Localization and Tracking of a Mobile Target for an Isogradient Sound Speed Profile

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Abstract—**In this paper, we analyze the problem of localizing and tracking a mobile node in an underwater environment with an isogradient sound speed profile (SSP). We will show that range-based localization algorithms are not so accurate in such an environment, and they should be replaced by time-based ones. Therefore, we relate the mobile node location to the travel time of a propagating sound wave from (to) an anchor node to (from) the mobile node. After obtaining sufficient time measurements, positioning can be achieved through multilateration. To accomplish this, we utilize the extended Kalman filter (EKF) for multilateration and tracking the mobile node's location in a recursive manner. Through several simulations, we will show that the proposed EKF algorithm performs superb in comparison with algorithms which assume a straight-line wave propagation in an underwater environment.**

I. Introduction

Exploring underwater environments has a variety of applications; early warning systems for natural disasters (e.g. tsunamis), ecosystem monitoring, oil drilling and military surveillance are a few examples of such applications [1]. Recently, advances in wireless sensor networks (WSNs) motivated system designers to think about using underwater acoustic sensor networks (UASNs) for data gathering and ocean explorations. To obtain a meaningful interpretation of the sensed data, we need the sensor positions either remotely or locally as in terrestrial WSNs. However, underwater acoustic communications is very challenging, and it is different from terrestrial wireless communications. Very low bit rate, low link quality, multi-path, time variability, and a depth-dependent sound speed profile (SSP) are the most challenging characteristics of underwater communications [2]. These characteristics also affect localization and tracking algorithms using UASNs. This necessitates the design and development of new localization and tracking algorithms.

In [3], the authors propose a centralized algorithm to overcome the severe multi-path property of the underwater environment caused by scattering from the seabed and ocean surface. In [4], a time difference of arrival based localization scheme for stationary UASNs is proposed which does not need time synchronization among network nodes. In [5], depth information as well as range measurements are used to localize a mobile node inside a three-dimensional (3D) area.

As stated before, one of the underwater localization challenges is the non-uniform SSP which varies with temperature, pressure, and salinity. Due to this property, an acoustic ray does not propagate along a straight line, but it bends. Even if the nodes are located at the same depth the distance between the two nodes in an underwater environment is not proportional to the wave travel time. However, in all the above mentioned underwater localization schemes, not only the propagation sound speed is assumed constant, but also the path of the sound ray is assumed to be a straight line, and these assumptions degrade the performance of underwater localization algorithms.

In contrast to the aforementioned algorithms, [6] considers a non-straight line propagation for range measurement. The depth information and SSP are used to compute the true horizontal distance between two nodes. Similar to [6], the authors of [7] consider a real wave propagation model for UASNs localization. They eliminate the underwater range computation by using a look up table (LUT), which relates the travel time information to the horizontal distance between two nodes. Recently, [8] evaluates the localization performance degradation of the straight-line propagation model compared to real propagation model. The main drawback of this work is the computational complexity, since the introduced algorithm is based on a numerical approach and uses exhaustive search.

In this work, we propose a UASN localization and tracking approach for an underwater medium with an isogradient SSP. Such an isogradient SSP can be found in a deep underwater environment [9][10]. To find a node's location, we analytically relate the position of a mobile node to the time of flight (ToF), and then, using at least four time measurements from four anchors we find the position of a node through multilateration. Since tracking is also important, we perform multilateration recursively by using the extended Kalman filter (EKF). The rest of the paper is organized as follows. In Section II, we explain how the position of two nodes is related to the ToF. We describe the localization/tracking network system in Section III. Next, in Section IV, we propose to use a EKF for underwater localization/tracking, and we evaluate its performance through several simulations in Section V. Finally, conclusions are drawn in Section VI.

