ANALYSIS OF ESPRIT BASED JOINT ANGLE-FREQUENCY ESTIMATION

Aweke N. Lemma[1], *Alle-Jan van der Veen and Ed F. Deprettere*

Delft University of Technology, Faculty of Information Technology
Mekelweg 4, 2628 CD Delft, The Netherlands
aweke@cas.et.tudelft.nl

ABSTRACT

High-resolution parameter estimation techniques have recently been applied to jointly estimate multiple signal parameters. In this work, we consider the problem of determining the directions and center frequencies of a number of narrow-band sources in a certain frequency band of interest. We present a joint angle-frequency estimation method, based on the multidimensional ESPRIT algorithm. A perturbation error analysis gives bounds on the parameter estimates, and provides optimal values for the temporal and spatial smoothing parameters. The analysis is shown to be consistent with simulation results.

1. INTRODUCTION

In the literature, a number of ESPRIT-based joint angle and frequency estimation methods have been proposed. In particular, Zoltowski et al. [1] discuss this problem in the context of radar applications and Haardt et al. [2] discuss the problem in the context of mobile communications for space division multiple access (SDMA) applications. A similar but simpler algorithm called JAFE (joint angle-frequency estimation) has been proposed by us in [3].

The objective of this paper is to give a comprehensive error analysis of this algorithm. The algorithm is a function of certain stacking (or smoothing) parameters, and the error analysis provides us with the optimal choices for these parameters.

2. MODEL

Suppose that there are d sources of interest, with complex baseband representations $s_i(t)$, for $i = 1, \cdots, d$. Let the band of interest have a center frequency f_c, and suppose that the i-th source has a carrier frequency of $f_c + f_i$. After demodulation to IF, the signal due to the i-th source is $e^{j2\pi f_i t} s_i(t)$, and the signal received at the k-th antenna $k = 1, \cdots, M$ is

$$x_k(t) = \sum_{i=1}^{d} a_k(\theta_i) e^{j2\pi f_i t} b_i s_i(t) + w_k(t)$$

where θ_i is the parameterization of the direction of arrival of the i-th signal, with respect to a common phase reference, $a_k(\theta)$ is the antenna response of the k-th antenna to a signal from direction θ, $b_i \in \mathbb{R}^+$ is the amplitude of the i-th signal, and $w_k(t)$ is noise. It is natural to stack the antenna outputs into a single vector $\boldsymbol{x}(t)$.

Further suppose that the narrow-band signals have a bandwidth of less than $\frac{1}{T}$, so that they can be sampled with a period T to satisfy the Nyquist rate. We normalize to $T = 1$. Let us say that the bandwidth of the band to be scanned is an integer number P times larger: after demodulation to IF we have to sample at a rate P (obviously we require $-\frac{P}{2} \le f_i < \frac{P}{2}$ to prevent aliasing). The data sample at the receiver is

$$\boldsymbol{x}(\tfrac{n}{P}) = \sum_{i=1}^{d} \boldsymbol{a}(\theta_i) b_i e^{j\frac{2\pi}{P} f_i n} s_i(\tfrac{n}{P}) + \boldsymbol{w}(\tfrac{n}{P})$$

where $\boldsymbol{a}(\theta_i)$ is the array response vector for the i-th source and $\boldsymbol{w}(\frac{n}{P}) \in \mathbb{C}^{M,1}$ is the noise vector collecting the samples of the noise terms at the output of each antenna element. In matrix form this can be written as

$$\boldsymbol{x}(\tfrac{n}{P}) = \boldsymbol{A}\boldsymbol{B}\boldsymbol{\Phi}^n \boldsymbol{s}(\tfrac{n}{P}) + \boldsymbol{w}(\frac{n}{P}) \qquad (1)$$

