Space-Time Block Coding for Frequency-Selective and Time-Varying Channels

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***Abstract*—In this work, we present a new space-time block code for time- and frequency-selective (doubly-selective) channels. It can be interpreted as the extension of the Alamouti code to doubly-selective channels, and relies on a joint time-frequency reversal of the transmitted sequences. Under certain channel conditions, the proposed space-time block code belongs to the class that achieves full spatial, delay, and Doppler diversity using a maximum likelihood (ML) receiver, as well as a linear zero-forcing (LZF) or linear minimum mean-squared error (LMMSE) receiver. For realistic doubly-selective channels, a real-valued linear data model is presented, for which different receiver structures can be developed.**

***Index Terms*—Space-time block coding, doubly-selective channels, delay diversity, Doppler diversity.**

I. Introduction

Modern wireless communication systems not only require high transmission rates giving rise to frequency-selectivity due to multipath propagation, but they also need to support high-mobility terminals and scatterers, which induce Doppler shifts. Advanced techniques are needed to accurately model time- and frequency-selective (doubly-selective) channels and to counteract the related performance degradation. However, doubly-selective channels can also provide multiplicative delay-Doppler diversity gains if the transceiver is properly designed [1], with diversity being an effective way to combat fading channels.

In the last decade, multi-antenna systems have attracted a lot of research interest for future wireless systems. The use of multiple transmit and/or receive antennas can significantly enhance communication system performances such as channel capacity and reliability [2]. Space-time block coding (STBC) [3], [4] has been introduced to achieve the spatial diversity offered by multiple transmit and/or receive antennas. However, as STBC is typically designed for flat-fading channels, the time- and frequency-selectivity will seriously degrade the system performance.

Among the papers considering time-selective channels, [6] designs STBC for purely time-selective channels by transforming the time-selective channels into frequency-selective channels, and by adjusting existing space-time code designs over frequency-selective MIMO channels to collect joint space-Doppler gains over purely time-selective MIMO channels. Further, [7] uses digital phase sweeping (DPS) to develop a space-time code that can achieve full space-delay-Doppler diversity for any number of transmit-receive antennas. However, to quantify the maximum Doppler diversity order, the above papers rely on a parsimonious critically sampled complex-exponential basis expansion model (CCE-BEM) for the underlying purely time-selective or doubly-selective channels [9].

In this paper, we develop a novel STBC for multi-antenna transmissions over doubly-selective channels. The proposed STBC is designed for a multiple-input single-output (MISO) system with 2 transmit antennas and 1 receive antenna, but it is straightforward to extend the ideas to a general multiple-input multiple-output (MIMO) system. The proposed technique can be interpreted as the extension of the Alamouti code to doubly-selective channels, and relies on a joint time-frequency reversal of the transmitted sequences. Assuming a block fading channel where the time-variation from subblock to subblock is modeled by a CCE-BEM, the proposed STBC belongs to the class that achieves full spatial, delay, and Doppler diversity using a maximum likelihood (ML) receiver, as well as a linear zero-forcing (LZF) or linear minimum mean-squared error (LMMSE) receiver. For realistic doubly-selective channels, which can not exactly be modeled by a block fading CCE-BEM channel, a real-valued linear data model is presented, for which different receiver structures can be developed. This paper assumes that the receiver has perfect channel state information (CSI), as well as perfect knowledge of the maximum delay spread τ_{max} and the maximum Doppler spread f_{max} which can be derived from the wireless transmission channel. The transmitter on the other hand has no access to CSI.

Notation: We use upper (lower) bold face letters to denote matrices (column vectors). $(\cdot)^*$, $(\cdot)^T$ and $(\cdot)^H$ represent complex conjugate, transpose and complex conjugate transpose (Hermitian), respectively. $\mathrm{E}\{x\}$ stands for the expectation with respect to x. a mod b gives the remainder of a divided by b. We use $[\mathbf{x}]_p$ to indicate the $(p+1)$st element of $\mathbf{x}$, and $[\mathbf{X}]_{p,q}$ to indicate the $(p+1, q+1)$st entry of $\mathbf{X}$. Further, we let $\mathbf{I}_N$ denote an $N \times N$ identity matrix and $\mathbf{0}_{M\times N}$ an $M \times N$ all-zero matrix. $\mathbf{F}_N$ denotes the unitary N-point DFT matrix with $[\mathbf{F}_N]_{p,q} = \frac{1}{\sqrt{N}} e^{-j\frac{2\pi}{N}pq}$. We use the symbol $\otimes$ to denote the Kronecker product between matrices. The $J \times J$ permutation matrices $\{\mathbf{P}_J^{(n)}\}_{n=0}^{J-1}$ are defined to perform a reversed cyclic shift, i.e., $[\mathbf{P}_J^{(n)}\mathbf{a}]_p = [\mathbf{a}]_{(J-p+n) \bmod J}$.

