

# Optimal Hard Fusion Strategies for Cognitive Radio Networks

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**Abstract**—Optimization of hard fusion spectrum sensing using the  $k$ -out-of- $N$  rule is considered. Two different setups are used to derive the optimal  $k$ . A throughput optimization setup is defined by minimizing the probability of false alarm subject to a probability of detection constraint representing the interference of a cognitive radio with the primary user, and an interference management setup is considered by maximizing the probability of detection subject to a false alarm rate constraint. It is shown that the underlying problems can be simplified to equality constrained optimization problems and an algorithm to solve them is presented. We show the throughput optimization and interference management setups are dual. The simulation results show the majority rule is optimal or near optimal for the desirable range of false alarm and detection rates for a cognitive radio network. Furthermore, an energy efficient setup is considered where the number of cognitive radios is to be minimized for the AND and the OR rule and a certain probability of detection and false alarm constraint. The simulation results show that the OR rule outperforms the AND rule in terms of energy efficiency.

**Index Terms**—Cooperative sensing, Cognitive radio, Energy detector, Hard decision fusion, Spectrum sensing.

## I. INTRODUCTION

Cooperative spectrum sensing is proposed as a solution for increasing the detection reliability of the cognitive radio network by exploiting the spatial diversity of multiple cognitive users [1]. In this paper, a cooperative sensing scheme is considered where each cognitive radio performs a periodic sensing, locally processes the accumulated data samples and sends the result to a fusion center (FC). Afterwards, the FC makes a final decision about the presence of the primary user using the local sensing results. Several fusion schemes have been proposed to combine the local sensing information of the secondary users [4], [5] that can be categorized as hard and soft fusion schemes. Due to its energy and bandwidth efficiency as well as comparable performance with the soft schemes, in this paper, we consider the hard decision fusion schemes. Several hard fusion techniques have been proposed in the literature [4], [3], [6]. Among them the OR and AND rule attract most of the attention, since these rules are easily implementable by simple logics. However, the OR and AND rule can be considered as special cases of the general  $k$ -out-of- $N$  rule when  $k$  is 1 or  $N$ , respectively.

There have been few works on the optimization of the  $k$ -out-of- $N$  rule in order to find the optimal  $k$ . In [3], the detection error probability is minimized in order to find the optimal  $k$  when the detection threshold is considered to be constant.

However, the weighting effects of the probability of primary user presence or absence are not considered in the error function. Furthermore, the detection error probability as the weighted sum of the probability of false alarm and detection does not have a meaningful interpretation from a cognitive radio perspective. In [6], the overall data rate of the cognitive radio network is maximized subject to an interference with the primary user in order to find the optimal  $k$ , sensing time and false alarm rate. However, optimization of the false alarm rate alone is not considered. False alarm rate optimization, not only increases the throughput of the cognitive radio but also decreases the overall switching time of the sensing module between the different frequency bands.

In this paper, we find the optimal value of  $k$  under two setups,

- A throughput maximization setup is defined by minimizing the global probability of false alarm ( $Q_f$ ) subject to a global probability of detection ( $Q_d$ ) constraint. The detection rate constraint in this problem represents the constraint on the interference of the cognitive user to the primary user activation.
- An interference management setup is defined by maximizing the global probability of detection ( $Q_d$ ) subject to a global probability of false alarm ( $Q_f$ ) constraint. The false alarm rate constraint in this problem, represents a lower bound on the throughput of the cognitive radio. Furthermore, upon a false alarm detection, a cognitive radio has to switch to another band to find a spectrum hole. Therefore, a false alarm rate constraint also puts an upper bound on the overall switching time of the cognitive radio among the different frequency bands.

We provide algorithms to find the optimal value of  $k$  for the underlying optimization problems. It is shown that in the desirable range of the false alarm and detection rates for cognitive radio networks ( $Q_d \geq 0.9$ ,  $Q_f \leq 0.1$ ), the majority rule is either optimal or near optimal.

