Compressive Sampling Based Differential Detection of Ultra Wideband Signals

Shahzad Gishkori*, Geert Leus*, Vincenzo Lottici†
* Faculty of EEMCS, Delft University of Technology, Delft, The Netherlands
† Dept. of Information Engineering, University of Pisa, Pisa, Italy
Emails: s.s.gishkori@tudelft.nl; g.j.t.leus@tudelft.nl; vincenzo.lottici@iet.unipi.it

***Abstract*—In this paper we focus on compressive sampling (CS) based ultra wideband (UWB) differential detection. We formulate an optimization problem to jointly recover the sparse received UWB signals as well as the differentially encoded data symbol. We utilize an alternating direction method of multipliers (ADMoM) to solve this joint optimization problem. Our proposed joint recovery method outperforms the straightforward separate recovery method, which recovers the sparse received UWB signals in a first step and then detects the differentially encoded symbol based on the recovered signals.**

***Index Terms*—compressive sampling, ultra-wideband, differential detection, ADMoM**

I. Introduction

Ultra wideband (UWB) communications has opened the door for short range communications with increased capacity, robustness against multipath fading, fine timing resolutions, etc [1]. A number of modulation techniques have been proposed to help achieve these goals. The most common modulation scheme is pulse position modulation (PPM) [2]. However, another useful non-coherent modulation scheme is the bi-phase modulation scheme in which case the pulse is inverted to create an opposite phase [2], [3]. This is also known as the differential modulation (DM) scheme. Since DM is an antipodal modulation method, it has a 3 dB gain in power efficiency over PPM [2]. Noncoherent detectors for differentially modulated UWB have been proposed in literature, e.g., in [4] and [5]. In [4], classical differential detection (DD) has been considered, whereas in [5], a performance improvement is obtained by using multiple symbol differential detection (MSDD). The performance of this MSDD scheme is comparable to the conventional Rake receiver. Moreover, to alleviate the strict conditions of timing recovery for this MSDD scheme, a novel approach has been proposed in [6] to rely on symbol-level synchronization only.

One big hurdle though in the implementation of any detection scheme for UWB (including [6]) is the high sampling rate required for an all-digital implementation. Since UWB signals have an extremely high bandwidth, the Nyquist sampling criterion demands analog to digital converters (ADCs) with a high sampling rate. Thus the realization of all-digital low-power UWB receivers becomes quite difficult. In the past few years, the idea of compressive sampling (CS) [7], [8] has been advocated as an effective means to reduce the sampling rate of sparse signals much below the Nyquist rate without large performance losses. The basic concept is to represent the signal with only a few measurements via random projections that are carried out in the analog domain. Based on these measurements, the sparse signal is then reconstructed through any sparse recovery method. Since the received UWB signals can be considered sparse, a CS based approach might be very useful here, and this has already been demonstrated for coherent UWB receivers in [13].

In this paper we propose a CS based detector for differentially modulated UWB signals. We formulate our cost function as a composition of sparse regularized least squares errors for the sparse received UWB signals and a DD error. We then provide an efficient method, namely alternating direction method of multipliers (ADMoM) [9], to minimize the cost function. As a benchmark, we also consider the straightforword approach in which we first minimize the sparse regularized least squares errors for the sparse received UWB signals, and then apply DD based on the recovered signals.

Notation: ℓ_p norm of a vector $\mathbf{x}$ is denoted as $||\mathbf{x}||_p = (\sum_{i=0}^{N-1} |x_i|^p)^{1/p}$, $[.]^T$is the transpose, $[.]^H$ is the hermitian, $\mathbf{I}$ is the identity matrix, $\text{sign}(x_i)$ is the sign function which takes values -1 and 1 depending on the polarity of x_i, and $\mathcal{E}(.)$ is the expected value.

II. Signal Model

Let s_k (-1 or $+1$) be the transmitted symbol at time instant k. Assuming a symbol period of T, the received signal during the kth symbol period can then be written as

$$r(t + kT) = h(t)s_k + v(t + kT),\ t \in [0, T),$$

where $v(t)$ is the additive noise and $h(t)$ is the composite channel impulse response, including the transmitted pulse and the receive filter. Note that we implicitly assume that the support of $h(t)$ is included in $[0, T)$, which obviously puts some constraints on synchronization and the delay spread of the channel. For DM, the transmitted symbols s_k can be written as

$$s_k = b_k s_{k-1},$$

where b_k (-1 or $+1$) is the data symbol at time instant k. Suppose now that the Nyquist rate is given by $1/T_s = N/T$, and that $r(t + kT)$ is sampled within $[0, T)$ at Nyquist rate,

This work is supported in part by NWO-STW under the VICI program (project 10382).