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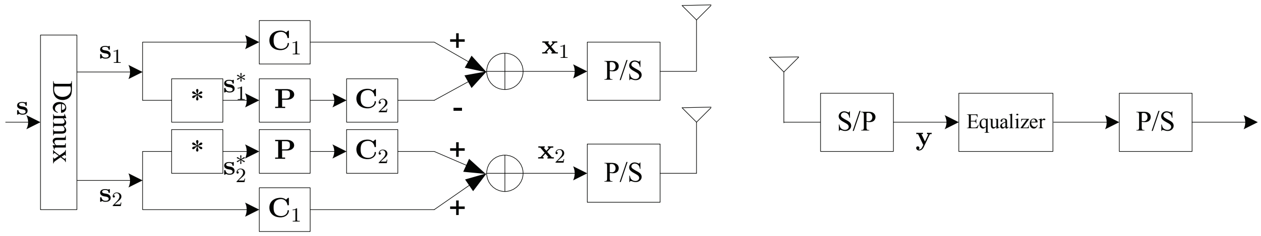

Fig. 1. System model of the proposed STBC system.

II. System Model

We focus on a discrete-time baseband-equivalent description. Suppose $x_t[n]$ is the symbol sequence transmitted over the t-th transmit antenna. The received signal can then be written as

$$y[n] = \sum_{t=1}^{2} \sum_{l=0}^{L} h_t[n;l] x_t[n-l] + \eta[n], \tag{1}$$

where $h_t[n;l]$ is the order-L time- and frequency-selective channel from the t-th transmit antenna to the receive antenna, and $\eta[n]$ is the additive noise. Suppose now that the STBC has a length of N. In order to avoid inter block interference (IBI), we design our STBC codewords in such a way that the last L symbols within each codeword are zero (as shown in the next section). Since IBI is then avoided, the equalizer at the receiver can be designed for each codeword separately. For simplicity reasons, we here focus on the first codeword; the other codewords can be treated in a similar fashion. Parsing $x_t[n]$ and $y[n]$ into blocks of length N, with $\mathbf{x}_t = [x_t[0], x_t[1], \ldots, x_t[N-1]]^T$ and $\mathbf{y} = [y[0], y[1], \ldots, y[N-1]]^T$, respectively, the input-output relationship can be expressed as

$$\mathbf{y} = \sum_{t=1}^{2} \mathbf{H}_t \mathbf{x}_t + \boldsymbol{\eta}, \tag{2}$$

where $\mathbf{H}_t$ is the $N \times N$ channel matrix with $[\mathbf{H}_t]_{n,n'} = h_t[n;(n-n') \bmod N]$ (we may use the modulo operator here since every codeword has L zeros at the end), and $\boldsymbol{\eta} = [\eta[0], \eta[1], \ldots, \eta[N-1]]^T$. For simplicity, we assume that $\boldsymbol{\eta}$ is a circular complex Gaussian noise vector with zero mean and covariance matrix $\mathrm{E}\{\boldsymbol{\eta}\boldsymbol{\eta}^H\} = \sigma_\eta^2 \mathbf{I}_N$.

III. Space-Time Block Coding

Orthogonal STBC [3], [4] has been designed to achieve the spatial diversity offered by multiple transmit and/or receive antennas. The STBC schemes proposed in [3], [4] are designed for flat-fading channels, which lead to a performance degradation in time- and frequency-selective channels. Our goal is to design efficient STBC schemes to counteract the effects of doubly-selective channels. We assume a *block fading CCE-BEM* channel, which is defined as a block fading channel where the time-variation from subblock to subblock is modeled by a CCE-BEM. In this section, we develop and analyze the STBC under this block fading CCE-BEM channel model, and in the next section, we show how to decode the STBC for real-life channels, which do not exactly fit this block fading CCE-BEM channel model.