Furthermore, we find the optimal number of cognitive users  $N$  under an energy efficient setup. In [3] the number of cognitive users is minimized for a detection error probability constraint. Such an optimization problem has the same drawbacks as mentioned above for finding the  $k$  that minimizes the error probability. In this paper, the number of cognitive users is to be minimized for a certain probability of detection and

false alarm constraint. For this setup we only considered two special cases: the AND and the OR rule. It is shown that the OR rule is much more energy efficient than the AND rule.

The remainder of the paper is organized as follows. In Section II, we present the considered cooperative spectrum sensing scheme and derive the sensing parameters of the system including the global probability of detection and false alarm. In Section III, the underlying optimization problems are presented and analyzed. Furthermore, the algorithms to solve the problems are also presented. We show the simulation results in Section IV and finally draw our conclusions in Section V.

## II. SYSTEM MODEL

A parallel cooperative spectrum sensing configuration is considered. Each cognitive radio senses the spectrum in periodic sensing slots and sends the result to the FC. There are several decision fusion approaches available in the literature including hard and soft fusion schemes. Here, a hard fusion scheme is employed by the FC to make a final decision by considering the attained binary results from the cognitive radios. Note that hard schemes have a much higher energy and bandwidth efficiency than soft schemes, while they give a comparable performance in terms of the detection reliability [1].

Each cognitive radio solves a binary hypothesis testing problem by making a decision about the presence or absence of the primary user, denoted by  $H_0$  and  $H_1$ , respectively. Denoting  $Y[n]$  to be the  $n$ -th received sample at each cognitive radio,  $W[n]$  to be the noise and  $X[n]$  the primary user signal, the data model that is used for such a hypothesis testing problem is given by

$$\begin{aligned} H_0 : Y[n] &= W[n], \quad n = 1, \dots, M; \\ H_1 : Y[n] &= X[n] + W[n], \quad n = 1, \dots, M; \end{aligned} \quad (1)$$

where the noise is assumed to be additive white Gaussian noise (AWGN) with zero mean and variance  $\sigma_w^2$  and the signal is assumed to be an i.i.d random variable with average signal energy of  $\sigma_x^2$ . In this paper, an energy detector is employed by each cognitive user. The energy detector calculates the energy of the  $M$  accumulated signal samples by  $T = \sum_{n=1}^M (Y[n])^2$ . For a large number of samples, we can use the central limit theorem to approximate the test statistic as Gaussian [2]:

$$\begin{aligned} H_0 : T &\sim \mathcal{N}(M\sigma_w^2, 2M\sigma_w^4) \\ H_1 : T &\sim \mathcal{N}(M(\sigma_w^2 + \sigma_x^2), 2M(\sigma_w^2 + \sigma_x^2)^2) \end{aligned} \quad (2)$$

The binary hypothesis testing problem is solved by comparing the resulting decision statistic with a predetermined threshold  $\lambda$ . The cognitive radio selects  $H_1$  if  $T \geq \lambda$  and  $H_0$  otherwise.

Denote  $P_f$  and  $P_d$  to be the respective local probability of false alarm and detection. The probability of false alarm indicates the false detection of the primary user in case the primary user is absent while the probability of detection

indicates the reliability of the primary user detection. The analytical expressions for  $P_f$  and  $P_d$  are given by

$$P_f = Q\left(\frac{\lambda - M\sigma_w^2}{\sqrt{2M\sigma_w^4}}\right) \quad P_d = Q\left(\frac{\lambda - M(\sigma_w^2 + \sigma_x^2)}{\sqrt{2M(\sigma_w^2 + \sigma_x^2)^2}}\right) \quad (3)$$

Denote  $\gamma = \frac{\sigma_x^2}{\sigma_w^2}$  to be the signal-to-noise ratio (SNR) of the primary user measured at the secondary user (cognitive radio) of interest. In this paper, a network of  $N$  identical cognitive radios is considered that measure the same SNR. Based on the threshold of the energy detector, each cognitive radio makes a local sensing decision about the presence or absence of a primary user of interest. The local sensing decision is sent to the FC and then the FC makes a global decision about the primary user presence or absence.