where $\boldsymbol{\Phi} = \text{diag}\{\phi_i\}_{i=1}^d$, $\phi_i = e^{j\frac{2\pi}{P} f_i}$, $\boldsymbol{B} = \text{diag}\{b_i\}_{i=1}^d$ is a signal gain matrix, $\boldsymbol{A}$ is an $M \times d$ matrix collecting the d steering vectors, and the vector $\boldsymbol{s}(t)$ is a stack of the d signals, where each signal has a unit amplitude. In the remainder of the paper, unless it is necessary to write it explicitly, the diagonal matrix $\boldsymbol{B}$ in the data model is absorbed by $\boldsymbol{s}(t)$, in which case the amplitude of the i-th signal is equal to b_i instead of 1. Assume that we have collected N samples of the array output $\boldsymbol{x}(t)$ at a rate P into the $M \times N$ data matrix $\boldsymbol{X}$, i.e.,

$$\boldsymbol{X} = \boldsymbol{A} \left[\; \boldsymbol{s}(0) \;\; \boldsymbol{s}(\tfrac{1}{P}) \;\; \cdots \;\; \boldsymbol{s}(\tfrac{N-1}{P}) \; \right] + \boldsymbol{W} \in \mathbb{C}^{M,N}, \qquad (2)$$

where $\boldsymbol{W} \in \mathbb{C}^{M,N}$ is a matrix collecting N samples of the $M \times 1$ array noise vector. Let $\boldsymbol{S}(k) \in \mathbb{C}^{d,N-m+1}$ be given as

$$\boldsymbol{S}(k) = \left[\; \boldsymbol{s}(\tfrac{k}{P}) \;\; \boldsymbol{s}(\tfrac{1}{P}) \;\; \cdots \;\; \boldsymbol{s}(\tfrac{N-m+k}{P}) \; \right]$$

Then an *m-factor* temporally smoothed data matrix $\boldsymbol{X}_m$ is constructed as

$$\boldsymbol{X}_m = \begin{bmatrix} \boldsymbol{A}\boldsymbol{S}(0) \\ \vdots \\ \boldsymbol{A}\boldsymbol{S}(m-1) \end{bmatrix} + \boldsymbol{W}_m \in \mathbb{C}^{mM,N-m+1} 4 \qquad (3)$$

$\boldsymbol{W}_m$ represents the noise term constructed from $\boldsymbol{W}$ in a similar way as $\boldsymbol{X}_m$ is obtained from $\boldsymbol{X}$. Assume that the signals are narrow band, i.e.,

$$\boldsymbol{S}(0) \approx \boldsymbol{S}(1) \approx \cdots \approx \boldsymbol{S}(m-1) = \boldsymbol{S}.$$

In this case, $\boldsymbol{X}_m$ has the factorization

$$\boldsymbol{X}_m \approx \begin{bmatrix} \boldsymbol{A} \\ \boldsymbol{A}\boldsymbol{\Phi} \\ \vdots \\ \boldsymbol{A}\boldsymbol{\Phi}^{m-1} \end{bmatrix} \boldsymbol{S} =: \boldsymbol{A}_m \boldsymbol{S} + \boldsymbol{W}_m \qquad (4)$$

where $\boldsymbol{A}_m$ is referred to as the extended array steering matrix.

[1]The research of Aweke N. Lemma is supported by TNO-FEL, The Hague, The Netherlands.

2.1. The Estimation Algorithm

At this point, we have obtained a model with much the same structure as in the classical ESPRIT algorithm, but with $\boldsymbol{A}$ replaced by $\boldsymbol{A}_m$. The estimation of the parameters and the construction of the beamformer can now follow the same strategy as well. First note that the rank of $\boldsymbol{X}_m$ is only d, since this is the number of rows of $\boldsymbol{S}$. We compute the SVD of $\boldsymbol{X}_m$, i.e. $\boldsymbol{X}_m =: \boldsymbol{U}_s \boldsymbol{\Sigma}_s \boldsymbol{V}_s$ where $\boldsymbol{U}_s$ has d columns, spanning the column space of $\boldsymbol{X}_m$. Thus for some non-singular $d \times d$ matrix $\boldsymbol{T}$,

$$\boldsymbol{U}_s = \boldsymbol{A}_m \boldsymbol{T}^{-1}.$$

We begin the estimation of the parameters by defining two types of selection matrices: a pair to select submatrices for estimating $\boldsymbol{\Phi}$, and a pair for estimating $\boldsymbol{\Theta} = \text{diag}\{\theta_i\}_{i=1}^d$:

$$\begin{cases} \boldsymbol{J}^x(\phi) := [\boldsymbol{I}_{m-1} \quad \boldsymbol{0}_1] \otimes \boldsymbol{I}_M \\ \boldsymbol{J}^y(\phi) := [\boldsymbol{0}_1 \quad \boldsymbol{I}_{m-1}] \otimes \boldsymbol{I}_M, \end{cases} \tag{5}$$

$$\begin{cases} \boldsymbol{J}^x(\theta) := \boldsymbol{I}_m \otimes [\boldsymbol{I}_{M-1} \quad \boldsymbol{0}_1] \\ \boldsymbol{J}^y(\theta) := \boldsymbol{I}_m \otimes [\boldsymbol{0}_1 \quad \boldsymbol{I}_{M-1}]. \end{cases} \tag{6}$$

To estimate $\boldsymbol{\Phi}$, we take submatrices consisting of the first and the last $M(m-1)$ rows of $\boldsymbol{U}_s$, respectively, i.e.,

$$\boldsymbol{U}_{x,\phi} = \boldsymbol{J}^x(\phi)\boldsymbol{U}_s\,, \qquad \boldsymbol{U}_{y,\phi} = \boldsymbol{J}^y(\phi)\boldsymbol{U}_s\,,$$

whereas to estimate $\boldsymbol{\Theta}$ we stack, for all m blocks, its first and last $M-1$ rows, respectively:

$$\boldsymbol{U}_{x,\theta} = \boldsymbol{J}^x(\theta)\boldsymbol{U}_s\,, \qquad \boldsymbol{U}_{y,\theta} = \boldsymbol{J}^y(\theta)\boldsymbol{U}_s\,.$$

These data matrices have the structures

$$\begin{cases} \boldsymbol{U}_{x,\phi} = \boldsymbol{A}'\boldsymbol{T}^{-1} \\ \boldsymbol{U}_{y,\phi} = \boldsymbol{A}'\boldsymbol{\Phi}\boldsymbol{T}^{-1} \end{cases} \qquad \begin{cases} \boldsymbol{U}_{x,\theta} = \boldsymbol{A}''\boldsymbol{T}^{-1} \\ \boldsymbol{U}_{y,\theta} = \boldsymbol{A}''\boldsymbol{\Theta}\boldsymbol{T}^{-1} \end{cases}, \tag{7}$$

where $\boldsymbol{A}'$ and $\boldsymbol{A}''$ are both submatrices of $\boldsymbol{A}_m$. If dimensions are such that these are low-rank factorizations, then

$$\begin{aligned} \boldsymbol{E}_\phi &:= \boldsymbol{U}_{x,\phi}^\dagger \boldsymbol{U}_{y,\phi} = \boldsymbol{T}\boldsymbol{\Phi}\boldsymbol{T}^{-1} \\ \boldsymbol{E}_\theta &:= \boldsymbol{U}_{x,\theta}^\dagger \boldsymbol{U}_{y,\theta} = \boldsymbol{T}\boldsymbol{\Theta}\boldsymbol{T}^{-1}, \end{aligned} \tag{8}$$

It is seen that the matrices $\boldsymbol{E}_\phi$ and $\boldsymbol{E}_\theta$ are jointly diagonalizable by the same matrix $\boldsymbol{T}$. There are several algorithms to compute this joint diagonalization, e.g. by mans of QZ iteration [4] or Jacobi iterations [5]. For this to work, it is necessary that each submatrix in (7) has at least d rows. After $\boldsymbol{T}$ has been found, we also have estimates of $\{(\theta_i, \phi_i)\}$ for each of the d sources. This provides us with angle and frequency estimates:

$$\alpha_i = \text{asin}\left(\frac{\arg(\theta_i)}{2\pi\Delta}\right), \qquad f_i = \frac{P}{2\pi}\arg(\phi_i).$$