A. Block Fading CCE-BEM Channel Model

We assume that within the span of one STBC codeword, the doubly-selective channel behaves like a block fading channel, where the fading from subblock to subblock can be described by a CCE-BEM. Assume for instance that the span of the STBC codeword can be split into $2P'$ subblocks of length K', i.e., $N = 2P'K'$. Every channel is then assumed to be constant within every subblock of length K' and to vary over the $2P'$ subblocks as a CCE-BEM, which uses $2Q+1$ complex exponential basis functions to model the time variation over the $2P'$ subblocks:

$$h_t[n;l] = \sum_{q=-Q}^{Q} e^{j2\pi(\lfloor n/K' \rfloor)q/(2P')} h_{t,q}[l], \tag{3}$$

where $h_{t,q}[l]$ is the q-th CCE-BEM coefficient of the l-th channel tap within the STBC codeword. The $(L+1)(2Q+1)$ BEM coefficients $\{\{h_{t,q}[l]\}_{l=0}^{L}\}_{q=-Q}^{Q}$ remain constant during each length-N block, and are allowed to change over different length-N blocks. The $2Q+1$ CCE-BEM basis functions used to capture the time variations are the same for every length-N block. Q can be regarded as the discrete Doppler spread index with frequency-domain resolution $1/(NT)$, and it needs to satisfy $Q/(NT) \geq f_{max}$.

Under the block fading CCE-BEM channel model, the channel matrix $\mathbf{H}_t$ can be written as

$$\mathbf{H}_t = \sum_{q=-Q}^{Q} (\boldsymbol{\Lambda}_{2P',q} \otimes \mathbf{I}_{K'}) \mathbf{H}_{N,t,q}, \tag{4}$$

where $\boldsymbol{\Lambda}_{2P',q}$ is the $2P' \times 2P'$ diagonal matrix given by $[\boldsymbol{\Lambda}_{2P',q}]_{p,p} = e^{j2\pi pq/(2P')}$ and $\mathbf{H}_{N,t,q}$ is the $N \times N$ circulant matrix given by $[\mathbf{H}_{N,t,q}]_{n,n'} = h_{t,q}[(n-n') \bmod N]$.

B. Code Design

In [8], it has been shown how to generate orthogonal full-diversity subchannels in case the CCE-BEM is adopted to model the time-variation of the channel from sample to sample. It has been introduced there to develop a multi-user communications scheme where users remain orthogonal after propagation over a doubly-selective channel and where the full delay-Doppler diversity of a doubly-selective channel is enabled. Similarly, the same transmission scheme can be used to generate two orthogonal full-diversity subchannels in case the CCE-BEM is used to model the time-variation of the channel from subblock to subblock. This will be the basis of our STBC design.

Assuming $K' > L$ and $P' > 2Q$, and defining $K = K' - L$ and $P = P' - 2Q$, let us introduce the channel-independent $N \times PK$ spreading matrices $\mathbf{C}_u$ and $N \times P'K'$ despreading matrices $\mathbf{D}_u$ defined as [8]

$$\mathbf{C}_u = [\mathbf{F}_{2P'}^H(\mathbf{c}_u \otimes \mathbf{T}_2)] \otimes \mathbf{T}_1, \quad (5)$$

$$\mathbf{D}_u = [\mathbf{F}_{2P'}^H(\mathbf{c}_u \otimes \mathbf{I}_{P'})] \otimes \mathbf{I}_{K'}, \quad (6)$$

where $\mathbf{T}_1 = [\mathbf{I}_K, \mathbf{0}_{K\times L}]^T$ is the $K' \times K$ zero padding matrix, $\mathbf{T}_2 = [\mathbf{0}_{P\times Q}, \mathbf{I}_P, \mathbf{0}_{P\times Q}]^T$ is the $P' \times P$ two-sided zero inserting matrix, and $\{\mathbf{c}_u\}_{u=1}^2$ is an arbitrary set of 2 orthonormal code vectors. Notice that the last L rows of the spreading matrix $\mathbf{C}_u$ are set to zero, which avoids the IBI. Similar to [8], it is possible to show that these spreading matrices $\mathbf{C}_u$ and despreading matrices $\mathbf{D}_u$ can be used to create two orthogonal full-diversity subchannels under the assumption of a block fading CCE-BEM channel model. The composite channel matrix consisting of the block fading CCE-BEM channel as well as the spreading and despreading operations can be expressed as [8]

$$\mathbf{D}_{u'}^H \mathbf{H}_t \mathbf{C}_u = \begin{cases} \boldsymbol{\mathcal{H}}_t \mathbf{T}, & u' = u; \\ \mathbf{0}_{P'K'\times PK}, & u' \neq u, \end{cases} \quad (7)$$

where $\boldsymbol{\mathcal{H}}_t = \sum_{q=-Q}^{Q} \mathbf{J}_{P',q} \otimes \mathbf{H}_{K',t,q}$ and $\mathbf{T} = \mathbf{T}_2 \otimes \mathbf{T}_1$, with $\mathbf{J}_{P',q}$ the $P' \times P'$ circulant matrix given by $[\mathbf{J}_{P',q}]_{p,p'} = \delta[(p - p' - q) \bmod P']$ and $\mathbf{H}_{K',t,q}$ the $K' \times K'$ circulant matrix given by $[\mathbf{H}_{K',t,q}]_{k,k'} = h_{t,q}[(k - k') \bmod K']$.

