

ORTHOGONAL WAVELET DIVISION MULTIPLEXING FOR WIDEBAND TIME-VARYING CHANNELS

Tao Xu*, Geert Leus*, and Urbashi Mitra†

* Fac. EEMCS, Delft University of Technology, Mekelweg 4, 2628CD, Delft, Netherlands

† Ming Hsieh Department of Electrical Engineering, University of Southern California, Los Angeles, USA

ABSTRACT

Block transmission of multi-scale orthogonal wavelet division multiplexing (OWDM) is proposed for signaling over wideband linear time-varying channels (LTV). Such channels are best modeled by multi-scale, multi-lag (MSML) models and the proposed OWDM designs are tailored to such channels. Given this signaling, the effective channel matrix for the received signal is banded, allowing for the modification of prior methods of equalization for orthogonal frequency division multiplexing over narrowband LTV channels. Performance of such equalizers and signaling is provided via simulation and shown to offer good performance coupled with high spectral efficiency over previously proposed designs.

Index Terms— Wideband channels, wavelets, OFDM, OWDM.

1. INTRODUCTION

Wideband linear time-varying (LTV) channels are of interest in a variety of wireless communication scenarios including underwater acoustic systems and wideband terrestrial radio frequency systems such as spread-spectrum or ultrawideband. Due to the nature of wideband propagation, such channels exhibit some fundamental differences relative to so-called *narrowband* channels. In particular, it has been shown that multi-scale, multi-lag (MSML) channel descriptions offer improved modeling of LTV wideband channels over multi-Doppler-shift, multi-lag models [1, 2]. Orthogonal frequency-division multiplexing (OFDM) has been examined for wideband channels. Approaches include splitting the wideband LTV channel into parallel narrowband LTV channels [3] or assuming a simplified model which reduces the wideband LTV channel to a narrowband LTV channel with a carrier frequency offset [4].

Receivers for single-scaled wavelet-based pulses for wideband MSML channels are presented in [1, 2], and a similar waveform is adopted in spread-spectrum systems [5] over wideband channels modeled by wavelet transforms; while [6] considers equalizers for block transmissions in wideband MSML channels. In order to achieve better realistic channel matching, single-scaled rational wavelet modulation was designed in [7]. However these schemes all employ single-scale modulation and thus do not maximize the spectral efficiency. In particular, herein, we shall focus on a form of orthogonal wavelet division multiplexing (OWDM) which has been previously examined in [8] but for additive white Gaussian noise channels. While we shall employ such a modulation over wideband MSML channels, new equalizer designs are necessary.

The main contributions of this work is a particularization of OWDM to a MSML channel model using multi-scale block transmission, identifying proper wavelets for the wideband LTV channel

and providing equalizer designs for our system. In particular, we observe that our signaling and the channel yield a received signal with an effective channel matrix which is banded. As such, equalization methods designed for combatting inter-carrier interference due to the use of OFDM in narrowband LTV channels (e.g. [9, 10]) can be exploited as we do so herein.

Notation: Upper (lower) bold-face letters stand for matrices (vectors); Superscript H denotes Hermitian, $*$ conjugate, T transpose, and \dagger matrix pseudo-inverse. We reserve j for the imaginary unit, $\lceil \cdot \rceil$ for integer ceiling, $\lfloor \cdot \rfloor$ for integer flooring, $[\mathbf{A}]_{k,m}$ for the (k, m) th entry of the matrix \mathbf{A} , $\text{diag}(\mathbf{x})$ for a diagonal matrix with \mathbf{x} on its main diagonal, and $i_{\text{mod}/k}$ for the modulus of i divided by k . δ_k represents a delta function which is equal to one only if $k = 0$ and zero otherwise.