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II. Exact Time of Flight Computation

We consider the problem of tracing a ray between a mobile and an anchor node in a 3D environment with an isogradient sound speed where the SSP is only dependent on depth, and has the following form

$$C(z) = b + az, \tag{1}$$

where z denotes the depth, b indicates the sound speed at the surface, and a is a constant depending on the environment. Without loss of generality for the ToF computation between the two nodes, we assume that the z axis crosses the anchor node. Therefore, due to the cylindrical symmetry around the z axis we can transfer the ToF computation problem into the plane which includes both nodes and the z axis as is shown in Fig. 1. In this figure, $r^{\mathrm{M}} - r^{\mathrm{A}}$ represents the horizontal distance between the anchor and the mobile node, and it can be written as

$$r^{\mathrm{M}} - r^{\mathrm{A}} = \sqrt{(x^{\mathrm{M}} - x^{\mathrm{A}})^2 + (y^{\mathrm{M}} - y^{\mathrm{A}})^2}, \tag{2}$$

where x^{M}, y^{M}, x^{A}, and y^{A} indicate the x-coordinate and y-coordinate of respectively the mobile node and anchor node in a 3D environment. Since the z-axis is assumed to cross the anchor node we have $r^A = 0$, but we keep it in our formulation for representation purposes.

Acoustic propagation is usually modeled using a ray-tracing approach which is a valid approximation for the aforementioned isogradient SSP underwater environment. Ray-tracing is guided by Snell's law which is given by [7]

$$\frac{\cos\theta}{C(z)} = \frac{\cos\theta^{\mathrm{A}}}{C(z^{\mathrm{A}})} = \frac{\cos\theta^{\mathrm{M}}}{C(z^{\mathrm{M}})} = l; \qquad \theta \in \left[\frac{-\pi}{2}, \frac{\pi}{2}\right], \tag{3}$$

where θ^{M} and θ^{A} are the ray angles at the mobile node and anchor node locations, respectively, as illustrated in Fig. 1. z^{A} and z^{M} represent the depth of the anchor node and the mobile node, respectively, and l is constant along a ray traveling between the nodes. Moreover, the parameters θ and z represent the angle and depth of a given point along the ray. From Fig. 1, we can write

$$dr = \frac{dz}{\tan\theta}, \tag{4a}$$

$$dt = \frac{ds}{C(z)} = \frac{dz}{C(z)\sin\theta}, \tag{4b}$$

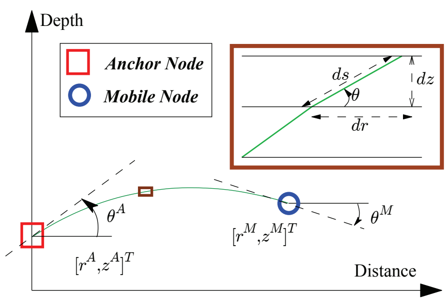

Fig. 1. Ray path between a mobile node and an anchor node.

where s is the arc length of a ray traveling between the mobile node and the anchor node, and t is the wave travel time between these two nodes. Moreover, using (1) and (3), and by taking derivatives with respect to (w.r.t) z and θ, we can write

$$dz = -\frac{1}{la}\sin\theta \; d\theta. \tag{5}$$

Now, by substituting (5) into (4a), and integrating w.r.t θ we have

$$r^{\mathrm{M}} - r^{\mathrm{A}} = -\frac{1}{al}\left(\sin\theta^{\mathrm{M}} - \sin\theta^{\mathrm{A}}\right), \tag{6}$$

for the horizontal distance, and for the vertical distance between the two nodes we can write

$$z^{\mathrm{M}} - z^{\mathrm{A}} = \frac{1}{al}\left(\cos\theta^{\mathrm{M}} - \cos\theta^{\mathrm{A}}\right). \tag{7}$$

Dividing (7) by (6), considering $r^{\mathrm{M}} \neq r^{\mathrm{A}}$ we end up with

$$\frac{z^{\mathrm{M}} - z^{\mathrm{A}}}{r^{\mathrm{M}} - r^{\mathrm{A}}} = -\frac{\cos\theta^{\mathrm{M}} - \cos\theta^{\mathrm{A}}}{\sin\theta^{\mathrm{M}} - \sin\theta^{\mathrm{A}}}, \text{ for } r^{\mathrm{M}} \neq r^{\mathrm{A}}. \tag{8}$$