The proposed STBC now proceeds as follows. We start by demultiplexing a $2PK \times 1$ data vector $\mathbf{s}$ into two $PK \times 1$ data subvectors $\mathbf{s}_1$ and $\mathbf{s}_2$, where the data symbols are assumed to be circular complex with zero mean and covariance matrix $\mathrm{E}\{\mathbf{s}\mathbf{s}^H\} = \sigma_s^2 \mathbf{I}_{2PK}$. On the first and second antenna, we then send

$$\begin{aligned} \mathbf{x}_1 &= \mathbf{C}_1\mathbf{s}_1 - \mathbf{C}_2\mathbf{P}\mathbf{s}_2^*, \\ \mathbf{x}_2 &= \mathbf{C}_1\mathbf{s}_2 + \mathbf{C}_2\mathbf{P}\mathbf{s}_1^*, \end{aligned} \quad (8)$$

where $\mathbf{P}$ is a $PK \times PK$ permutation matrix that will be determined later on. The received signal can now be expressed as

$$\begin{aligned} \mathbf{y} &= \mathbf{H}_1\mathbf{x}_1 + \mathbf{H}_2\mathbf{x}_2 + \boldsymbol{\eta} \\ &= \mathbf{H}_1\mathbf{C}_1\mathbf{s}_1 - \mathbf{H}_1\mathbf{C}_2\mathbf{P}\mathbf{s}_2^* + \mathbf{H}_2\mathbf{C}_1\mathbf{s}_2 + \mathbf{H}_2\mathbf{C}_2\mathbf{P}\mathbf{s}_1^* + \boldsymbol{\eta}. \end{aligned} \quad (9)$$

Applying the despreading operations $\mathbf{D}_1$ and $\mathbf{D}_2$ at the receiver, we obtain

$$\bar{\mathbf{y}}_1 = \mathbf{D}_1^H\mathbf{y} = \boldsymbol{\mathcal{H}}_1\mathbf{T}\mathbf{s}_1 + \boldsymbol{\mathcal{H}}_2\mathbf{T}\mathbf{s}_2 + \bar{\boldsymbol{\eta}}_1, \quad (10)$$

$$\bar{\mathbf{y}}_2 = \mathbf{D}_2^H\mathbf{y} = \boldsymbol{\mathcal{H}}_2\mathbf{T}\mathbf{P}\mathbf{s}_1^* - \boldsymbol{\mathcal{H}}_1\mathbf{T}\mathbf{P}\mathbf{s}_2^* + \bar{\boldsymbol{\eta}}_2. \quad (11)$$

Since the $N \times P'K'$ despreading matrix $\mathbf{D}_u$ is a tall matrix and $\mathbf{D}_u^H\mathbf{D}_u = \mathbf{I}_{P'K'}$ [8], $\bar{\boldsymbol{\eta}}_u$ is still a circular complex Gaussian noise vector with zero mean and covariance matrix $\mathrm{E}\{\bar{\boldsymbol{\eta}}_u\bar{\boldsymbol{\eta}}_u^H\} = \sigma_\eta^2\mathbf{I}_{P'K'}$.

We wish to be able to decode the two multiplexed transmitted data streams $\mathbf{s}_1$ and $\mathbf{s}_2$ separately at the receiver, similar to the scalar case for Alamouti decoding [3]. In order to achieve that, we now have to find the $PK \times PK$ permutation matrix $\mathbf{P}$ such that there exists a $P'K' \times P'K'$ permutation matrix $\mathbf{P}'$ for which

$$\mathbf{P}'\boldsymbol{\mathcal{H}}_t\mathbf{T}\mathbf{P} = \boldsymbol{\mathcal{H}}_t^T\mathbf{T}. \quad (12)$$