2. WIDEBAND CHANNEL MODEL

A wideband LTV channel can be described by a general MSML model [11],

$$r(t) = \int_0^\infty \int_{-\infty}^\infty h(\alpha, \tau) \sqrt{\alpha} x(\alpha(t - \tau)) d\tau d\alpha + v(t), \quad (1)$$

where $x(t)$ is the transmitted signal, $r(t)$ is the received signal, $v(t)$ is the time domain white noise with σ^2 as its power spectral density, and $h(\alpha, \tau)$ is the wideband channel spreading function [11]. This model reflects the fact that the received signal $r(t)$ can be represented by a superposition of different delayed (by τ) and scaled (by α) versions of the transmitted signal ($\sqrt{\alpha}$ is a normalization factor). Due to practical restrictions, τ and α can be limited to $\tau \in [0, \tau_{max}]$ and $\alpha \in [1, \alpha_{max}]$ without loss of generality by appropriately delaying and scaling the received signal. The parameters $\tau_{max} > 0$ and $\alpha_{max} - 1 > 0$ respectively represent the *delay spread* and *Doppler scale spread*.

We assume that the wideband transmitted signal has a bandwidth of W_* and a Mellin support of M_*^1 . Then (1) can be approximated by the following finite-dimensional discrete MSML model ([1, 2]):

$$r(t) = \sum_{r=0}^{R_*} \sum_{l=0}^{L_*(r)} h_{r,l} a_*^{r/2} x(a_*^r t - lT_*) + v(t), \quad (2)$$

where $T_* = 1/W_*$ is the arithmetic time resolution and $a_* = e^{1/M_*} > 1$ is the geometric scale resolution of the model. Further, the channel coefficients are given by $h_{r,l} = \hat{h}(a_*^r, lT_*/a_*^r)$ with $\hat{h}(\alpha, \tau)$ defined in [1] as the doubly-smoothed version of $h(\alpha, \tau)$. Finally, the scale order R_* is given by $R_* = \lceil \log_{a_*}(\alpha_{max}) \rceil$, and the delay order for the r th scale $L_*(r)$ is given by $L_*(r) =$

This work is supported in part by NWO-STW under the VICI program (project 10382).

¹The Mellin support is the scale analogy of the Doppler spread for narrowband LTV channels – further details and definitions can be found in [12].

$\lceil a_*^r \tau_{max} / T_* \rceil$. In this work, we assume as in [1] that the discrete MSML model is well-matched and thus there is negligible difference between (1) and (2).

3. SCALE LAYERED TRANSMISSION

As previously noted, most prior work on wideband LTV systems employed transmitted signals at a single scale (or a single delay); herein we generalize it to multiple scales and delays to increase spectral efficiency by using an OWDM scheme. By further imposing inter-scale and inter-delay orthogonality via wavelet signaling, we can simplify receiver processing. However, it should be observed that just like in the narrowband time-varying case, we cannot perfectly jointly diagonalize across delay and scale.

3.1. Signaling Scheme

We start from a unit-energy orthogonal mother wavelet $\psi(t)$ with unity orthogonality time shift (we call this base time from now on) and a base scale of a , i.e. $\int_{-\infty}^{\infty} \psi(a^k t - n) \psi^*(a^{k'} t - n') dt = \delta_{k-k'} \delta_{n-n'}$. A symbol is modulated onto the k th scale and n th delay via the unit-energy pulse

$$\psi_{k,n}(t) = \sqrt{\frac{a^k}{T}} \psi(a^k t / T - n),$$

which has a base time of T/a^k and the base scale $a > 1$. It is easy to show that $\int_{-\infty}^{\infty} \psi_{k,n}(t) \psi_{k',n'}^*(t) dt = \delta_{k-k'} \delta_{n-n'}$.

A critical element of our system is the assumption that we can properly match the scales and delays of our signaling to that of the channel, that is, the base time T/a^k and base scale a of $\psi_{k,n}(t)$ are matched to the time resolution T_* and scale resolution a_* of the channel, which on its turn is determined by the bandwidth and the Mellin support of the transmitted signal, respectively. Thus, $\psi_{k,n}(t)$ should have a bandwidth of a^k/T and a Mellin support of $1/\ln a$ (equivalently $\psi(t)$ should have a bandwidth of 1 and a Mellin support of $1/\ln a$). Not all wavelet families satisfy these constraints; in particular, band-pass natured wavelets often violate our constraints. However, some orthogonal wavelet families possess good frequency and Mellin localization (e.g., Shannon wavelets [13]) and as such approximately satisfy our constraints and thus are candidates for our signaling system. We assume for simplicity that the desired matching can be achieved without any errors.