Furthermore, by substituting (1) into (3) we can write

$$\frac{b + az^{\mathrm{M}}}{b + az^{\mathrm{A}}} = \frac{\cos\theta^{\mathrm{M}}}{\cos\theta^{\mathrm{A}}}. \tag{9}$$

Now, by applying the change of variables $\theta^{\mathrm{A}} = \beta_0 + \alpha_0$, and $\theta^{\mathrm{M}} = \beta_0 - \alpha_0$, (8) and (9) can be modified to

$$\frac{z^{\mathrm{M}} - z^{\mathrm{A}}}{r^{\mathrm{M}} - r^{\mathrm{A}}} = \tan\beta_0, \text{ for } r^{\mathrm{M}} \neq r^{\mathrm{A}}, \tag{10}$$

$$\frac{b + az^{\mathrm{M}}}{b + az^{\mathrm{A}}} = \frac{1 - \tan\beta_0\tan\alpha_0}{1 + \tan\beta_0\tan\alpha_0}. \tag{11}$$

For the exceptional condition where $z^{\mathrm{A}} = z^{\mathrm{M}}$, (11) is not valid and should be modified into

$$\tan\alpha_0 = \frac{1}{2}\frac{a(r^{\mathrm{M}} - r^{\mathrm{A}})}{b + az^{\mathrm{M}}}, \text{ for } z^{\mathrm{M}} = z^{\mathrm{A}}, \tag{12}$$

which is extracted from (6). Now, by integrating (4b) w.r.t θ, the ToF can be calculated as

$$t = -\frac{1}{a}\left(\ln\frac{1 + \sin\theta^{\mathrm{M}}}{\cos\theta^{\mathrm{M}}} - \ln\frac{1 + \sin\theta^{\mathrm{A}}}{\cos\theta^{\mathrm{A}}}\right). \tag{13}$$

Up to now, the ToF for an isogradient SSP can be computed using (13) by first calculating β_0 from (10), substituting it in (11) and computing α_0, and consequently θ^A and θ^M. Since we will adopt the EKF for localization and tracking of a mobile node, in addition to the ToF as a function of the node locations, we also need the derivatives of the ToF w.r.t the mobile location. To derive $\frac{\partial t}{\partial r^{\mathrm{M}}}$ and $\frac{\partial t}{\partial z^{\mathrm{M}}}$ using (13) we take the following partial derivatives

$$\frac{\partial t}{\partial r^{\mathrm{M}}} = -\frac{1}{a}\left(\frac{1}{\cos\theta^{\mathrm{M}}}\frac{\partial\theta^{\mathrm{M}}}{\partial r^{\mathrm{M}}} - \frac{1}{\cos\theta^{\mathrm{A}}}\frac{\partial\theta^{\mathrm{A}}}{\partial r^{\mathrm{M}}}\right), \tag{14}$$

$$\frac{\partial t}{\partial z^{\mathrm{M}}} = -\frac{1}{a}\left(\frac{1}{\cos\theta^{\mathrm{M}}}\frac{\partial\theta^{\mathrm{M}}}{\partial z^{\mathrm{M}}} - \frac{1}{\cos\theta^{\mathrm{A}}}\frac{\partial\theta^{\mathrm{A}}}{\partial z^{\mathrm{M}}}\right). \tag{15}$$