2.2. Spatio-temporally smoothed Data Model

Consider an M element antenna array and assume that we divide the array into L sub-arrays. Let M_L be the number of antenna elements in the subarrays, and let for $l = 1, \ldots, L$, the selection matrix $\boldsymbol{J}_l \in \mathbb{R}^{mM_L, mM}$ select part of the data matrix $\boldsymbol{X}_m$ that corresponds to the l-th subarray. Then, an (m, L) factor spatio-temporally smoothed data matrix $\boldsymbol{X}_{m,L}$ is constructed as

$$\boldsymbol{X}_{m,L} = \left[\, \boldsymbol{J}_1\boldsymbol{X}_m \;\; \boldsymbol{J}_2\boldsymbol{X}_m \;\; \cdots \;\; \boldsymbol{J}_L\boldsymbol{X}_m \,\right] \in \mathbb{C}^{mM_L, L(N-m+1)}. \tag{9}$$

Using the structure of $\boldsymbol{X}_m$ from (3), this can be factored as

$$\boldsymbol{X}_{m,L} = \left[\, \boldsymbol{J}_1\boldsymbol{A}_m \;\cdots\; \boldsymbol{J}_L\boldsymbol{A}_m \,\right] \begin{bmatrix} \boldsymbol{S} & & \\ & \ddots & \\ & & \boldsymbol{S} \end{bmatrix} + \boldsymbol{W}_{m,L},$$

where $\boldsymbol{W}_{m,L}$ is a noise term which has also been shuffled in a similar way as $\boldsymbol{X}_{m,L}$. Let $\boldsymbol{A}'_m = \boldsymbol{J}_1\boldsymbol{A}_m \in \mathbb{C}^{mM_L, d}$, then from the shift invariance structure of $\boldsymbol{A}_m$ it follows that, for $k = 1, \ldots, L$

$$\boldsymbol{J}_k\boldsymbol{A}_m = \boldsymbol{J}_1\boldsymbol{A}_m\boldsymbol{\Theta}^{k-1} =: \boldsymbol{A}'_m\boldsymbol{\Theta}^{k-1}.$$

Thus, $\boldsymbol{X}_{m,L}$ can be written in a compact form as

$$\boldsymbol{X}_{m,L} = \boldsymbol{A}'_m \left[\, \boldsymbol{S} \;\; \boldsymbol{\Theta}\boldsymbol{S} \;\cdots\; \boldsymbol{\Theta}^{L-1}\boldsymbol{S} \,\right] + \boldsymbol{W}_{m,L}. \tag{10}$$

This data model contains the dual shift invariance property needed by the JAFE algorithm. Thus, the angle-frequency pairs may be estimated in the usual way (by considering shift invariance pairs).

2.3. Whitening as the JAFE Processing Stage

The spatio-temporal smoothing procedure introduces correlation between the noise terms in the different rows of the data matrix. In many cases this correlation causes degradation as it tends to reduce the degree of averaging that could have been obtained had the noise been white. In this context the JAFE algorithm can be preceded with a whitening filter.

Consider the noise part of spatio-temporally smoothed data matrix given in 9. Let the the SVD of the noise covariance matrix $\boldsymbol{R}_{w,w} = \boldsymbol{W}_{m,L}\boldsymbol{W}_{m,L}^H$ be given by

$$\boldsymbol{R}_{w,w} = \boldsymbol{U}_n\boldsymbol{\Sigma}_n\boldsymbol{V}_n^H.$$

Then, the Whitened data matrix is derived as (viz. [6])

$$\boldsymbol{X}'_{m,L} = \boldsymbol{U}_n\boldsymbol{\Sigma}_n^{-1/2}\boldsymbol{U}_n^H\boldsymbol{X}_{m,L}. \tag{11}$$

Let the SVD of $\boldsymbol{X}'_{m,L}$ be given by $\boldsymbol{X}'_{m,L} = \boldsymbol{U}'\boldsymbol{\Sigma}'\boldsymbol{V}'$, and let $\boldsymbol{U}'_s$ be the d dominant columns of $\boldsymbol{U}'$ corresponding to the d largest singularvalues, then the JAFE algorithm is implemented by requiring dual shift invariance property on $\boldsymbol{U}_s = \boldsymbol{U}_n\boldsymbol{\Sigma}_n^{1/2}\boldsymbol{U}_n^H\boldsymbol{U}'_s$. The effect of whitening on the parameter estimation errors is summarized in the simulation results.