Since $\boldsymbol{\mathcal{H}}_t$ is a block circulant matrix of circulant matrices, it is easy to show similar to [5] that the permutation matrices that satisfy this property are given by $\mathbf{P} = \mathbf{P}_P^{(P-1)} \otimes \mathbf{P}_K^{(K-1)}$, and $\mathbf{P}' = \mathbf{P}_{P'}^{(P'-1)} \otimes \mathbf{P}_{K'}^{(K-1)}$. Note that $\mathbf{P}$ actually represents a $PK \times PK$ element reversal, where due to the structure of the spreading matrices, the first part $(\mathbf{P}_P^{(P-1)})$ can be interpreted as a frequency reversal and the second part $(\mathbf{P}_K^{(K-1)})$ as a time reversal. As a result, (11) can be rewritten as

$$\mathbf{P}'\bar{\mathbf{y}}_2^* = \boldsymbol{\mathcal{H}}_2^H\mathbf{T}\mathbf{s}_1 - \boldsymbol{\mathcal{H}}_1^H\mathbf{T}\mathbf{s}_2 + \mathbf{P}'\bar{\boldsymbol{\eta}}_2^*, \quad (13)$$

Now applying $\mathbf{F} = \mathbf{F}_{P'} \otimes \mathbf{F}_{K'}$ to (10) and (13), we get

$$\mathbf{F}\bar{\mathbf{y}}_1 = \boldsymbol{\mathcal{G}}_1\mathbf{F}\mathbf{T}\mathbf{s}_1 + \boldsymbol{\mathcal{G}}_2\mathbf{F}\mathbf{T}\mathbf{s}_2 + \mathbf{F}\bar{\boldsymbol{\eta}}_1, \quad (14)$$

$$\mathbf{F}\mathbf{P}'\bar{\mathbf{y}}_2^* = \boldsymbol{\mathcal{G}}_2^*\mathbf{F}\mathbf{T}\mathbf{s}_1 - \boldsymbol{\mathcal{G}}_1^*\mathbf{F}\mathbf{T}\mathbf{s}_2 + \mathbf{F}\mathbf{P}'\bar{\boldsymbol{\eta}}_2^*. \quad (15)$$

In these formulas, $\boldsymbol{\mathcal{G}}_t = \mathbf{F}\boldsymbol{\mathcal{H}}_t\mathbf{F}^H$ is a $P'K' \times P'K'$ diagonal matrix. Stacking (14) and (15), we obtain the following relationship

$$\begin{aligned} \check{\mathbf{y}} &= \begin{bmatrix} \mathbf{F}\bar{\mathbf{y}}_1 \\ \mathbf{F}\mathbf{P}'\bar{\mathbf{y}}_2^* \end{bmatrix} = \begin{bmatrix} \boldsymbol{\mathcal{G}}_1 & \boldsymbol{\mathcal{G}}_2 \\ \boldsymbol{\mathcal{G}}_2^* & -\boldsymbol{\mathcal{G}}_1^* \end{bmatrix} \begin{bmatrix} \mathbf{F}\mathbf{T}\mathbf{s}_1 \\ \mathbf{F}\mathbf{T}\mathbf{s}_2 \end{bmatrix} + \begin{bmatrix} \mathbf{F}\bar{\boldsymbol{\eta}}_1 \\ \mathbf{F}\mathbf{P}'\bar{\boldsymbol{\eta}}_2^* \end{bmatrix} \\ &= \boldsymbol{\mathcal{G}} \begin{bmatrix} \mathbf{F}\mathbf{T}\mathbf{s}_1 \\ \mathbf{F}\mathbf{T}\mathbf{s}_2 \end{bmatrix} + \check{\boldsymbol{\eta}}. \end{aligned} \quad (16)$$

Similar to [5], if we define $\boldsymbol{\mathcal{G}}_{12} = (\boldsymbol{\mathcal{G}}_1^*\boldsymbol{\mathcal{G}}_1 + \boldsymbol{\mathcal{G}}_2^*\boldsymbol{\mathcal{G}}_2)^{1/2}$ and apply the unitary matrix $\mathbf{U} = \boldsymbol{\mathcal{G}}(\mathbf{I}_2 \otimes \boldsymbol{\mathcal{G}}_{12}^{-1})$, we obtain

$$\mathbf{U}^H\check{\mathbf{y}} = \begin{bmatrix} \boldsymbol{\mathcal{G}}_{12}\mathbf{F}\mathbf{T}\mathbf{s}_1 \\ \boldsymbol{\mathcal{G}}_{12}\mathbf{F}\mathbf{T}\mathbf{s}_2 \end{bmatrix} + \mathbf{U}^H\check{\boldsymbol{\eta}}. \quad (17)$$

Note that if $\boldsymbol{\mathcal{G}}_1$ and $\boldsymbol{\mathcal{G}}_2$ share a common zero, we can still design a unitary $\mathbf{U}$ without compromising the validity of (17), by replacing either one of the common zeros by a one in the formula for $\mathbf{U}$, as explained in [5].