The above equations depend on the partial derivatives of the ray angles at the mobile and the anchor location. These partial derivatives can be computed from (8) and (9) as

$$\frac{\partial \theta^{\mathrm{M}}}{\partial r^{\mathrm{M}}} + \frac{\partial \theta^{\mathrm{A}}}{\partial r^{\mathrm{M}}} = -\frac{z^{\mathrm{M}} - z^{\mathrm{A}}}{\left(r^{\mathrm{M}} - r^{\mathrm{A}}\right)^2} \frac{\left(\sin\theta^{\mathrm{M}} - \sin\theta^{\mathrm{A}}\right)^2}{1 - \cos\left(\theta^{\mathrm{M}} - \theta^{\mathrm{A}}\right)}, \tag{16a}$$

$$\frac{\partial \theta^{\mathrm{M}}}{\partial r^{\mathrm{M}}} - \frac{b + az^{\mathrm{M}}}{b + az^{\mathrm{A}}} \frac{\sin\theta^{\mathrm{A}}}{\sin\theta^{\mathrm{M}}} \frac{\partial \theta^{\mathrm{A}}}{\partial r^{\mathrm{M}}} = 0, \tag{16b}$$

$$\frac{\partial \theta^{\mathrm{M}}}{\partial z^{\mathrm{M}}} + \frac{\partial \theta^{\mathrm{A}}}{\partial z^{\mathrm{M}}} = \frac{1}{r^{\mathrm{M}} - r^{\mathrm{A}}} \frac{\left(\sin\theta^{\mathrm{M}} - \sin\theta^{\mathrm{A}}\right)^2}{1 - \cos\left(\theta^{\mathrm{M}} - \theta^{\mathrm{A}}\right)}, \tag{17a}$$

$$\frac{\partial \theta^{\mathrm{M}}}{\partial z^{\mathrm{M}}} - \frac{b + az^{\mathrm{M}}}{b + az^{\mathrm{A}}} \frac{\sin\theta^{\mathrm{A}}}{\sin\theta^{\mathrm{M}}} \frac{\partial \theta^{\mathrm{A}}}{\partial z^{\mathrm{M}}} = -\frac{a}{b + az^{\mathrm{A}}} \frac{\cos\theta^{\mathrm{A}}}{\sin\theta^{\mathrm{M}}}, \tag{17b}$$

where (16a) and (17a) are calculated from (8), and (16b) and (17b) are derived from (9). Observe that (16) and (17) are linear in $\frac{\partial \theta^{\mathrm{M}}}{\partial z^{\mathrm{M}}}$, $\frac{\partial \theta^{\mathrm{A}}}{\partial z^{\mathrm{M}}}$, $\frac{\partial \theta^{\mathrm{M}}}{\partial r^{\mathrm{M}}}$, $\frac{\partial \theta^{\mathrm{A}}}{\partial r^{\mathrm{M}}}$, and can thus simply be solved in closed form. By computing these values for each anchor and substituting them into (14) and (15), we are able to compute all measured ToFs' derivatives w.r.t the mobile node position, in which $\frac{\partial t}{\partial x^{\mathrm{M}}}$ and $\frac{\partial t}{\partial y^{\mathrm{M}}}$ can be derived as

$$\frac{\partial t}{\partial x^{\mathrm{M}}} = \frac{\partial t}{\partial r^{\mathrm{M}}} \frac{x^{\mathrm{M}} - x^{\mathrm{A}}}{r^{\mathrm{M}} - r^{\mathrm{A}}}, \tag{18a}$$

$$\frac{\partial t}{\partial y^{\mathrm{M}}} = \frac{\partial t}{\partial r^{\mathrm{M}}} \frac{y^{\mathrm{M}} - y^{\mathrm{A}}}{r^{\mathrm{M}} - r^{\mathrm{A}}}. \tag{18b}$$

III. Localization/Tracking Network

We consider a synchronized underwater network consisting of N anchor nodes with known locations and one mobile node, where the extension to multiple mobile nodes is straightforward. To be able to localize the mobile node in a recursive manner (sometimes referred to as tracking), we exploit an EKF to estimate and track the position. Let us denote the location of the mobile node at time instant k as $\mathbf{x}_k = [x_k, y_k, z_k]^T$, and the corresponding state vector for the EKF as $\mathbf{s}_k = [\mathbf{x}_k^T, \dot{\mathbf{x}}_k^T]^T$, which contains both the location and velocity of the mobile node at time instant k.