3. A PERFORMANCE ANALYSIS

In this section, we give a behavioral analysis of the JAFE algorithm. As described in section 2.1, the JAFE algorithm involves three main steps, namely, SVD (singularvalue decomposition) of the data matrix, diagonalization of a set of EVD (eigenvalue decomposition) problems and the transformation of the eigenvalues into signal parameters. The first step, which is equivalent to finding the EVD of the data covariance matrix, is well studied in the literature [7–11], for the case of white Gaussian noise contaminated data model. In our case, however, since some data stacking techniques have been employed, the noise is no more white. Thus, first we will derive the eigenvalue estimation error for the JAFE data model and then proceed to the other two JAFE stages. Analysis of the last two steps, in the context of white Gaussian noise, have been presented in [12] and [13]. The results obtained here could be seen as the generalization of these results.

3.1. Eigenvectors of the Data Covariance Matrix

Following, we give fundamental results in forms of theorems. Because of shortage of space the proofs are not given, they will appear in the Journal version of the paper.

Theorem 3.1 *Consider an M-element antenna array impinged by* d *far field narrowband signals. Let the* (m,L) *factor spatio-temporally smoothed data matrix $\boldsymbol{X}_{m,L}$ be as given in (9), $\boldsymbol{R}_{m,L} = \frac{1}{L(N-m+1)}\boldsymbol{X}_{m,L}\boldsymbol{X}_{m,L}^H$ be the finite sample data covariance matrix, and $\boldsymbol{U} \in \mathbb{C}^{mM_L, mM_L}$ and $\boldsymbol{\Sigma} \in \mathbb{C}^{mM_L, mM_L}$ be such that the eigenvalue decomposition of $\bar{\boldsymbol{R}}_{m,L} = E\{\boldsymbol{R}_{m,L}\}$ is given by*

$$\bar{\boldsymbol{R}}_{m,L} = \boldsymbol{U}\boldsymbol{\Sigma}^2\boldsymbol{U}^H.$$

Let $\boldsymbol{U}_s$ be the first d *columns of $\boldsymbol{U}$ and $\boldsymbol{u}_j$ be the j-th column of $\boldsymbol{U}_s$, where $j \leq d$. Moreover, let $\Delta\boldsymbol{u}_j$ represent a noise caused perturbation on $\boldsymbol{u}_j$, $\sigma_k^2 = \bar{\sigma}_k^2 + \sigma_n^2$ be the k-th eigenvalue of $\bar{\boldsymbol{R}}_{m,L}$, where $\bar{\sigma}_k^2$ is the k-th noise free eigenvalue and σ_n^2 is the noise contribution. Let $\boldsymbol{\Phi} = diag\{\phi_i\}_{i=1}^d$, and $\boldsymbol{\Theta} = diag\{\phi_i\}_{i=1}^d$ be the parameterizations of the center frequencies and direction of arrivals of the d signals. Moreover, let*

- *$\boldsymbol{Z}^x$ be a Toeplitz matrix with all the elements equal to zero, except for those unity valued entries on the x-th parallel to the main diagonal,*
- *$\mu := \min(m, N-m)$ and $\eta := \min(M_L, L)$*
- *$\boldsymbol{Q} := \boldsymbol{T}^{-1}$, $\boldsymbol{q}_x^H$ and $\boldsymbol{t}_x$ be the x-th row and column of the matrices $\boldsymbol{Q}^H$ and $\boldsymbol{T}^H$, respectively. Note that $\boldsymbol{q}_x^H\boldsymbol{t}_y = 0$, for $x \neq y$ and $\boldsymbol{q}_x^H\boldsymbol{t}_y = 1$ for $x = y$.*
- $\Upsilon_{x,y,v,w} := \sum_{h=-\mu}^{\mu}\sum_{r=-\eta}^{\eta} \boldsymbol{u}_x^H\boldsymbol{Z}^{hM_L+r}\boldsymbol{u}_y\boldsymbol{q}_v^H\boldsymbol{\Phi}^h\boldsymbol{\Theta}^r\boldsymbol{t}_w$
- $\Omega_{x,y,v,w} := \sum_{h=-\mu}^{\mu}\sum_{r=-\eta}^{\eta} \boldsymbol{u}_x^H\boldsymbol{Z}^{-hM_L-r}\boldsymbol{u}_y\boldsymbol{q}_v^H\boldsymbol{\Phi}^{-h}\boldsymbol{\Theta}^{-r}\boldsymbol{t}_w$
- $\Psi_{x,y,v,w} := \sum_{h=-\mu}^{\mu}\sum_{r=-\eta}^{\eta} \boldsymbol{u}_x^H\boldsymbol{z}^{hM_L+r}\boldsymbol{u}_y\boldsymbol{u}_v^H\boldsymbol{z}^{-hM_L-r}\boldsymbol{u}_w$