leading to $\mathbf{r}_k = [r(kT), r(T_s + kT), \ldots, r((N-1)T_s + kT)]^T$. We then obtain the model

$$\begin{aligned}\mathbf{r}_k &= \mathbf{h}s_k + \mathbf{v}_k \\ &= \mathbf{x}_k + \mathbf{v}_k,\end{aligned} \tag{1}$$

where $\mathbf{h} = [h(0), h(T_s), \ldots, h((N-1)T_s)]^T$ and $\mathbf{v}_k$ is similarly defined as $\mathbf{r}_k$. For simplicity we will assume that $\mathbf{v}_k$ is a zero-mean white vector sequence with coveriance matrix $\mathcal{E}(\mathbf{v}_k\mathbf{v}_k^T) = \sigma^2\mathbf{I}$. The instantaneous SNR is defined as

$$\eta = \frac{\|\mathbf{h}\|_2^2 \mathcal{E}(s_k^2)}{\mathcal{E}(\|\mathbf{v}_k\|_2^2)} = \frac{\|\mathbf{h}\|_2^2}{N\sigma^2}. \tag{2}$$

In UWB, $\mathbf{x}_k$ is generally sparse due to the fact that the channel is sparse and/or the delay spread of the channel is much smaller than the symbol period. Thus according to the CS theory [7], [8] it can be represented by M linear measurements with $M \ll N$. These measurements are generally obtained through analog processing of $r(t)$ [14], but for convenience, we model them here as an operation that is carried out on the Nyquist rate sampled version of $r(t)$.

The compressed received signal can be modeled as

$$\begin{aligned}\mathbf{y}_k &= \mathbf{\Phi}_k \mathbf{r}_k \\ &= \mathbf{\Phi}_k \mathbf{x}_k + \mathbf{\Phi}_k \mathbf{v}_k \\ &= \mathbf{\Phi}_k \mathbf{x}_k + \mathbf{n}_k,\end{aligned} \tag{3}$$

where the $M \times N$ matrix $\mathbf{\Phi}_k$ is the transform operator or measurement matrix at time instant k with M linear functionals as its rows. Note that this matrix $\mathbf{\Phi}_k$ changes with time in its most general form, but this is not required.

The compression ratio δ is defined as $\delta = M/N$ where $\delta \in (0, 1]$. It has a direct impact on the performance of CS. A higher value of δ implies a higher value of M and hence a better result but obviously we would like to have acceptable performance with minimum M so as to decrease the sampling rate.

III. Signal Recovery

Let us first look at a single time instant k, and focus on the reconstruction of $\mathbf{x}_k$. To simplify notation, we omit the time index k here. A naive way to reconstruct $\mathbf{x}$ is by adopting ordinary least squares (OLS). An estimate for $\mathbf{x}$ is then obtained as

$$\hat{\mathbf{x}}_{\mathrm{ols}} = \arg\min_{\mathbf{x}} \|\mathbf{y} - \mathbf{\Phi}\mathbf{x}\|_2^2 \tag{4}$$

but since $\mathbf{\Phi}$ is fat ($M \ll N$) and thus not full column rank, the solution is not unique. One way to circumvent this problem is to use Tikhonov's regularization which penalizes the OLS cost function with a quadratic penalty (ℓ_2 regularization), leading to

$$\hat{\mathbf{x}}_{\mathrm{reg}} = \arg\min_{\mathbf{x}} \|\mathbf{y} - \mathbf{\Phi}\mathbf{x}\|_2^2 + \lambda \|\mathbf{x}\|_2^2, \tag{5}$$

where λ is the Lagrangian constant. Though this regularization results in a unique solution, it pays no respect to the sparsity of $\mathbf{x}$. Tibshirani proposed in [10] a solution to this problem under the name least-absolute shrinkage and selection operator (LASSO), by suggesting an ℓ_1 regularization term. The estimate can then be written as

$$\hat{\mathbf{x}}_{\mathrm{lasso}} = \arg\min_{\mathbf{x}} \|\mathbf{y} - \mathbf{\Phi}\mathbf{x}\|_2^2 + \lambda \|\mathbf{x}\|_1, \tag{6}$$

where λ is again the Lagrangian constant. The ℓ_1 regularization forces sparsity and some of the coefficients of $\hat{\mathbf{x}}$ will turn out to be exactly zero. This technique has been extensively used for reconstructing sparse signals.