In general, a discrete-time linear movement process model can be considered as

$$\mathbf{s}_k = \mathbf{\Phi}\mathbf{s}_{k-1} + \mathbf{w}_k, \tag{19}$$

where the matrix $\mathbf{\Phi}$ relates the state of the previous time instant to the current one, and $\mathbf{w}_k$ represents i.i.d Gaussian process noise with covariance matrix $\mathbf{Q}_k$.

The idea behind the proposed scheme can be explained as follows. Classical approaches in the literature simply try to calculate the distances corresponding to the ToF measurements by assuming a near-constant wave velocity for the whole environment, while this assumption does not really hold for an underwater environment. Therefore, we directly work with the ToF measurements.

It is noteworthy that we can further improve the accuracy of our estimates by the help of a depth measurement in cases where this information can be acquired. However, for the network to be able to extract the depth of the mobile node, the node will have to transmit a signal containing the depth information to the anchors which itself is resource-demanding due to the bandwidth limitations of the underwater channel. In order to make this process more bandwidth efficient, we suppose that the mobile node transmits the depth information in every ρ'th transmission frame. On the other hand, scenarios can be considered where the mobile node itself requires to know its location information. Then, we can consider that depth information is always available. Since mobile node velocity measurements are avoided here, the measurement model under consideration can be described as

$$\mathbf{t}_k = \mathbf{h}(\mathbf{s}_k) + \mathbf{v}_k, \tag{20}$$

$$\tilde{z}_k = z_k + v_k \qquad \text{if} \;\; \mathrm{mod}(k, \rho) = 0, \tag{21}$$

where $\mathbf{h}(.) = [h_1(.), h_2(.), \ldots, h_N(.)]^T$ is the function relating the state of the mobile node, $\mathbf{s}_k$ (actually only the location $\mathbf{x}_k$ is required) to the wave travel times between the mobile node and the N anchors, $\mathbf{t}_k = [t_{k,1}, \ldots, t_{k,N}]^T$. $\mathbf{v}_k$ and v_k represent the i.i.d. Gaussian noise of the measurements with covariance matrix $\sigma_t^2 \mathbf{I}_{N \times N}$ and variance σ_z^2, respectively . In the following, we explain how we can utilize the EKF for localization and tracking of a mobile node in an underwater environment.

IV. EKF-ESSP: EKF Exploiting Exact SSP

The EKF algorithm for underwater tracking considering the exact SSP (EKF-ESSP) is shown in Algorithm 1. In the algorithm, $\mathbf{P}_k$, $\mathbf{R} = \sigma_t^2 \mathbf{I}_{N \times N}$, and $\mathbf{Q}_k$ are the covariance matrix of the error in the state estimate, the measurement noise, and the process noise, respectively. To linearize the measurement equations, we compute the Jacobian matrix of $\mathbf{h}(.)$ as

$$\mathbf{H} = \begin{bmatrix} \frac{\partial t_1}{\partial x} & \frac{\partial t_1}{\partial y} & \frac{\partial t_1}{\partial z} & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & 0 & 0 & 0 \\ \frac{\partial t_N}{\partial x} & \frac{\partial t_N}{\partial y} & \frac{\partial t_N}{\partial z} & 0 & 0 & 0 \end{bmatrix}, \tag{22}$$

where t_n is the ToF between the mobile node and the n'th anchor node. The Jacobian matrix must be evaluated for time instant k as $\mathbf{H}_k = \mathbf{H}|_{x=\hat{x}_k^-, y=\hat{y}_k^-, z=\hat{z}_k^-}$. Following the derivation of the EKF if depth measurements are available, $\mathbf{H}_k$ and $\mathbf{R}$ should be modified to $\breve{\mathbf{H}}_k$ and $\breve{\mathbf{R}}$ as

$$\breve{\mathbf{H}}_k = \begin{bmatrix} \mathbf{H}_k \\ [0, 0, 1, 0, 0, 0] \end{bmatrix},$$

$$\breve{\mathbf{R}} = \begin{bmatrix} \mathbf{R} & \mathbf{0}_{N \times 1} \\ \mathbf{0}_{N \times 1}^T & \sigma_z^2 \end{bmatrix}.$$