In conclusion, by applying complex conjugations and linear unitary matrix operations, we can separate the two substreams, leading to two matrix equations of the form

$$\mathbf{z}_u = \boldsymbol{\mathcal{G}}_{12}\mathbf{F}\mathbf{T}\mathbf{s}_u + \boldsymbol{\zeta}_u = \mathbf{H}\mathbf{s}_u + \boldsymbol{\zeta}_u, \quad (18)$$

where $\mathbf{z}_u(\boldsymbol{\zeta}_u)$ represents the corresponding part of $\mathbf{U}^H\check{\mathbf{y}}(\mathbf{U}^H\check{\boldsymbol{\eta}})$ in (17). Every stream can then be decoded using your favorite decoder. Note that all these derivations only hold under the assumption of a block fading CCE-BEM channel model.

Similar to [8], we can show that if a (near-)ML decoder is used, the proposed STBC enables the maximum space-delay-Doppler diversity that the doubly-selective channel can offer, which is multiplicative in the degrees of freedom of the channel in space, time and frequency dimensions. The proof is an extension of that in [8], and the details can be found in [12]. Using the results of [11], we can even show that if a LZF or LMMSE decoder is used, this full diversity order can still be achieved. It is shown in [11] that if $\det(\mathbf{H}^H\mathbf{H}) > 0, \forall\mathbf{H}$, i.e., $\mathbf{H}$ has full column rank for any channel realization, then the LZF and LMMSE decoder can obtain the same diversity order as the ML decoder.

IV. Proposed Receiver for Realistic Channels

The STBC design and analysis discussed before is based on the block fading CCE-BEM channel model. The nice algebraic structure of this block fading CCE-BEM channel model allows us to extend the Alamouti code to doubly-selective channels enabling the full space-delay-Doppler diversity, as mentioned in Section III-B. Although this channel model was useful to design and analyze our STBC, it does not perfectly model real-life doubly-selective channels under all circumstances [10]. Hence, the receiver processing discussed in Section III-B can only be applied if we approximate the true channel by its best possible fit to a block fading CCE-BEM channel. The related modeling error will of course introduce a bit-error-rate (BER) performance floor at medium to high SNR, and this floor will increase with the Doppler spread. To avoid this floor, we will next propose an alternative receiver for the proposed STBC that is suitable for realistic doubly-selective channels, which do not rely on any specific channel model, so that there is no channel modeling error. More specifically, we will present a real-valued linear data model for the proposed STBC on which any existing receiver structure can be applied.

Defining the $N \times PK$ matrix $\mathbf{K}_{t,u}$ as $\mathbf{K}_{t,u} = \mathbf{H}_t\mathbf{C}_u$, the received vector $\mathbf{y}$ can be written as

$$\mathbf{y} = \mathbf{K}_{1,1}\mathbf{s}_1 - \mathbf{K}_{1,2}\mathbf{P}\mathbf{s}_2^* + \mathbf{K}_{2,1}\mathbf{s}_2 + \mathbf{K}_{2,2}\mathbf{P}\mathbf{s}_1^* + \boldsymbol{\eta}. \quad (19)$$

Further defining

$$\tilde{\mathbf{K}}_{t,u} = \begin{bmatrix} \Re(\mathbf{K}_{t,u}) & -\Im(\mathbf{K}_{t,u}) \\ \Im(\mathbf{K}_{t,u}) & \Re(\mathbf{K}_{t,u}) \end{bmatrix}, \quad (20)$$

$$\tilde{\tilde{\mathbf{K}}}_{\mathbf{t},\mathbf{u}} = \begin{bmatrix} \Re(\mathbf{K}_{t,u}) & \Im(\mathbf{K}_{t,u}) \\ \Im(\mathbf{K}_{t,u}) & -\Re(\mathbf{K}_{t,u}) \end{bmatrix}, \quad (21)$$

we can write $\tilde{\mathbf{y}} = [\Re\{\mathbf{y}^T\}, \Im\{\mathbf{y}^T\}]^T$ as