Several hard fusion schemes are presented in [4]. Due to its implementation simplicity and frequent utilization in the current proposed cognitive radio networks, we focus on the  $k$ -out-of- $N$  fusion rule for combining the local binary decisions. In this case, the FC decides that the primary user is present when  $k$  or more received local decisions are in support of the presence of the primary user, else the FC announces that the primary user is absent and the relevant spectrum band can be employed by the cognitive radios. When  $k = 1$ , the fusion rule becomes an OR-fusion rule and if  $k = N$ , it becomes an AND rule.

Denote  $D_i$  to be the local binary sensing decision of the  $i$ -th cognitive radio, e.g.,  $D_i = 0$  for  $H_0$  and  $D_i = 1$  for  $H_1$ . Thus the resulting  $k$ -out-of- $N$  binary hypothesis testing problem at the FC is given by  $I = \sum_{i=1}^N D_i < k$  for  $H_0$  and  $I = \sum_{i=1}^N D_i \geq k$  for  $H_1$ . By choosing a common threshold,  $\lambda$  for the energy detector at each secondary user, the global probabilities of detection  $Q_d$  and false alarm  $Q_f$  at the FC can be obtained as follows

$$\begin{aligned} Q_d &= \sum_{i=k}^N \binom{N}{i} P_d^i (1 - P_d)^{N-i} \\ Q_f &= \sum_{i=k}^N \binom{N}{i} P_f^i (1 - P_f)^{N-i} \end{aligned} \quad (4)$$

In the following section we are going to define the underlying problems in order to find the optimal  $k$ -out-of- $N$  rule for different setups.

## III. ANALYSIS AND PROBLEM FORMULATION

The performance of a cognitive radio network can be enhanced either by minimizing the global probability of false alarm or maximizing the global probability of detection depending on the priorities of the cognitive radios. A high probability of detection represents a low interference to the primary user signal while a low probability of false alarm represents a high throughput for the cognitive radio. With these two degrees of freedom, we can select  $k$  in the  $k$ -out-of- $N$  rule to meet the requirements of throughput and interference. In this section, we formulate two distinct optimization problems

for two possible setups. A throughput optimization setup and an interference management setup.

Furthermore, we consider the energy efficiency optimization of the cognitive radio network by minimizing  $N$  subject to a certain detection performance constraint.

#### A. Throughput optimization setup

1) *Problem formulation:* The global probability of false alarm,  $Q_f$  determines the throughput of the cognitive radio network. Minimizing  $Q_f$  improves the chances of utilizing the spectrum. However, we have to keep the secondary user interference to the primary users below a certain level that is determined by a lower bound on the probability of detection. Hence, we can formulate an optimization problem for minimizing  $Q_f$ , subject to an allowable interference level as follows

$$\begin{aligned} \min_{k, \lambda} Q_f(k, \lambda) \\ \text{s.t. } Q_d \geq \alpha, 1 \leq k \leq N \end{aligned} \quad (5)$$

With the following theorem, we can reduce the optimization problem (5) to an equality constraint problem.

*Theorem 1:* The optimal value of  $Q_f$  is attained for  $Q_d = \alpha$ .

**Proof:** Assuming  $k$  to be a constant,  $Q_f$  is a monotonically decreasing function of  $\lambda$ . Therefore, the optimal  $Q_f$  is attained for the highest  $\lambda$  in the feasible set of (5).

Furthermore,  $Q_d$  is a monotonically decreasing function of  $\lambda$ . Therefore, the highest  $\lambda$  in the feasible set of (5) is attained for  $Q_d = \alpha$ .