The lower bound of the mean squared error (MSE) of estimation for any discrete time filtering problem, like the proposed EKF, can be computed via the posterior Cramér-Rao bound (PCRB) [11]. The recursive PCRB derived in [12] provides a formula for updating the posterior Fisher information matrix (FIM) from one time instant to the next. The posterior FIM sequence $\mathbf{J}_k$ for a linear process and a non-linear measurement model can be computed as

$$\mathbf{J}_k = (\mathbf{Q}_k + \mathbf{\Phi}\mathbf{J}_{k-1}^{-1}\mathbf{\Phi}^T)^{-1} + \bar{\mathbf{H}}_k^T \mathbf{R}_k^{-1} \bar{\mathbf{H}}_k \tag{23}$$

where all the parameters have been defined earlier except $\bar{\mathbf{H}}_k$

Algorithm 1 EKF

Start with an initial location guess
for $k = 1$ to K **do**
  Next state:
    $\hat{\mathbf{s}}_k^- = \mathbf{\Phi}\hat{\mathbf{s}}_{k-1}$
  Next error covariance:
    $\mathbf{P}_k^- = \mathbf{\Phi}\mathbf{P}_{k-1}\mathbf{\Phi}^T + \mathbf{Q}_k$
  if z info. is not available: **then**
    Compute the Kalman gain:
      $\mathbf{K}_k = \mathbf{P}_k^- \mathbf{H}_k^T (\mathbf{H}_k \mathbf{P}_k^- \mathbf{H}_k^T + \mathbf{R})^{-1}$
    Update the state:
      $\hat{\mathbf{s}}_k = \hat{\mathbf{s}}_k^- + \mathbf{K}_k(\mathbf{t}_k - \mathbf{h}(\hat{\mathbf{s}}_k^-))$
    Update the error covariance:
      $\mathbf{P}_k = (\mathbf{I} - \mathbf{K}_k \mathbf{H}_k)\mathbf{P}_k^-$
  else
    Compute the Kalman gain:
      $\breve{\mathbf{K}}_k = \mathbf{P}_k^- \breve{\mathbf{H}}_k^T (\breve{\mathbf{H}}_k \mathbf{P}_k^- \breve{\mathbf{H}}_k^T + \breve{\mathbf{R}})^{-1}$
    Update the state:
      $\hat{\mathbf{s}}_k = \hat{\mathbf{s}}_k^- + \breve{\mathbf{K}}_k([\mathbf{t}_k^T, \tilde{z}_m]^T - [\mathbf{h}(\hat{\mathbf{s}}_k^-)^T, \hat{z}^-]^T)$
    Update the error covariance:
      $\mathbf{P}_k = (\mathbf{I} - \breve{\mathbf{K}}_k \breve{\mathbf{H}}_k)\mathbf{P}_k^-$
  end if
end for

which is the measurement Jacobian matrix evaluated at the true location of the mobile node. It is noteworthy that since we basically estimate the location of the mobile node and not its velocity, the PCRB of our location estimates will correspond to sum of the first three diagonal elements of $\mathbf{J}_k^{-1}$

$$\mathrm{PCRB}_k = \sum_{i=1}^{3} \left[\mathbf{J}_k^{-1}\right]_{ii}. \tag{24}$$

V. Numerical Results

In this section, we will conduct several simulations to evaluate the performance of our proposed algorithm in an isogradient environment. We assume that the sound speed at the surface is $b = 1480$ and it increases as a linear function of depth with a steepness of $a = 0.1$. Further, we consider that four anchors are located on the vertices of a cube with edge length 100m, in which one vertex is located at the origin of the Cartesian coordinate system as depicted in Fig. 2. The movement model is chosen to be a random walk with a sampling time step of $T_s = 10$s. The matrix $\mathbf{\Phi}$ as defined earlier is then given by

$$\mathbf{\Phi} = \begin{bmatrix} \mathbf{I}_{3\times3} & T_s\mathbf{I}_{3\times3} \\ \mathbf{0}_{3\times3} & \mathbf{I}_{3\times3} \end{bmatrix},$$