For differentially modoulated UWB signals, the decision is made on two consecutive symbols (assume the time instants $k = 1$ and $k = 2$ are considered). The sparse received UWB signals then have to be reconstructed and their polarities have to be compared in order to make a decision. Therefore, there are basically two cost functions to consider. One is related to the ℓ_1 regularized OLS cost function for the two consecutive symbol periods:

$$\begin{aligned}C_{\mathrm{a}} = &\frac{1}{2}\|\mathbf{y}_1 - \mathbf{\Phi}_1\mathbf{x}_1\|_2^2 + \lambda_1\|\mathbf{x}_1\|_1 \\ &+ \frac{1}{2}\|\mathbf{y}_2 - \mathbf{\Phi}_2\mathbf{x}_2\|_2^2 + \lambda_2\|\mathbf{x}_2\|_1,\end{aligned} \tag{7}$$

whereas the other one is related to the differential detection error:

$$C_{\mathrm{b}} = \frac{1}{2}\|\mathbf{x}_1 - b_2\mathbf{x}_2\|_2^2. \tag{8}$$

There are now two ways to proceed. We can follow the rather straightforward separate recovery approach, in which we first minimize (7) over $\mathbf{x}_1$ and $\mathbf{x}_2$ and then minimize (8) over b_2 using the previously estimated received UWB signals $\mathbf{x}_1$ and $\mathbf{x}_2$. Both problems are convex, which makes this approach easy to solve. But since it is a two-step approach, it might not be the most optimal thing to do. We will come back to the separate recovery approach in Section V. In this paper, we advocate a joint recovery approach, where we jointly try to estimate $\mathbf{x}_1$, $\mathbf{x}_2$, and b_2. To achieve this, we will minimize a weighted sum of (7) and (8):

$$\begin{aligned}C = &\frac{1}{2}\|\mathbf{y}_1 - \mathbf{\Phi}_1\mathbf{x}_1\|_2^2 + \lambda_1\|\mathbf{x}_1\|_1 \\ &+ \frac{1}{2}\|\mathbf{y}_2 - \mathbf{\Phi}_2\mathbf{x}_2\|_2^2 + \lambda_2\|\mathbf{x}_2\|_1 \\ &+ \frac{\alpha}{2}\|\mathbf{x}_1 - b_2\mathbf{x}_2\|_2^2\end{aligned} \tag{9}$$

where α is the weighting factor. Clearly (9) is not convex, and thus we cannot use ordinary convex optimization techniques to determine $\mathbf{x}_1$, $\mathbf{x}_2$, and b_2. In the next section, we derive an algorithm that can lead us to a local minimum of (9).

Apart from the both aforementioned scenarios, the cost function can also be expressed in the form of compressed symbols. Since the aim of differential detection is to make a decision on the polarity of the next symbol, we can write the cost function as

$$C_{comp} = \frac{1}{2}\|\mathbf{y}_1 - b_2\mathbf{y}_2\|_2^2 \tag{10}$$

which has a simple closed form solution

$$\hat{b}_2 = \mathrm{sign}(\mathbf{y}_2^{\mathrm{T}}\mathbf{y}_1) \tag{11}$$

Clearly this approach does not take any advantage of the sparsity of the signal and does not involve the reconstruction of $\mathbf{x}_1$ and $\mathbf{x}_2$, therefore its performance may not be optimal. We shall compare the performance of (11) with our proposed method by means of simulations.

IV. JOINT RECOVERY

Augmented Lagrangian methods have been suggested in literature for optimizing cost functions which are not strictly convex [9]. It adds a quadratic penalty to the cost function and it is then solved by using the so-called method of multipliers which carries out successive minimizations of the cost function till convergence. These successive approximations can result in an optimal solution. In [9] one such method of multipliers has been suggested which they named as alternating direction method of multipliers (ADMoM). In ADMoM, the augmented Lagrangian is minimized with respect to one set of variables and then with respect to the other set of variables and finally the multipliers are updated. This cycle continues till convergence. The application of ADMoM in sparse signal recovery has for instance been successfully demonstrated in [11]. The context in [11] is different but their cost function is actually very much related to our convex cost function (7). Here, we will apply ADMoM to our non-convex cost function (9).

Let us first introduce two $N \times 1$ auxiliary vectors $\mathbf{u}_1$ and $\mathbf{u}_2$ and rewrite the cost function (9) as

$$\begin{aligned} C = &\frac{1}{2}\left\|\mathbf{y}_1 - \mathbf{\Phi}_1\mathbf{x}_1\right\|_2^2 + \lambda_1\left\|\mathbf{u}_1\right\|_1 \\ &+ \frac{1}{2}\left\|\mathbf{y}_2 - \mathbf{\Phi}_2\mathbf{x}_2\right\|_2^2 + \lambda_2\left\|\mathbf{u}_2\right\|_1 \\ &+ \frac{\alpha}{2}\left\|\mathbf{x}_1 - b_2\mathbf{x}_2\right\|_2^2, \end{aligned} \tag{12}$$