$$\begin{aligned} \tilde{\mathbf{y}} &= \tilde{\mathbf{K}}_{1,1}\tilde{\mathbf{s}}_1 - \tilde{\tilde{\mathbf{K}}}_{1,2}(\mathbf{I}_2 \otimes \mathbf{P})\tilde{\mathbf{s}}_2 + \tilde{\mathbf{K}}_{2,1}\tilde{\mathbf{s}}_2 \\ &\quad + \tilde{\tilde{\mathbf{K}}}_{2,2}(\mathbf{I}_2 \otimes \mathbf{P})\tilde{\mathbf{s}}_1 + \tilde{\boldsymbol{\eta}} \\ &= \begin{bmatrix} \tilde{\mathbf{K}}_{1,1} + \tilde{\tilde{\mathbf{K}}}_{2,2}(\mathbf{I}_2 \otimes \mathbf{P}) & \tilde{\mathbf{K}}_{2,1} - \tilde{\tilde{\mathbf{K}}}_{1,2}(\mathbf{I}_2 \otimes \mathbf{P}) \end{bmatrix} \begin{bmatrix} \tilde{\mathbf{s}}_1 \\ \tilde{\mathbf{s}}_2 \end{bmatrix} + \tilde{\boldsymbol{\eta}} \\ &= \tilde{\mathbf{K}}\tilde{\mathbf{s}} + \tilde{\boldsymbol{\eta}}, \end{aligned} \quad (22)$$

where $\tilde{\mathbf{s}}_i = [\Re\{\mathbf{s}_i^T\}, \Im\{\mathbf{s}_i^T\}]^T, i = 1, 2$. On this real-valued data model, one can then apply any decoder, from a (near-)ML decoder to a LZF or LMMSE decoder. In this paper, we only consider the LMMSE decoder, and the estimated transformed symbol sequence is then given by

$$\hat{\tilde{\mathbf{s}}} = \tilde{\mathbf{K}}^H(\tilde{\mathbf{K}}\tilde{\mathbf{K}}^H + \frac{\sigma_s^2}{\sigma_n^2}\mathbf{I}_{2N})^{-1}\tilde{\mathbf{y}}. \quad (23)$$

From $\hat{\tilde{\mathbf{s}}}$, the original transmitted symbols can be recovered.

Note that when using (23), the spreading codes $\mathbf{c}_u$ do not strictly have to be orthogonal. However, if the block fading CCE-BEM channel model holds, we have shown in the previous section that orthogonal codes remain orthogonal after propagation, leading to an STBC with full spatial, delay, and Doppler diversity (note that (23) is then equivalent to the receiver processing of Section III, and thus its diversity analysis would be the same). As a consequence, we expect that if a more general realistic channel model holds, we can still benefit from using orthogonal codes. Although this realistic channel model will not fully maintain the orthogonality of the codes after propagation, it will approximately do so, and we still expect to obtain a diversity benefit. So we will always consider orthogonal codes in this work.

V. Simulation Results

In this section, the proposed STBC is examined and compared with other coding schemes by simulations. We only consider a system with two transmit antennas and one receive antenna. The maximum channel delay spread is set to $L = 2$. The channel taps from each transmit antenna to the receive antenna are i.i.d. complex Gaussian distributed with zero mean and variance $\mathrm{E}\{|h_t[n,l]|^2\} = 1/(L+1)$ (i.e., uniform power delay profile) and they follow Jakes' Doppler profile. QPSK symbols are used for transmission. The 2 orthonormal code vectors are set to $\mathbf{c}_1 = [1/\sqrt{2},\ 1/\sqrt{2}]^T$ and $\mathbf{c}_2 = [1/\sqrt{2},\ -1/\sqrt{2}]^T$, which are the columns of the 2×2 unitary Hadamard matrix. The normalized Doppler spread is defined as $f_d = \frac{vf}{c}T$, where v denotes the mobile velocity, f is the carrier frequency, and c is the speed of light. The receiver applies the LMMSE decoder of (23).

Test Case 1: We first compare the proposed STBC with the DPS algorithm of [7] for doubly-selective channels. The symbol block lengths are set to $P = 27$ and $K = 8$. The frequency domain guard band length is set to $Q = 3$, which is large enough for the Doppler spread used in this simulation, for both approaches. The spectral efficiency of the proposed STBC is $\varepsilon = 0.65$, which is higher than the spectral efficiency of DPS $\varepsilon_{DPS} = 0.54$, meaning that we disfavor our approach. We use the LMMSE decoder for both algorithms. It is clearly shown in Fig. 2 that the proposed STBC can achieve a better BER performance due to a larger coding gain. The diversity order is almost the same for both approaches, which increases as the Doppler spread increases.