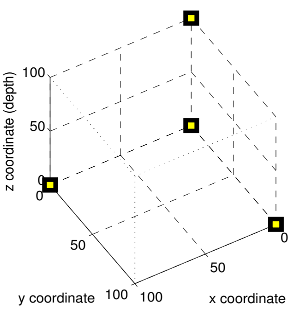

Fig. 2. Anchors' positions

and the process noise covariance matrix, which is assumed to be time-independent and only affecting the velocity, is given by

$$\mathbf{Q} = \begin{bmatrix} \mathbf{0}_{3\times3} & \mathbf{0}_{3\times3} \\ \mathbf{0}_{3\times3} & \mathrm{diag}(\sigma_{\dot{x}}^2, \sigma_{\dot{y}}^2, \sigma_{\dot{z}}^2) \end{bmatrix},$$

where we assume that $\sigma_{\dot{x}} = \sigma_{\dot{y}} = 10^{-2}$, and $\sigma_{\dot{z}} = 10^{-3}$.

For all simulations, we set the initial location guess of the Kalman filter to a point where it is $[30\mathrm{m}, 30\mathrm{m}, 30\mathrm{m}]^T$ apart from the actual location of the mobile node. For each run, we consider $K = 500$ movement steps, and we compute the positioning root mean squared error (RMSE) between the actual and estimated trajectories according to the following formula

$$\mathrm{RMSE} = \sqrt{\frac{1}{K - K_1 + 1} \sum_{k=K_1}^{k=K} \mathrm{E}\left[\left\|\hat{\mathbf{x}}_k - \mathbf{x}_k^{real}\right\|^2\right]}, \tag{25}$$

where we try to avoid transient effects by setting K_1 to a large number, e.g. $K_1 = 300$.

In addition, for the sake of further comparison, we also consider an ordinary EKF which considers a constant sound speed defined as the average sound speed between two given nodes (EKF-ASSP). In the EKF-ASSP, it is assumed that the sound waves propagate with the same speed everywhere inside the environment. Furthermore, in the EKF-ASSP, the distance between two nodes is estimated via the measured ToFs, $t = d/\bar{v}$, where $\bar{v}$ is a given constant wave velocity. In our simulations, we simply take $\bar{v}$ as the average speed over the region where the deepest and the shallowest anchors are located. Hence, we simply set $\bar{v} = [C(\max_n z_n^{\mathrm{A}}) + C(\min_n z_n^{\mathrm{A}})]/2$.

In Fig. 3, we depict a sample tracking result of the proposed and the EKF-ASSP algorithms. In this simulation, we set $\sigma_t =$ 1ms, and we consider that depth information is available once every 10 time instants with a noise std of σ_z = 1m. It is shown that the proposed algorithm converges well to the real trajectory. However, the EKF-ASSP algorithm always has an offset from the real trajectory, and this offset increases as the mobile node gets further away from the center of gravity of the anchors.

To quantitatively evaluate the performance of the proposed algorithm compared to the EKF-ASSP, we run a number of simulations where in all of the simulations we average over 5000 independent Monte Carlo trials. All simulation parameters are the same as before. In Fig. 4, the horizontal axis indicates the distance between the initial location of the mobile node and the center of gravity of the anchors. The initial depth and the x-coordinate of the mobile node are exactly equal to the depth and x-coordinate of the center of gravity of the anchors, which are both 50m for all simulations. As can be seen, for larger distances the performance of the EKF-ASSP significantly degrades while the EKF-ESSP is not so much affected by the distance and attains the PCRB. In Fig. 5, we investigate the effect of the measurement noise on the algorithms under consideration. Here, the horizontal axis represents the noise std on the ToF measurements. As is clear from the figure, the performance of the EKF-ESSP