which we will then minimize subject to the equality constraints $\mathbf{u}_1 = \mathbf{x}_1$ and $\mathbf{u}_2 = \mathbf{x}_2$. The augmented Lagrangian function $\mathcal{L}(\mathbf{x}_1, \mathbf{x}_2, \mathbf{u}_1, \mathbf{u}_2, \mathbf{l}_1, \mathbf{l}_2, b_2)$ can then be expressed as

$$\begin{aligned} \mathcal{L} = &\frac{1}{2}\left\|\mathbf{y}_1 - \mathbf{\Phi}_1\mathbf{x}_1\right\|_2^2 + \lambda_1\left\|\mathbf{u}_1\right\|_1 \\ &+ \frac{1}{2}\left\|\mathbf{y}_2 - \mathbf{\Phi}_2\mathbf{x}_2\right\|_2^2 + \lambda_2\left\|\mathbf{u}_2\right\|_1 \\ &+ \mathbf{l}_1^H(\mathbf{x}_1 - \mathbf{u}_1) + \mathbf{l}_2^H(\mathbf{x}_2 - \mathbf{u}_2) \\ &+ \frac{c}{2}\left(\left\|\mathbf{x}_1 - \mathbf{u}_1\right\|_2^2 + \left\|\mathbf{x}_2 - \mathbf{u}_2\right\|_2^2\right) \\ &+ \frac{\alpha}{2}\left\|\mathbf{x}_1 - b_2\mathbf{x}_2\right\|_2^2 \end{aligned} \tag{13}$$

where vectors $\mathbf{l}_1$ and $\mathbf{l}_2$ are the Lagrange multipliers and c is an arbitrary constant. The successive approximations for the solutions can then be written as

$$\mathbf{x}_1^{(i)} = \arg\min_{\mathbf{x}_1} \mathcal{L}(\mathbf{x}_1, \mathbf{x}_2^{(i-1)}, \mathbf{u}_1^{(i-1)}, \mathbf{l}_1^{(i-1)}, b_2^{(i-1)}) \tag{14}$$

$$\mathbf{x}_2^{(i)} = \arg\min_{\mathbf{x}_2} \mathcal{L}(\mathbf{x}_1^{(i-1)}, \mathbf{x}_2, \mathbf{u}_2^{(i-1)}, \mathbf{l}_2^{(i-1)}, b_2^{(i-1)}) \tag{15}$$

$$b_2^{(i)} = \arg\min_{b_2} \mathcal{L}(\mathbf{x}_1^{(i)}, \mathbf{x}_2^{(i)}, b_2) \tag{16}$$

$$\mathbf{u}_1^{(i)} = \arg\min_{\mathbf{u}_1} \mathcal{L}(\mathbf{x}_1^{(i)}, \mathbf{u}_1, \mathbf{l}_1^{(i-1)}) \tag{17}$$

$$\mathbf{u}_2^{(i)} = \arg\min_{\mathbf{u}_2} \mathcal{L}(\mathbf{x}_2^{(i)}, \mathbf{u}_2, \mathbf{l}_2^{(i-1)}) \tag{18}$$

and the resulting ADMoM iterations are

$$\begin{aligned} \mathbf{x}_1^{(i)} = &\left(\mathbf{\Phi}_1^H\mathbf{\Phi}_1 + (c+\alpha)\mathbf{I}\right)^{-1} \\ &\times \left(\mathbf{\Phi}_1^H\mathbf{y}_1 + c\mathbf{u}_1^{(i-1)} - \mathbf{l}_1^{(i-1)} + \alpha b_2^{(i-1)}\mathbf{x}_2^{(i-1)}\right) \end{aligned} \tag{19}$$

$$\begin{aligned} \mathbf{x}_2^{(i)} = &\left(\mathbf{\Phi}_2^H\mathbf{\Phi}_2 + (c+\alpha[b_2^{(i-1)}]^2)\mathbf{I}\right)^{-1} \\ &\times \left(\mathbf{\Phi}_2^H\mathbf{y}_2 + c\mathbf{u}_2^{(i-1)} - \mathbf{l}_2^{(i-1)} + \alpha b_2^{(i-1)}\mathbf{x}_1^{(i-1)}\right) \end{aligned} \tag{20}$$

$$b_2^{(i)} = \frac{\mathbf{x}_2^{(i)T}\mathbf{x}_1^{(i)}}{\|\mathbf{x}_2^{(i)}\|_2^2} \tag{21}$$

$$\mathbf{u}_1^{(i)} = \text{shrink}\left(\mathbf{x}_1^{(i)} + \frac{\mathbf{l}_1^{(i-1)}}{c}, \frac{\lambda_1}{c}\right) \tag{22}$$