Now assume  $\exists(k^*, \lambda^*) : Q_d^* > \alpha$  and  $Q_f^*$  is optimal in terms of (5). According to the above statement, there is a  $\lambda^{**} > \lambda^*$  for which  $Q_d^{**} = \alpha$  and  $Q_f^{**} < Q_f^*$  which is a contradiction. Hence, the optimal  $Q_f$  is attained for  $Q_d = \alpha$ .  $\square$

2) *Optimization algorithm:* Exploiting Theorem 1, we can find for every  $k$  a  $\lambda = g(k)$  that satisfies the constraint. Therefore, we can rewrite our problem in a single variable  $k$

$$\begin{aligned} \min_k Q_f(k) \\ \text{s.t. } 1 \leq k \leq N \end{aligned}$$

where  $Q_f(k) \triangleq Q_f(k, g(k))$ . This problem can be solved by an exhaustive search in  $k$ .

For many cases we observed that  $Q_f(k)$  is a convex sequence of  $k$ . Since a mathematical investigation of the problem is very complicated, we can not make a general claim about the convexity of (6) at this stage, but it is a subject of further work. In case it really is convex, the local minimum is the global minimum. This way any gradient descent optimization can be used to find the minimum point.

#### B. Interference management setup

1) *Problem formulation:* The global probability of detection,  $Q_d$  indicates the measure of the interference of the system. Maximization of  $Q_d$  results in a smaller interference of the cognitive radios to the primary user. Hence, we are interested in solving

$$\begin{aligned} \max_{k, \lambda} Q_d(k, \lambda) \\ \text{s.t. } Q_f \leq \beta, 1 \leq k \leq N \end{aligned} \quad (6)$$

By the following theorem, we can show that the optimization problem (6) can be reduced to an equality constraint problem.

*Theorem 2:* The optimal value of  $Q_d$  is attained for  $Q_f = \beta$ .

**Proof:** The proof is similar to the one for Theorem 1.

2) *Optimization algorithm:* Similar to the throughput optimization setup, since for all  $k$ , there is a  $\lambda = h(k)$  that satisfies the constraint, we can simplify (6) as

$$\begin{aligned} \max_k Q_d(k) \\ \text{s.t. } 1 \leq k \leq N \end{aligned}$$

where  $Q_d(k) \triangleq Q_d(k, h(k))$ . This problem can again be solved by an exhaustive search in  $k$ .

For many cases we observed that  $Q_d(k)$  is a concave sequence of  $k$ . In this case, any gradient descent algorithm ends up in a local minimum which is also the global minimum.

#### C. Duality

The following theorem shows that the interference management and throughput optimization setups are dual to each other.

*Theorem 3:* If  $(Q_f^*, k^*, \lambda^*)$  is the optimal solution of the throughput optimization problem for a detection rate constraint  $\alpha$ ,  $(Q_d^* = \alpha, k^*, \lambda^*)$  is the optimal solution of the interference management problem for a false alarm rate constraint  $\beta = Q_f^*$ .

**Proof:** To prove Theorem 3, we employ the counter example technique. Assume  $(Q_d^* = \alpha, k^*, \lambda^*)$  is not the optimal solution for the following problem

$$\begin{aligned} \max_{k, \lambda} Q_d \\ \text{s.t. } Q_f \leq Q_f^*, 1 \leq k \leq N \end{aligned}$$

which by using Theorem 2 can be reduced to an equality constraint problem. Therefore,  $\exists(k' \neq k^*, \lambda' \neq \lambda^*) : Q_d' > \alpha$  and  $Q_f' = Q_f^*$ . Since the probability of detection is an increasing function of the probability of false alarm,  $\exists(k', \lambda'') : Q_d'' = \alpha$  and  $Q_f'' < Q_f^*$ . This means  $\exists(k', \lambda'')$  that satisfies the throughput optimization problem constraint and gives a lower probability of false alarm than  $Q_f^*$  which is a contradiction. Hence,  $(k', \lambda')$  is not the optimal point for (7) and thus  $(Q_d^* = \alpha, k^*, \lambda^*)$  is the optimal solution.  $\square$