Assuming that $\bar{\boldsymbol{R}}_{m,L}$ has distinct eigenvalues, the covariance of the eigenvector estimation error $E\{\Delta\boldsymbol{u}_j\Delta\boldsymbol{u}_p^H\}$ (to the first order approximation) is given by

$$E\{\Delta\boldsymbol{u}_v\Delta\boldsymbol{u}_w^H\} \approx \frac{\sigma_n^2}{L(N-m+1)}\times \sum_{\substack{j=1\\j\neq v}}^{mM_L}\sum_{\substack{k=1\\k\neq w}}^{mM_L}\frac{\bar{\sigma}_v^2\Omega_{j,k,v,w} + \sigma_n^2\Psi_{j,k,v,w}}{(\sigma_v^2-\sigma_j^2)(\sigma_w^2-\sigma_k^2)}\boldsymbol{u}_j\boldsymbol{u}_k^H + \sum_{\substack{j=1\\j\neq v}}^{d}\sum_{\substack{k=1\\k\neq w}}^{d}\frac{\bar{\sigma}_j^2\Upsilon_{v,w,j,k}}{(\sigma_v^2-\sigma_j^2)(\sigma_w^2-\sigma_k^2)}\boldsymbol{u}_j\boldsymbol{u}_k^H. \quad (12)$$

Lemma 3.1 *For whitened spatio-temporally smoothed data matrix the covariance of the eigenvector estimation error $E\{\Delta\boldsymbol{u}_v\Delta\boldsymbol{u}_w^H\}$ (to the first order approximation) is given by*

$$E\{\Delta\boldsymbol{u}_v\Delta\boldsymbol{u}_v^H\} \approx \frac{1}{L(N-m+1)}\times \left(\sum_{\substack{j=1\\j\neq v}}^{d}\frac{\sigma_v^2\sigma_j^2 - \bar{\sigma}_v^2\bar{\sigma}_j^2}{(\sigma_v^2-\sigma_j^2)^2}\boldsymbol{u}_j\boldsymbol{u}_j^H + \sigma_n^2\sigma_v^2\sum_{j=d+1}^{mM_L}\frac{\boldsymbol{u}_j\boldsymbol{u}_j^H}{(\sigma_v^2-\sigma_j^2)^2}\right), \quad (13)$$

for $w = v$ and $E\{\Delta\boldsymbol{u}_v\Delta\boldsymbol{u}_w^H\} = 0$ for $w \neq v$.

3.2. The Shift Invariance Parameters

The following theorems, whose proofs are omitted because of shortage of space, summarize the major results.

Theorem 3.2 *Consider an $M \times N$ data matrix $\boldsymbol{X}$ collected at the output of an M element ULA. Assume that an (m, L) factor spatio-temporally smoothed data matrix $\boldsymbol{X}_{m,L}$ is constructed from $\boldsymbol{X}$ as discussed in section 2.2, and that the signal parameters are estimated from $\boldsymbol{X}_{m,L}$. If the sources are well separated both in angle and frequency, the variance of the parameterized frequency estimation error (denoted by σ_ϕ^2) is given by*

$$\sigma_\phi^2 \approx \frac{1}{SNR}\left(\frac{2}{(m-1)^2(N-m+1)L(M-L+1)}\right) \quad (14)$$

and the variance of the parameterized DOA estimation error (denoted by σ_θ^2) is given by

$$\sigma_\theta^2 \approx \frac{1}{SNR}\left(\frac{2}{m(N-m+1)L(M-L)^2}\right) \quad (15)$$