$$\mathbf{u}_2^{(i)} = \text{shrink}\left(\mathbf{x}_2^{(i)} + \frac{\mathbf{l}_2^{(i-1)}}{c}, \frac{\lambda_2}{c}\right) \tag{23}$$

and updates for the Lagrange multipliers are

$$\mathbf{l}_1^{(i)} = \mathbf{l}_1^{(i-1)} + c\left(\mathbf{x}_1^{(i)} - \mathbf{u}_1^{(i)}\right) \tag{24}$$

$$\mathbf{l}_2^{(i)} = \mathbf{l}_2^{(i-1)} + c\left(\mathbf{x}_2^{(i)} - \mathbf{u}_2^{(i)}\right) \tag{25}$$

where shrink$(.,.)$ is the soft thresholding operator which can be defined as in [10]

$$\begin{aligned} \text{shrink}\,(z,\gamma) &= \text{sign}(z)(|z|-\gamma)_+ \\ &= \begin{cases} z-\gamma & \text{if } z>0 \text{ and } \gamma<|z| \\ z+\gamma & \text{if } z<0 \text{ and } \gamma<|z| \\ 0 & \text{if } \gamma \geq |z| \end{cases}. \end{aligned} \tag{26}$$

The initial values for ${\mathbf{u}_1}^{(0)}$, ${\mathbf{u}_2}^{(0)}$, ${\mathbf{l}_1}^{(0)}$, ${\mathbf{l}_2}^{(0)}$ and $b_2^{(0)}$ can be chosen arbitrarily, and for ${\mathbf{x}_1}^{(0)}$ and ${\mathbf{x}_2}^{(0)}$, different types of warm starts can be chosen. Convergence results for ADMoM can be found in [9] which basically state that as i goes to infinity, convergence to a local optimum is obtained. Note that in practice though very few iterations are needed to get good results. Thus we can solve (9) by using (19-25). Note that the finite-alphabet estimate for b_2, which we will denote as $\hat{b}_2$, is finally obtained by taking the sign of $b_2^{(i)}$ obtained in the last iteration.

V. Separate Recovery

We compare the performance of our joint recovery approach with the separate recovery approach, which has been discussed earlier. In a first step, we have to minimize (7) for $\mathbf{x}_1$ and $\mathbf{x}_2$, which can be solved for example by using CVX (a package for specifying and solving convex programs [12]). However, to make a fair comparison between the two approaches, it is more appropriate that we solve (7) by ADMoM as well. Hence, in this section, we will propose the ADMoM solution for (7).

Using a similar method as in Section IV, we can write the augmented Lagrangian function $\mathcal{L}(\mathbf{x}_1, \mathbf{x}_2, \mathbf{u}_1, \mathbf{u}_2, \mathbf{l}_1, \mathbf{l}_2)$ for (7) as

$$\begin{aligned}\mathcal{L} = & \frac{1}{2}\left\|\mathbf{y}_1 - \mathbf{\Phi}_1\mathbf{x}_1\right\|_2^2 + \lambda_1\left\|\mathbf{u}_1\right\|_1 \\ & + \frac{1}{2}\left\|\mathbf{y}_2 - \mathbf{\Phi}_2\mathbf{x}_2\right\|_2^2 + \lambda_2\left\|\mathbf{u}_2\right\|_1 \\ & + {\mathbf{l}_1}^H(\mathbf{x}_1 - \mathbf{u}_1) + {\mathbf{l}_2}^H(\mathbf{x}_2 - \mathbf{u}_2) \\ & + \frac{c}{2}\left(\left\|\mathbf{x}_1 - \mathbf{u}_1\right\|_2^2 + \left\|\mathbf{x}_2 - \mathbf{u}_2\right\|_2^2\right) \end{aligned} \quad (27)$$

and the ADMoM iterations can be derived as

$$\mathbf{x}_1^{(i)} = \left(\mathbf{\Phi}_1^H\mathbf{\Phi}_1 + c\mathbf{I}\right)^{-1}\left(\mathbf{\Phi}_1^H\mathbf{y}_1 + c\mathbf{u}_1^{(i-1)} - \mathbf{l}_1^{(i-1)}\right) \quad (28)$$

$$\mathbf{x}_2^{(i)} = \left(\mathbf{\Phi}_2^H\mathbf{\Phi}_2 + c\mathbf{I}\right)^{-1}\left(\mathbf{\Phi}_2^H\mathbf{y}_2 + c\mathbf{u}_2^{(i-1)} - \mathbf{l}_2^{(i-1)}\right) \quad (29)$$

$$\mathbf{u}_1^{(i)} = \text{shrink}\left(\mathbf{x}_1^{(i)} + \frac{\mathbf{l}_1^{(i-1)}}{c}, \frac{\lambda_1}{c}\right) \quad (30)$$