#### D. Energy efficient setup

Although the detection performance of a cognitive radio network enhances with the number of cognitive radios so does the energy consumption. Furthermore, having a flexible  $k$ -out-of- $N$  fusion rule incurs a high implementation complexity to the system. Current standards also force the cognitive radio to behave above a certain detection performance in terms of a lower bound on the probability of detection and an upper

bound on the probability of false alarm [7]. Hence, it is necessary to consider an energy efficient mechanism to reduce the energy consumption of the system while maintaining a standard detection reliability. We define our energy efficient problem so as to minimize the number of cooperative cognitive users to attain a required probability of detection and false alarm for a fixed  $k$  as follows

$$\begin{aligned} & \min_N N \\ & \text{s.t. } Q_d \geq \alpha \text{ and } Q_f \leq \beta. \end{aligned} \quad (7)$$

Apparently the optimal  $N$  is attained with the minimum  $N$  in the feasible set of (7). In this paper, we consider the solution of the problem for two special cases: the AND and the OR rule. The extension of the problem for all  $k$ 's is a subject of further work.

We can show the optimal  $N$  for the AND rule is the minimum solution of the following inequality problem

$$Q\left(\frac{M\sigma_x^2 + Q^{-1}(\alpha^{1/N})\sqrt{2M(\sigma_x^2 + \sigma_w^2)^2}}{\sqrt{2M\sigma_w^4}}\right) \leq \beta^{1/N} \quad (8)$$

while for the OR rule, the optimal  $N$  is the minimum solution of the following inequality problem

$$Q\left(\frac{M\sigma_x^2 + Q^{-1}(\alpha')\sqrt{2M(\sigma_x^2 + \sigma_w^2)^2}}{\sqrt{2M\sigma_w^4}}\right) \leq \beta' \quad (9)$$

where  $\alpha' = 1 - (1 - \alpha)^{1/N}$  and  $\beta' = 1 - (1 - \beta)^{1/N}$ . The optimal  $N$  can be found by an exhaustive search in  $N$  from 1 to the first value that satisfies these inequalities.

#### IV. SIMULATION RESULTS

A network of 10 cognitive users is considered for the simulations. Each cognitive radio accumulates  $M = 275$  samples for the local decision and the SNR of the primary user at each cognitive radio is assumed to be  $\gamma = -7$  dB.

Fig. 1 shows that  $Q_f$  is a convex sequence of  $k$  when  $Q_d$  is constant for the setup that is considered for the simulations.

Fig. 2a considers the throughput optimization setup. The probability of false alarm is minimized for different detection rate constraints. We can see that the optimal value of  $k$  decreases with an increasing probability of detection constraint  $\alpha$  and in the range above 0.9 which is the desirable region for a cognitive radio, the majority fusion rule becomes optimal. Furthermore, we can see that at low values of the detection rate, the AND rule is near optimal.

Fig. 2b considers the interference management setup. The probability of detection is maximized for different false alarm rate constraints. We can see that the optimal value of  $k$  decreases with an increasing probability of false alarm constraint  $\beta$  and in the range below 0.1 which is the desirable region for a cognitive radio, the majority fusion rule becomes near optimal. Furthermore, we can see that at high values of the false alarm rate, the OR rule is near optimal.

In Fig. 3a, the variation of the optimal value of  $k$  is considered with the number of cognitive users for the interference management setup. Different values of the false alarm rate

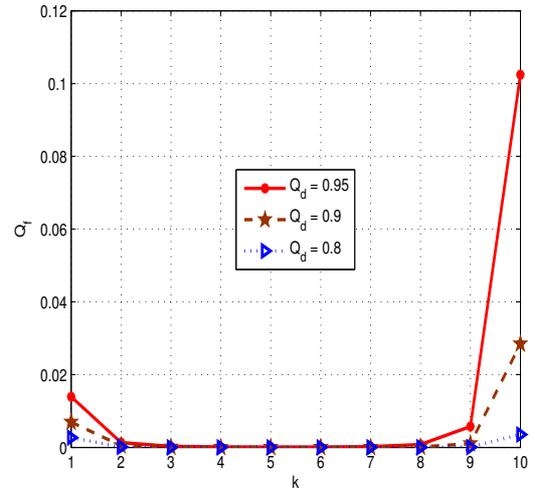


Fig. 1: Convexity of  $Q_f$  in  $k$  for constant  $Q_d$ .

constraint are considered for the simulations. We can see that for different numbers of cognitive users, the majority rule is either optimal or near optimal. Furthermore, it is shown that decreasing the false alarm rate constraint either increases or maintains the optimal  $k$ .