Optimizing the above two functions with respect to the data stacking parameters, we obtain optimum values for m and L.

$$L_{opt} = \begin{cases} \frac{M}{3}, & \text{for } \theta \\ \frac{M+1}{2}, & \text{for } \phi \end{cases} \qquad m_{opt} = \begin{cases} \frac{N+1}{2}, & \text{for } \theta \\ \frac{2N+3}{3}, & \text{for } \phi \end{cases}.$$

4. SIMULATION EXAMPLES

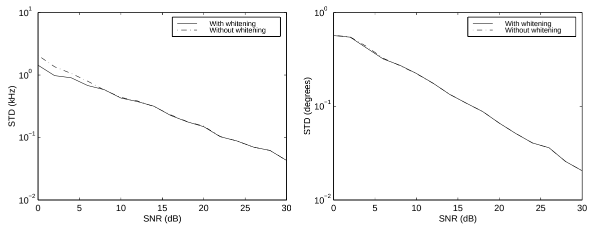
In this simulation example, we consider a 4 element ULA with baseline separation of $\Delta = 1/2$. We assume that two far field, equal power signals s_1 and s_2 are impinging on the antenna array. The DOA and center frequency of s_1 are $\alpha_1 = 10$ Degrees and $f_1 = 2$MHz, and those of s_2 are $\alpha_2 = 55$ Degrees and $f_2 = 5$MHz respectively. The source signals are narrow-band (25 kHz) amplitude modulated signals. The data is sampled at a rate of 20 MHz, and the processing is done over $N = 32$ time samples. All simulation results are based on 100 Monte-Carlo runs. The behaviors are summarized in Fig. 2 through Fig. 4. In Fig. 3 and Fig. 4, while keeping the rest of the parameters fixed at their original values, the DOA and center frequency of s_2 are varied to generate behaviors as functions of angular and frequency separations, respectively.

The effect of whitening on the frequency and DOA estimation errors is shown in Fig. 1. The results show that whitening improves the frequency estimation accuracy at low SNR regions, but has insignificant effect on the DOA estimation accuracy.

Fig. 2(a) shows how temporal smoothing affects the parameter estimation errors. From the plots, one can clearly see the agreement between the theoretically predicted and the simulated results. As predicted, the DOA estimation error is minimum for $m = N/2$ and the frequency estimation error is minimum for $m = 2N/3$. In

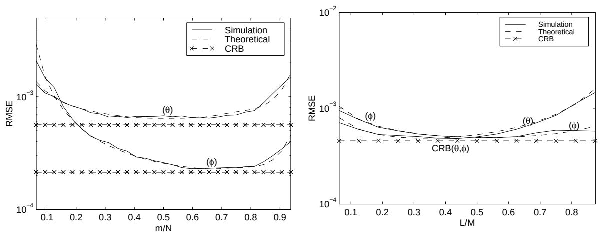
Fig. 3, it is shown that, apart from improving the estimation accuracy, temporal smoothing also provides robustness against rank loss when there exist multiple signals with the same DOA.

The effect of spatial smoothing on the estimation errors is summarized in Fig. 2(b). The simulation was run using a ULA with $M = 16$ elements, $N = 16$, $m = m_o = 10$ and SNR $=$ 20dB. The DOAs and the center frequencies of the two sources under consideration are the same as before. It is seen that the theoretically predicted behavior perfectly agrees with the simulation results. As predicted, the parameterized DOA and frequency estimation errors are minimum for $L = \frac{M}{3}$ and $L = \frac{M+1}{2}$, respectively. Moreover, in Fig. 4 it is seen that, apart from performance improvement, spatial smoothing achieves rank restoration when several (two in this case) signals have the same center frequencies.