We underscore that the base time (and thus bandwidth) of each $\psi_{k,n}(t)$ is different per scale. We equivalently refer to scale or *layer* in the context of the signaling scheme. Hence, for the k th layer, the system model of (2), ignoring the additive noise, is adapted to the appropriate time and scale resolutions, i.e., we select $a_* = a$ and $T_* = T/a^k$ in (2), which leads to

$$r_k(t) = \sum_{r=0}^R \sum_{l=0}^{L(r+k)} h_{r,l}(k) a^{r/2} x_k(a^r t - lT/a^k), \quad (3)$$

where $x_k(t)$ is the transmitted signal at the k th layer (scale), $r_k(t)$ is the received signal at the k th layer, and $h_{r,l}(k) = \hat{h}(a^r, lT/a^{r+k})$. We further have $R = \lceil \log_a(\alpha_{max}) \rceil$ and $L(r) = \lceil a^r \tau_{max} / T \rceil$.

We generate the transmitted signal $x_k(t)$ for the k th layer as

$$x_k(t) = \sum_n s_{k,n} \psi_{k,n}(t), \quad (4)$$

where the symbol sequence $s_{k,n}$ modulates the shaping pulses $\psi_{k,n}(t)$ at a symbol rate of a^k/T . Substituting (4) into (3), we

then obtain

$$\begin{aligned} r_k(t) &= \sum_{r=0}^R \sum_{l=0}^{L(r+k)} h_{r,l}(k) a^{r/2} \sum_n s_{k,n} \psi_{k,n}(a^r t - lT/a^k) \\ &= \sum_{r=0}^R \sum_{l=0}^{L(r+k)} h_{r,l}(k) \sum_n s_{k,n} \sqrt{\frac{a^{(r+k)}}{T}} \psi\left(\frac{a^{r+k} t}{T} - l - n\right) \\ &= \sum_{r=0}^R \sum_{l=0}^{L(r+k)} h_{r,l}(k) \sum_n s_{k,n} \psi_{r+k, l+n}(t). \end{aligned}$$

Transmitting data on multiple layers, our OWDM (multi-layer) waveform can finally be described as

$$x(t) = \sum_k \sum_n s_{k,n} \psi_{k,n}(t), \quad (5)$$

Its corresponding received signal $r(t)$ can be expressed as

$$r(t) = \sum_k \sum_{r=0}^R \sum_{l=0}^{L(r+k)} h_{r,l}(k) \sum_n s_{k,n} \psi_{r+k, l+n}(t).$$

3.2. Block Transmission

We limit the number of layers to K , where K is related to the overall available transmission bandwidth, i.e., K and T are selected such that a^{K-1}/T matches the overall bandwidth. Further, the data on every layer in blocks of length N are separated by a cyclic prefix (CP) of length Z , thus facilitating block processing. Focusing on the first block of data, the transmitted signal can be written as

$$x(t) = \sum_{k=0}^{K-1} \sum_{n=0}^{N+Z-1} s_{k,n} \psi_{k,n}(t),$$

and its related received signal can be written as

$$r(t) = \sum_{k=0}^{K-1} \sum_{r=0}^R \sum_{l=0}^{L(r+k)} h_{r,l}(k) \sum_{n=0}^{N+Z-1} s_{k,n} \psi_{r+k, l+n}(t), \quad (6)$$

where

$$s_{k,n} = \begin{cases} b_{k,n-Z}, & \text{for } Z \leq n < N \\ b_{k,N+n-Z}, & \text{for } 0 \leq n < Z \end{cases}. \quad (7)$$

At the receiver, we will only consider the received data on the layers for $k \in \{0, 1, \dots, K-1\}$; To avoid interblock interference (IBI) on these layers, we need $Z \geq L(k)$, for all $k \in \{0, 1, \dots, K-1\}$, or in other words $Z \geq \lceil a^{K-1} \tau_{max} / T \rceil = \lceil a^{K-1} L(0) \rceil$.