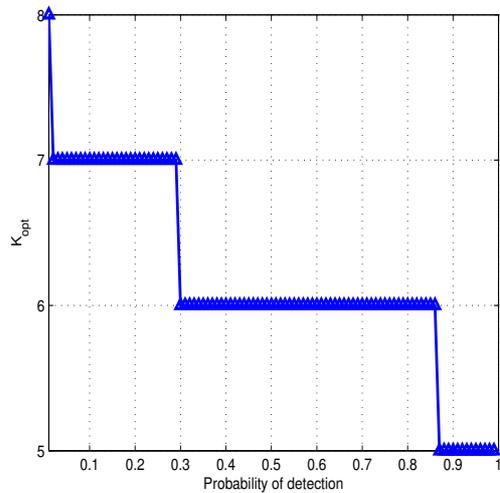
In Fig. 3b, the variation of the optimal value of  $k$  is considered with the number of cognitive users for the throughput optimization setup. Different values of the detection rate constraint are considered for the simulations. Similar to the previous scenario, we can see that for different numbers of cognitive users, the majority rule is either optimal or near optimal for a high probability of detection constraint. Furthermore, it is shown that increasing the detection rate constraint either decreases or maintains the optimal  $k$ .

Fig. 4a considers the energy efficient setup for the AND and OR rule. Two fixed  $\beta$ 's ( $\beta = 0.1, 0.05$ ) are considered for the simulation while the probability of detection constraint  $\alpha$  changes from 0.9 to 0.97. We can see that for different scenarios, the OR rule attains a lower optimal  $N$  than the AND rule, so it has a lower energy consumption for a certain detection performance constraint.

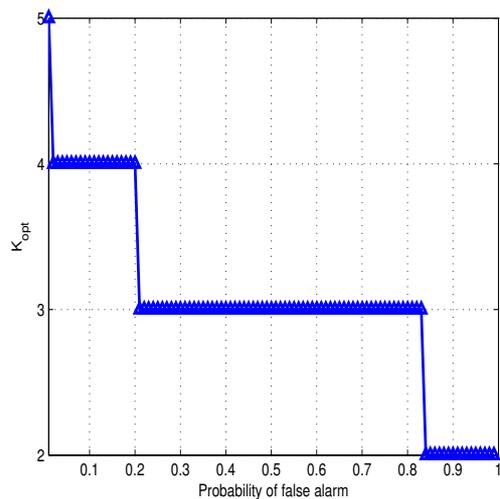
The energy efficient setup is also considered in Fig. 4b for two fixed  $\alpha$ 's ( $\alpha = 0.9, 0.95$ ), while the probability of false alarm constraint  $\beta$  changes from 0.01 to 0.1. As in the previous case, the OR rule gives a better performance in terms of energy efficiency than the AND rule.

#### V. CONCLUSION

In this paper, optimization of the  $k$ -out-of- $N$  rule is considered from three viewpoints: a throughput optimization setup, an interference management setup and an energy efficient setup. In the throughput optimization setup, the probability of false alarm is minimized subject to a detection rate constraint and in the interference management setup, maximization of the probability of detection constrained on the false alarm rate is considered. It is shown that in both cases the underlying



(a)

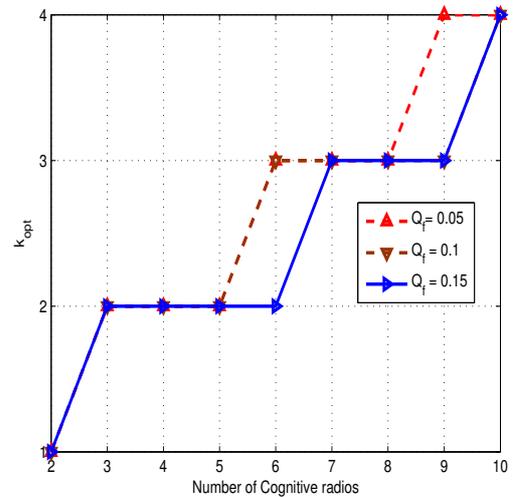


(b)