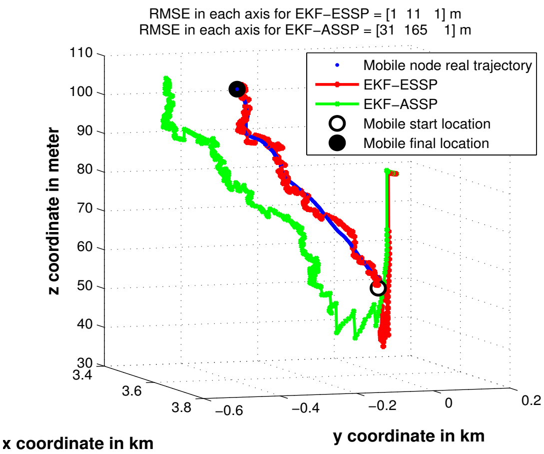

Fig. 3. Tracking comparison.

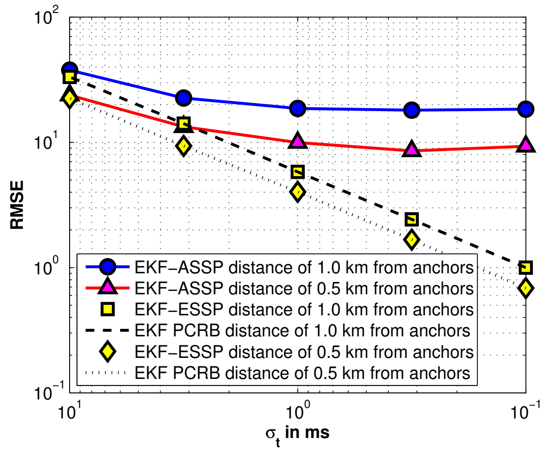

Fig. 5. Effect of the time measurement error on the proposed algorithm.

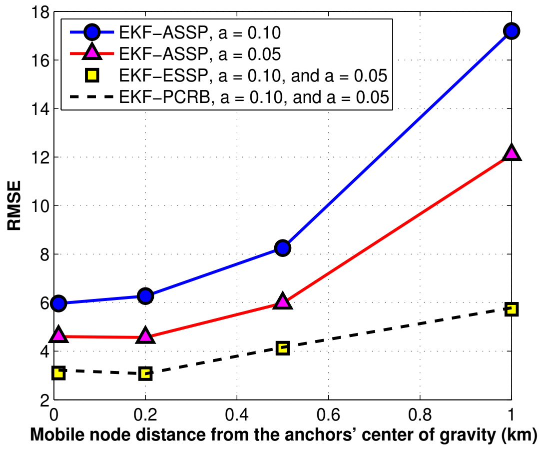

Fig. 4. Distance effect on the proposed and range-based algorithms.

constantly improves when increasing the ToF measurement accuracy (decreasing the noise std). While the EKF-ASSP does not show any improvement after a given noise std. Further, the performance of the EKF-ASSP gets even worse when the distance of the mobile node (in its initial location) from the center of gravity of the anchors increases. For large noise stds both algorithms have approximately the same performance, i.e., there is no advantage of the EKF-ESSP over the EKF-ASSP, and hence, in that case using the EKF-ASSP is preferred due to its lower complexity.

VI. Conclusions

In this paper, we have considered the problem of mobile node localization/tracking in an underwater environment with an isogradient SSP. We have shown that traditional terrestrial approaches for localization which are based on range measurements and a straight-line wave propagation model are not so accurate for such an environment. It is also shown that as the distance between two underwater nodes increases, the straight-line wave propagation model performs even worse since it does not follow the real propagation model. To solve this issue, we relate the ToF between two underwater nodes to their locations for an isogradient SSP, and formulate the localization problem as a time-based problem instead of a range-based one. Then, we use the extended Kalman filter with the proper formulation to solve the localization/tracking problem. It is shown that with this modification, our algorithm performs better than the algorithms based on a straight-line wave propagation model, especially for large distances. Although an isogradient SSP is not valid for all practical situations, the result can be used as an initial step towards more elaborate SSPs, since any given SSP can be modeled by several isogradient layers. This is the topic of further research.

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