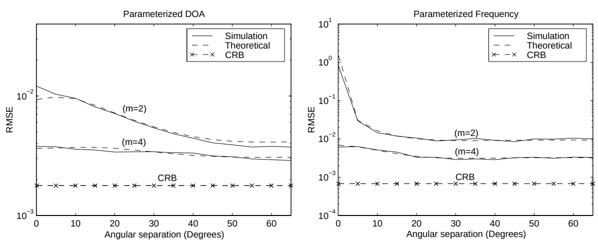
(a) (b)

Figure 1: The effect of whitening a) on frequency estimation error and b) on DOA estimation error.



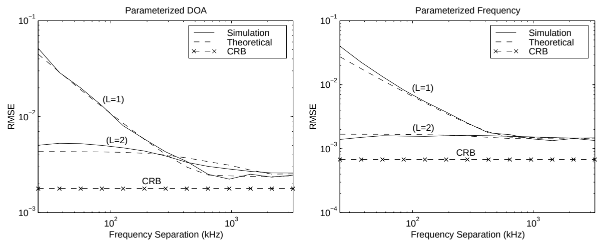
(a) (b)

Figure 2: The parameterized DOA and frequency estimation errors a) as functions of temporal smoothing factor m at SNR $= 30$dB and $L = 2$. b) as functions of spatial smoothing factor L at SNR $=$ 20dB and $m = 2$.



(a) (b)

Figure 3: Behavior of (a) the parameterized DOA and (b) the parameterized frequency estimation errors as functions of angular separation. (SNR $= 20$dB).



(a) (b)

Figure 4: Behavior of (a) the parameterized DOA and (b) the parameterized frequency estimation errors as functions of frequency separation. (SNR $= 20$dB.)

REFERENCES

[1] Michael D. Zoltowski and Cherian P. Mathews, "Real-time frequency and 2-D angle estimation with sub-nyquist spatio-temporal sampling," *IEEE Trans. Signal Processing*, vol. SP-42, pp. 2781–2794, October 1994.

[2] M. Haardt and J. A. Nossek, "3-D unitary ESPRIT for joint angle and carrier estimation," in *Proc. ICASSP'97*, (Munich, Germany), pp. 255–258, April 1997.

[3] Aweke N. Lemma, A.-J. van der Veen, and Ed F. Deprettere, "Joint angle-frequency estimation using multi-resolution ESPRIT," in *Proc. ICASSP'98*, (Seattle, Washington), May 1998.

[4] A.-J. van der Veen and A. Paulraj, "An analytical constant modulus algorithm," *IEEE Trans. Signal Processing*, vol. 44, pp. 1136–1155, May 1996.

[5] A.-J. van der Veen, P.B. Ober, and E.F. Deprettere, "Azimuth and elevation computation in high resolution DOA estimation," *IEEE Trans. Signal Processing*, vol. 40, pp. 1828–1832, July 1992.

[6] J. E. Hudson, *Adaptive Array Principles*. London: Institution of Electrical Engineers, 1981.

[7] T. W. Anderson, "Asymptotic theory for principal component analysis," *Annals Maths. Stat.*, vol. 34, pp. 122–148, 1963.

[8] D. R. Brillinger, *Time Series: Data Analysis and Theory*. New York: Holt, Rhinehart and Winston, 1975.

[9] D. J. Jeffries and D. R. Farrier, "Asymptotic results for eigenvector methods," *Proc. Inst. Elect. Eng.*, vol. 132, pp. 589–594, June 1985.

[10] Mostafa Kaveh and Arthur J. barabell, "The statistical performance of MUSIC and minimum-norm algorithms in resolving plane waves," *IEEE Trans. on Acoust. Speech and Signal processing*, vol. ASSP-34, pp. 331–341, April 1986.

[11] Harry Lee and Fu Li, "An eigenvector technique for detecting the number of emitters in a cluster," *IEEE Trans. SP*, vol. 42, pp. 2380–2388, September 1994.

[12] Bhaskar D. Rao and K. V. S. Hari, "Performance analysis of ESPRIT and TAM in determining the direction of arrival of plane waves in noise," *IEEE Trans. Acous. Speech and Signal Processing*, vol. 37, pp. 1990–1995, December 1989.

[13] P. Stoica and A. Nehorai, "Performance comparison of subspace rotation and MUSIC methods for direction estimation," *IEEE Tran. Signal Processing*, vol. 39, pp. 446–453, February 1991.