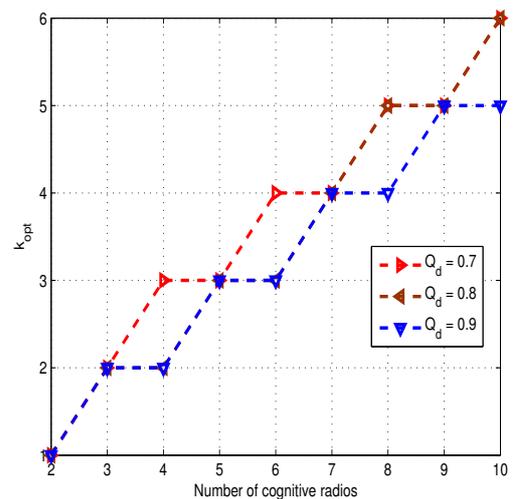
Fig. 2: a) Optimal  $k$  for the throughput optimization setup, b) Optimal  $k$  for the interference management setup.

inequality constrained problems can be simplified to equality constrained problems. We have shown that for the desirable values of the probability of false alarm and detection for the cognitive radio networks, the majority rule is either optimal or near optimal for different number of users. Furthermore, we proved that the throughput optimization and interference management setups are dual.

In the energy efficient setup, the number of cognitive users is minimized subject to a certain probability of detection and false alarm constraint. For this setup, we only considered the OR and the AND rule. It is shown that the OR rule has a higher energy efficiency than the AND rule under similar detection performance conditions.



(a)

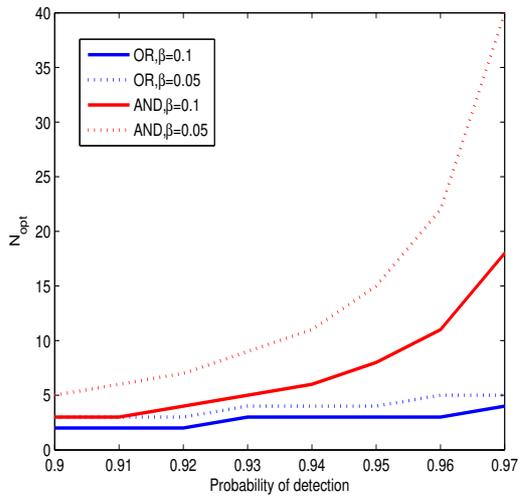


(b)

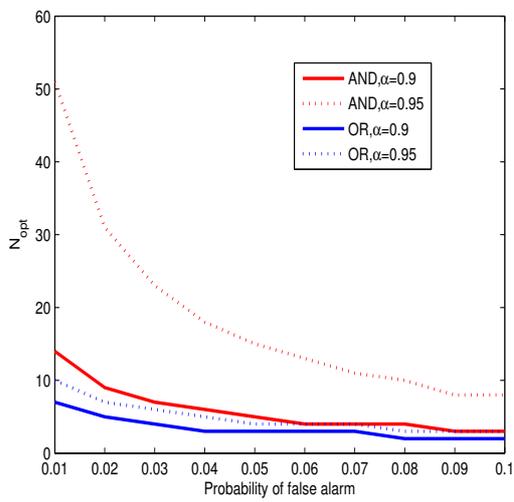
Fig. 3: a) Optimal  $k$  with the number of cognitive users for the interference management setup, b) Optimal  $k$  with the number of cognitive users for the throughput optimization setup.

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(a)



(b)

Fig. 4: a) Optimal  $N$  with the probability of detection, b) Optimal  $N$  with the probability of false alarm

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