compute tx-rx weights [3]. This criterion aims at minimizing the maximum of the MSEs over all subcarriers. Although it is not BER-optimal, this method is much more computationally attractive ; the performance degradation is marginal [3].

Results will be reported by means of plots giving the BER averaged among subcarriers and users versus the average SNR at each receive antenna. The BER averages were calculated over at least 10.000 channels realizations with 1 OFDM block being transmitted per channel realization.

Orthogonalization : Fig. 3 shows the average BER for the orthogonalization (*ortho1*) and improved orthogonalization (*ortho2*) schemes and a 2-equal-rate-user system with N_t=4 and N_r^1=N_r^2=2, 3. For *ortho1*, we observe paradoxically that the system with three receive antennas gives worse results than the system with two receive antennas, which we do not observe for *ortho2*. Indeed, as summarized by table IV, *ortho1* applied to the present scenario turns the 4×2 and 4×3 channels into 2×2 and 1×3 channels, respectively. This explains why users should preferably not use a third antenna. On the contrary, *ortho2* preserves the benefit of using an increasing number of receive antennas.

TABLE III

max-min-SINR ALGORITHM.

$\forall n, u$

1. initialize G_n^u with the single-user case
2. compute F_n^u by [12] satisfying (7)
3. compute SINR-optimal G_n^u (18)
4. go to step 2 until convergence

Fig. 3 features also different power allocation schemes. For *ortho1*, the uniform scheme allocates the same power to *each user*, while the non uniform one allocates power like (22) where λ_n^u is the maximum singular value of the enhanced channel $H_n^u {F_A}_n^u$. For *ortho2*, the uniform scheme allocates the same power to *each subcarrier* of each user : $P_n^u = P_t/(N \cdot U) \quad \forall n, u$ whereas the non uniform allocation is given by (23). The non uniform schemes take into account the quality of the channels by allocating more power to the worst channels. It can be observed that the gain for *ortho1* is almost negligible whereas the *ortho2* gain is significant. It simply comes from the fact that for *ortho1*, the allocation is made for the whole set of subcarriers whereas it is made on a subcarrier basis for *ortho2*.

$$P^u = \frac{P_t \cdot N}{\sum_{k=1}^{U} \left(\frac{1}{\sum_{n=1}^{N} \lambda_n^k} \right)} \cdot \frac{1}{\sum_{n=1}^{N} \lambda_n^u} \tag{22}$$

$$P_n^u = \frac{P_t \cdot N}{\sum_{k=1}^{U} \left(\frac{1}{\sum_{n=1}^{N} \lambda_n^k} \right)} \cdot \frac{1}{\lambda_n^u} \tag{23}$$

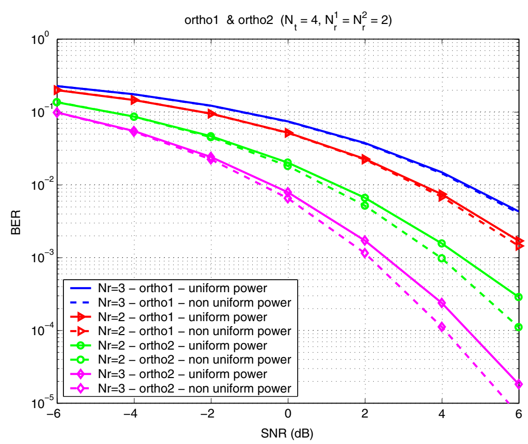

Fig. 3. *ortho1* and *ortho2* for various numbers of receive antennas and power allocation schemes.

min-MSE & ***max-min-SINR*** : Fig. 4 and 5 show the mean BER for various numbers of iterations for the same 2-user system as above with N_r^1=N_r^2=2. We can observe the good convergence properties of both algorithms. *min-MSE* converges slightly more rapidly than *max-min-SINR* but remember that this method may converge to a local minimum whereas

TABLE IV

CHANNELS DIMENSIONS BEFORE AND AFTER ORTHOGONALIZATION.

original ch.	ch. after *ortho1*	ch. after *ortho2*
4×2	2×2	3×2
4×3	1×3	3×3

the latter converges to the global optimum. By comparing the two figures, one can also notice that *max-min-SINR* achieves better performances as will be discussed in the next section.