$$\mathbf{u}_2^{(i)} = \text{shrink}\left(\mathbf{x}_2^{(i)} + \frac{\mathbf{l}_2^{(i-1)}}{c}, \frac{\lambda_2}{c}\right) \quad (31)$$

$$\mathbf{l}_1^{(i)} = \mathbf{l}_1^{(i-1)} + c\left(\mathbf{x}_1^{(i)} - \mathbf{u}_1^{(i)}\right) \quad (32)$$

$$\mathbf{l}_2^{(i)} = \mathbf{l}_2^{(i-1)} + c\left(\mathbf{x}_2^{(i)} - \mathbf{u}_2^{(i)}\right) \quad (33)$$

In a second step, we then minimize (8) over b_2 using the estimates $\mathbf{x}_1^{(i)}$ and $\mathbf{x}_2^{(i)}$ obtained in the last iteration of the first step. Taking into account the finite-alphabet constraint, this leads to

$$\hat{b}_2 = \text{sign}(\mathbf{x}_2^{(i)T}\mathbf{x}_1^{(i)}). \quad (34)$$

VI. Validation

Values of parameters α and λ can impact the optimal solution of (9). λ enforces sparsity and α puts more weight on the decision of b_2. A very large value of λ can eliminate important components of the symbol and a very small value can allow a lot of noise components to effect the decision. Similarly small values of α may not help much in making a right decision and large values can render sparsity irrelevant. One way of choosing the values for λ and α is by means of training and/or decision directed symbols. By observing the bit error rate (BER) results, appropriate values can then be selected.

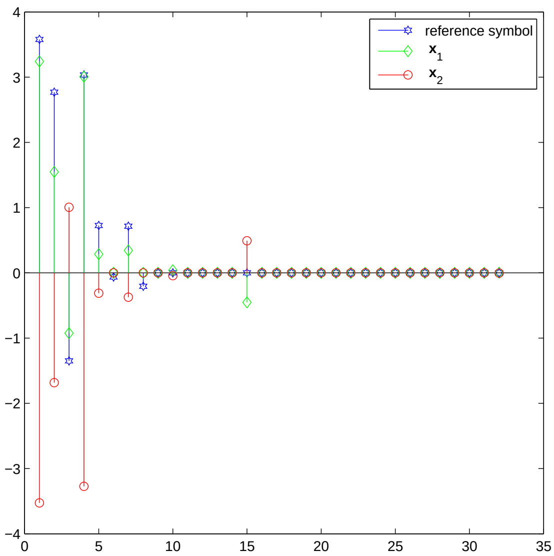

Figure 1. Reconstruction using the joint ADMoM detection approach method at $\eta = 4$ dB.

VII. Simulation Results

In this section, we shall illustrate the performance of our detection approach by means of simulations. For the purpose of illustration we consider a toy example. We assume that the received symbol can be represented with $N = 32$ Nyquist rate samples. The received symbol is compressed using a random Gaussian measurement matrix $\mathbf{\Phi}$ (where $\mathbf{\Phi} = \mathbf{\Phi}_1 = \mathbf{\Phi}_2$) with $M = N/2$ rows. So $\delta = 0.5$ and we are sampling at half the Nyquist rate.

From the reduced set of samples we reconstruct the two consecutive symbols as well as the data symbol using the joint ADMoM detection approach as described in Section IV. The initial values $\mathbf{u}_1^{(0)}$, $\mathbf{u}_2^{(0)}$, $\mathbf{l}_1^{(0)}$, $\mathbf{l}_2^{(0)}$, and $b_2^{(0)}$ are taken equal to zero. For $\mathbf{x}_1^{(0)}$ and $\mathbf{x}_2^{(0)}$, an ordinary least squares estimate has been utilized but solutions of other problems, e.g., the solutions of (5) or (6) can also be utilized as a warm start.

Figure 1 shows one realization of the reconstructed received UWB signals that were obtained using the joint ADMoM detection approach at an SNR of $\eta = 4$ dB and $\lambda = 0.1893$ (where $\lambda = \lambda_1 = \lambda_2$). We use the ADMoM iterations for a tolerance level of 10^{-4} with a maximum limit of 10000 iterations. Samples labeled as 'reference symbol' show the true received UWB signal without noise which contains the composite channel impulse response including the transmitted pulse and the receive filter. It provides a reference as to how well ADMoM can reconstruct the received symbol. Clearly, the reconstructed samples labeled as $\mathbf{x}_1$ and $\mathbf{x}_2$ seem to have the opposite polarity, so for this realization, we have $b_2 = -1$. We also notice that many small components of the noisy received UWB signals have been forced to zero. The nonzero reconstructed entries have an energy that is comparable to the