The multiple block transmission scheme is depicted in Fig. 1. The shaded area here depicts the effective data while the blank area represents the CPs. Fig. 1 shows that there is no IBI after CP removal. As previously noted, the multi-layer OWDM transmission offers improved spectral efficiency over a single-layered approach as considered in [7, 1] at the expense of a more complex receiver structure. If we assume K layers, our achieved spectral efficiency is $\sum_{k=0}^{K-1} a^k N / (NT) = \frac{a^K - 1}{(a-1)T}$. In contrast, the single-layered approach has a maximal efficiency of a^{K-1}/T (on the $(K-1)$ th layer) using the same overall bandwidth. Note that for $a > 1$, it is clear that the OWDM approach achieves an increased efficiency.

4. WIDEBAND RECEIVER DESIGN

In this section, we develop the receiver for the proposed OWDM transmission. The first part of the receiver is an extension of the RAKE receiver concept proposed in [1] and consists of a matched filter bank for the layers of $k \in \{0, 1, \dots, K-1\}$ as shown in Fig. 2. One may compare the discarding of the layers for $k \in \{K, \dots, K+$

support in the Mellin domain is more or less the same, i.e., $a \approx 2$. We further choose QPSK for the data constellation, $N = 128$, $K = 3$, and $Z = 8$, which satisfies the constraints $K > R$ and $N > Z \geq \lceil a^{K-1} L(0) \rceil$.

Fig. 3 shows the bit-error-rate (BER) performance of our system without any bit coding using the LMMSE equalizer from [9] adapted to (11). The receiver performs best for Channel I which has no Doppler effects, while a slight drop in BER performance can be witnessed when the delay or scale spread increases. A similar observation can be made for an OFDM system over three narrowband LTV channels modeled by a CE-BEM [15] (ignoring its modeling error), with the same Doppler shift order R and delay order $L(0)$. One may argue that the BER performance herein is much better than that in [9] even when the same Doppler shift order R is adopted. The reason for this is because we assume (6) is well-matched without any error, while [9] considers the modeling errors of the CE-BEM and thus has a performance drop compared with Fig. 3. This figure also compares the BER performance of our proposed wideband receiver with that of the matched filter without equalization. As expected, the inter-scale interference severely affects system performance. Fig. 4 shows the BER performance using the Turbo-I/II/III equalizers from [10] adapted to (11) for Channel II and Channel III (without coding). The performance curves behave similarly to those in [10], and it is known that with more iterations, the BER performance can be further improved.

6. CONCLUSIONS

A block transmission scheme based on multi-scale OWDM over wideband LTV channels is proposed. Our proposed signaling offers higher spectral efficiency over previously proposed methods for such channels. An associated equalizer which employs a matched filter bank coupled with a frequency-domain equalizer is presented. The output effective channel has strong similarities to that seen for narrowband LTV systems employing OFDM and thus, equalizers for such systems can be adapted to the current scenario resulting in good performance.

7. REFERENCES

- [1] Y. Jiang and A. Papandreou-Suppappola. Discrete time-scale characterization of wideband time-varying systems. *IEEE T. Signal. Proces.*, 54(4):1364–1375, 2006.
- [2] A. R. Margetts, P. Schniter, and A. Swami. Joint scale-lag diversity in wideband mobile direct sequence spread spectrum systems. *IEEE T. Wirel. Commun.*, 6(12):4308–4319, 2007.
- [3] G. Leus and P. van Walree. Multiband OFDM for covert acoustic communications. *IEEE J. Sel. Area. Comm.*, 26(9):1662–1673, 2008.
- [4] B. Li, S. Zhou, M. Stojanovic, L. Freitag, and P. Willett. Multicarrier communication over underwater acoustic channels with nonuniform doppler shifts. *IEEE J. Oceanic. Eng.*, 33(2):198–209, 2008.
- [5] M. Martone. Wavelet-based separating kernels for array processing of cellular ds/cdma signals in fast fading. *IEEE T. Commun.*, 48(6):979–995, 2000.
- [6] U. Mitra and G. Leus. Equalizers for multi-scale / multi-lag wireless channels. accepted by IEEE Globecom 2010, 2010.
- [7] L. Yu and L. B. White. Optimum receiver design for broadband doppler compensation in multipath/doppler channels with rational orthogonal wavelet signaling. *IEEE T. Signal. Proces.*, 55(8):4091–4103, 2007.
- [8] S. L. Linfoot, M. K. Ibrahim, and M. M. Al-Akaidi. Orthogonal wavelet division multiplex: An alternative to ofdm. *IEEE T Consum Electr.*, 53(2):278–284, 2007.