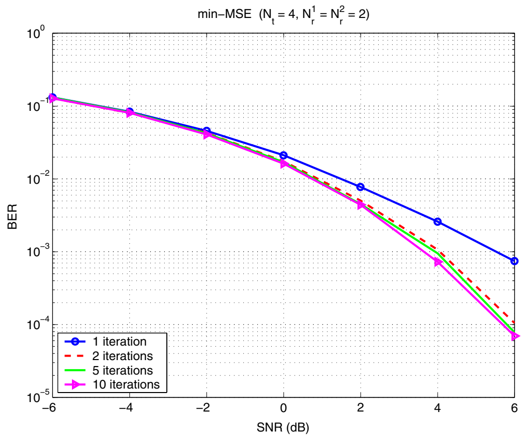

Fig. 4. *min-MSE* algorithm for various numbers of iterations.

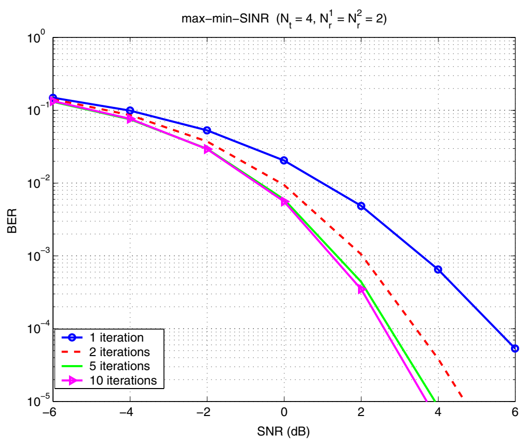

Fig. 5. *max-min-SINR* algorithm for various numbers of iterations.

V. Discussion and Conclusion

Fig. 6 provides an overall comparison of the presented schemes. A 3-user system with N_t=5 and N_r^u=2 $\forall u$ is now investigated. Note that the same conclusions can be drawn from the 2-user system by comparing figures 3 to 5. We used 10 iterations for *max-min-SINR* and *min-MSE* and a non uniform power allocation for both *ortho1* and *ortho2*.

As we can observe, the differences between the schemes are huge but complexity issues must be taken into account. Due to its stringent orthogonality constraints, *ortho1* leads to the worst results. The 5×2 channels are turned into 1×2 channels whereas *ortho2* turns them into 3×2 channels. However, *ortho1*'s complexity is much lower since it does not require any iterative process. Secondly, *min-MSE* and *ortho2* achieve close performances but the latter must be favored since *min-MSE* may converge to a local mininum. Finally, while requiring a computational burden similar to *min-MSE*, *max-min-SINR* exhibits much better performances. It comes from the fact that the mean BER is mainly dominated by the subcarriers with the worst SINR, which is optimized by *max-min-SINR* in contrast to *min-MSE*, which makes a global optimization.

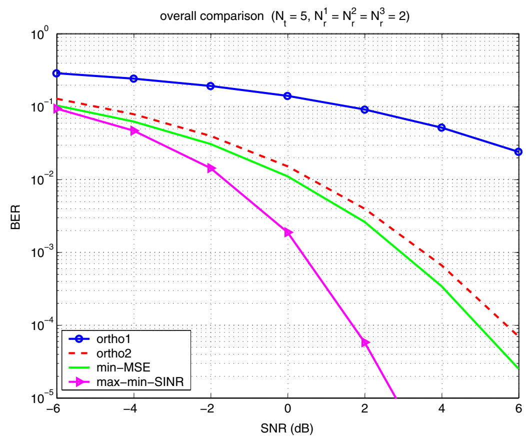

Fig. 6. Overall comparison of the presented schemes.

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