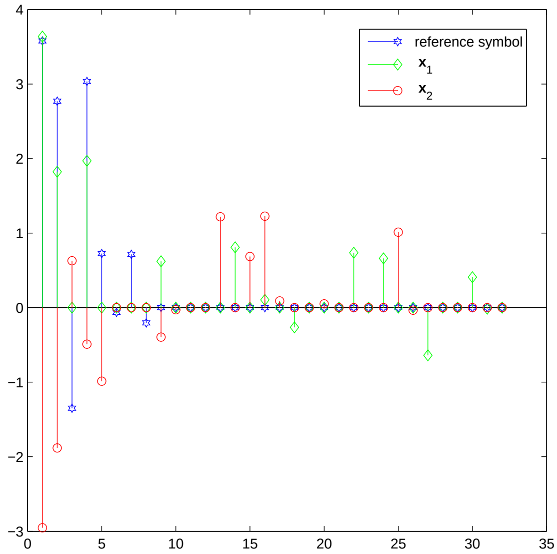

Figure 2. Reconstruction using separate ADMoM detection approach method at $\eta = 4$ dB.

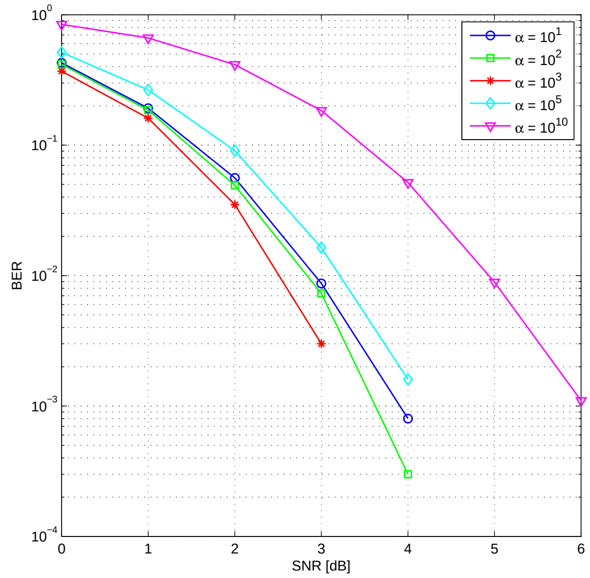

Figure 3. BER comparison of varying values of α for joint ADMoM detection approach.

reference signal.

Figure 2 shows the reconstructed symbols $\mathbf{x}_1$ and $\mathbf{x}_2$ by utilizing the separate ADMoM approach at an SNR of $\eta = 4$ dB and $\lambda = 0.1893$. Clearly the performance is not as optimal as that of the joint ADMoM approach.

For our simulations of joint ADMoM detection, we have verified the optimal α through bit error rate (BER) curves for different values of α by fixing $\lambda = 0.1893$. Figure 3 shows the BER results of the joint ADMoM detection for varying values of α. We can see that initially the BER decreases with increase in α but after $\alpha = 10^3$, it starts increasing. Thus minimum BER is achieved for an optimal value of $\alpha = 10^3$.

To demonstrate the performance of our detection approach, we show simulation results for the BER of both the joint and separate ADMoM detection approaches. We compare the ADMoM performance with the compressed detection approach (11) (i.e., without reconstructing $\mathbf{x}_1$ and $\mathbf{x}_2$) as well. Figure 4 compares the BER results for these three approaches. We take $\alpha = 10^3$ and $\lambda = 0.1893$. It is clear that the ADMoM approaches perform better than the detection through compressed symbols. Also the joint detection approach is performing even better than the separate detection approach. It is also clear that both ADMoM approaches are good enough to be used for differential detection of UWB signals.

To assess the performance of CS, we carry out simulations to find the BER for different values of M. We vary δ from 0.25 to 1, so M varies from 8 to 32 for $N = 32$. We plot BER curves for $\eta = 1, 2, 3$ dB. We see that even at such low SNR, we get reasonable performance.

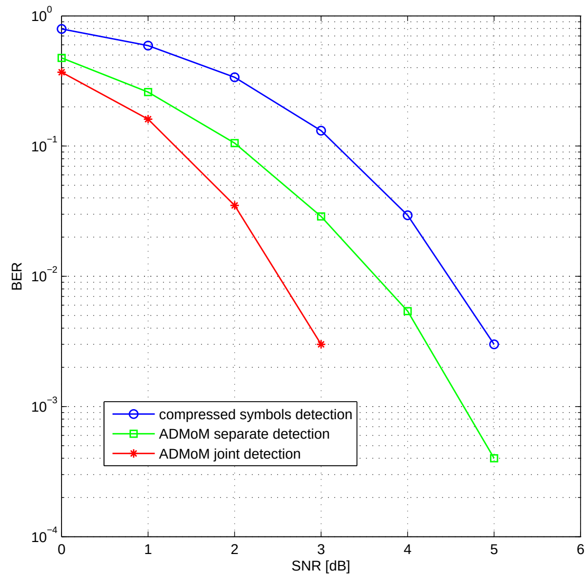

Figure 4. Instantaneous BER comparison for compressed, separate ADMoM and joint ADMoM detection approaches.