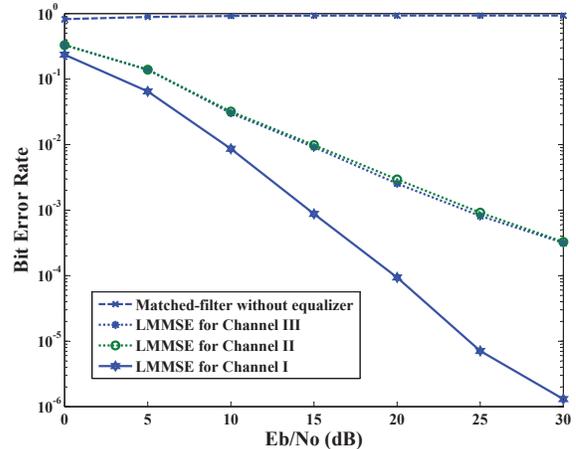


Fig. 3. Performance of the OWDM system using the proposed LMMSE equalizer adapted from [9].

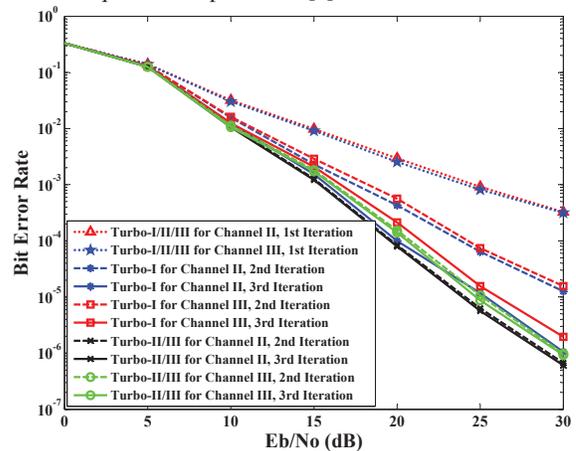


Fig. 4. Performance of the OWDM system using the proposed Turbo equalizer I/II/III adapted from [10].

- [9] L. Rugini, P. Banelli, and G. Leus. Simple equalization of time-varying channels for OFDM. *IEEE Commun. Lett.*, 9(7):619–621, 2005.
- [10] K. Fang, L. Rugini, and G. Leus. Low-complexity block turbo equalization for OFDM systems in time-varying channels. *IEEE T. Signal. Proces.*, 56(11):5555–5566, 2008.
- [11] L. H. Sibil, L.G. Weiss, and T.L. Dixon. Characterization of stochastic propagation and scattering via gabor and wavelet transforms. *Journal of Computational Acoustics*, 2(1):345–369, Jan. 1994.
- [12] P. Bertrand J. Bertand and J. P. Ovarlez. The Mellin transform. In A. D. Poularikas, editor, *The Transforms and Applications Handbook*, chapter 11, pages 829–885. CRC Press LLC, Boca Raton, Fla, USA, 1996.
- [13] C. Cattani. Shannon wavelets theory. *Mathematical Problems in Engineering*, 2008.
- [14] X. Ma and G. B. Giannakis. Maximum-diversity transmissions over doubly selective wireless channels. *IEEE T. Inform. Theory*, 49(7):1832–1840, 2003.
- [15] G. B. Giannakis and C. Tepedelenioglu. Basis expansion models and diversity techniques for blind identification and equalization of time-varying channels. *Proc. IEEE*, 86(10):1969–1986, 1998.