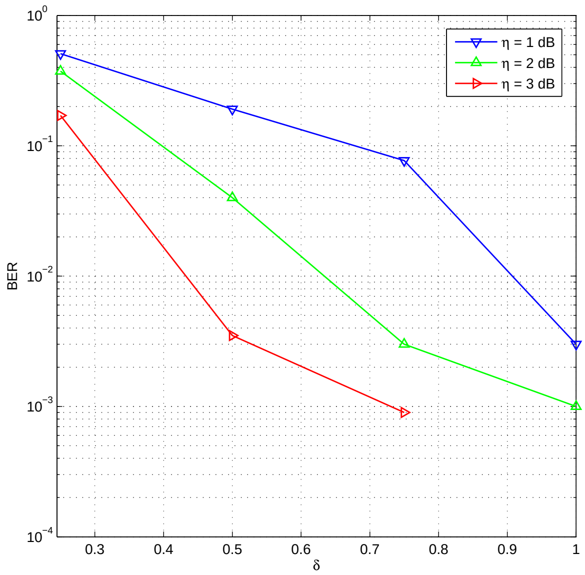

Figure 5. BER versus varying δ for SNR $\eta = 1, 2, 3$ dB

VIII. Conclusions

In this paper we have proposed compressive sampling based UWB differential detectors to reduce the sampling rate much below the Nyquist. We have also proposed joint and separate detection approaches based on the ADMoM method. We have shown with the help of simulations that our joint detection approach outperforms the separate and compressed symbol detection approaches.

References

[1] M. Z. Win and R. A. Scholtz, "Impulse radio: how it works," *IEEE Communications Letters*, vol. 2, no. 2, pp. 36-38, Feb. 1998.

[2] M. Ghavami, L. B. Michael and R. Kohno, *Ultra Wideband signals and systems in communication engineering*, 2nd ed. West Sussex, England: John Wiley and Sons, 2007.

[3] J. G. Proakis, *Digital Communications*, 4th ed. Avenue of Americas, NY; McGraw-Hill, 2001.

[4] M. Ho, S. Somayazulu, J. Foerster, S. Roy, "A differential detector for UWB communications system," *IEEE VTC*, vol. 4, pp. 1896-1900, May 2002.

[5] V. Lottici, Z. Tian, "Multiple symbol differential detection for UWB communications," *IEEE Trans. Wireless Commun.*, vol. 7, no. 5, pp. 1656-1666, 2008.

[6] V. Lottici, Z. Tian, G. Leus, "A novel approach to UWB data detection with symbol-level synchronization," *Physical Communication*, vol. 2, pp. 296-305, 2009.

[7] David L. Donoho, "Compressed Sensing," *IEEE Transactions on Information Theory*, Vol. 52, No. 4,April 2006.

[8] E. Candès, J. Romberg and T. Tao, "Robust Uncertainty Principles: Exact signal reconstruction from highly incomplete frequency information," *IEEE Transaction on Information Theroy*, Vol. 52, No. 2, pp. 489-509, February 2006.

[9] D.P. Bertsekas and J.N. Tsitsiklis, *Parallel and Distributed Computation: Numerical Methods*, Athena Scientific, 1997.

[10] R. Tibshirani, "Regression shrinkage and selection via the lasso", Technical Report, University of Toronto, 1994.

[11] D. Angelosante, G.B. Giannakis, "Multiple frequency-hopping signal estimation via sparse regression," *IEEE ICASSP*, 2010.

[12] M. Grant and S. Boyd. CVX: Matlab software for disciplined convex programming (web page and software). http://stanford.edu/ boyd/cvx, June 2009.

[13] Y. Vanderperren, W. Dehaene and G. Leus, "Performance analysis of a flexible subsampling receiver for pulsed UWB signals," *IEEE Trans. Wireless. Comm.* Aug. 2009.

[14] S. Kirolos, J. Laska, M. Wakin, M. Duarte, D. Baron, T. Ragheb, Y. Massoud, R. Baraniuk, "Analog-to-Information Conversion via Random Demodulation," in Proc. *IEEE Dallas Circuits and Systems Workshop* (DCAS), 2006.