Test Case 2: We next compare the proposed STBC with the STBC designed for frequency-selective channels in [5]. The zero-padding only STBC in [5] can actually be regarded as a special case of the proposed STBC with $P = 1$ and $Q = 0$. Without data symbol spreading and guards in the frequency domain ($P = 1$, $Q = 0$), a higher spectral efficiency can be achieved, and the block length can be made smaller, which also leads to a lower complexity. A natural question is then if we can ignore the time-selectivity and only use the STBCs designed for a frequency-selective channel in doubly-selective channels. To have the same spectral efficiency, we set

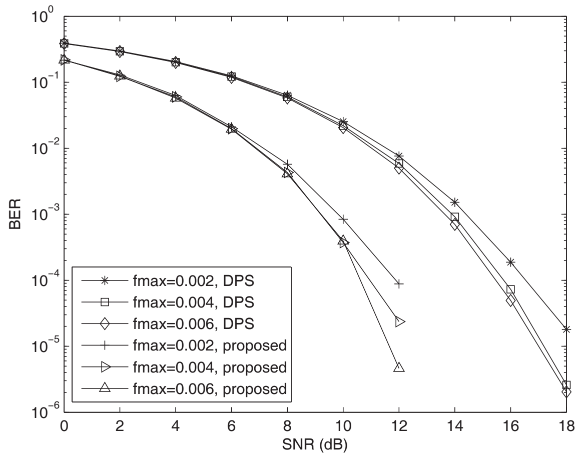

Fig. 2. BER comparison of proposed STBC with DPS [7].

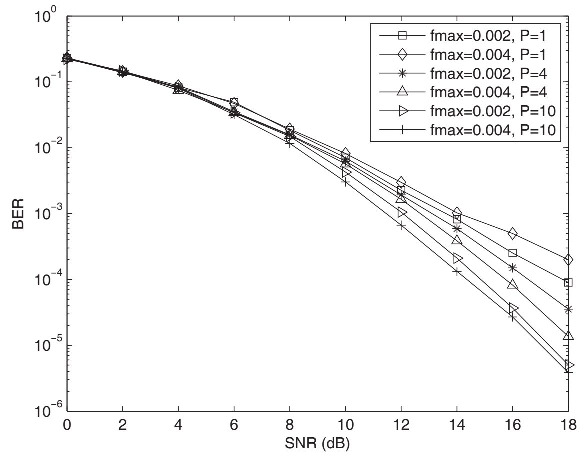

Fig. 3. BER comparison of proposed STBC with the STBC of [5] (case $P = 1$).

$Q = 0$ for the proposed STBC, and keep $K = 5$ fixed for both approaches. Since [5] considers a purely frequency-selective channel, the decoder of [5] relies on the fact that the channel is constant during the entire space-time codeword. To obtain a fair comparison, we simulate the approach of [5] by using our LMMSE receiver, with $P = 1$ and $Q = 0$, so that the STBC design is the same as in [5], but the receiver does not require the channel to be constant. The simulation results in Fig. 3 show that we get a better BER performance as we increase P. But the BER performance is worse compared to the $Q > 0$ case shown in Fig. 2. This is due to the interference related to the lack of frequency-domain guard bands. However, Doppler diversity can still be explored even without a frequency-domain guard band. As shown in the figure, when $P = 1$, i.e., when the data symbols are only spread in the time domain, a higher Doppler spread leads to a worse BER performance. When $P > 1$, a higher Doppler spread leads to a better BER performance, and as P increases, the BER becomes smaller due to an increasing Doppler diversity.

VI. Conclusions

We have developed a novel STBC for multi-antenna transmissions over doubly-selective channels. By spreading the data symbols in the space-time-frequency dimensions with appropriate guard bands, the proposed STBC can achieve the full spatial, delay, and Doppler diversity, using the ML receiver as well as using a LZF or LMMSE receiver, under a specific channel model. Further, a real-valued linear data model has been presented for realistic doubly-selective channels, for which different receiver structures can be developed. Simulation results have shown significantly improved performance by jointly exploring the space-delay-Doppler diversity in doubly-selective channels.

References

[1] A. M. Sayeed and B. Aazhang, "Joint Multipath-Doppler Diversity in Mobile Wireless Communications," *IEEE Trans. on Communications,* vol. 47, no. 1, pp. 123-132, Jan. 1999.

[2] L. Zheng and D.N.C. Tse, "Diversity and multiplexing: A fundamental tradeoff in mulitple antenna channels," *IEEE Trans. on Inform. Theory*, vol. 49, no. 5, pp. 1073-1096, May. 2003.

[3] S. M. Alamouti, "A simple transmit diversity technique for wireless communications," *IEEE J. Select. Areas Commun.*, vol. 16, no. 8, pp. 1451-1458, Oct. 1998.

[4] V. Tarokh, H. Jafarkhani, and A. R. Calderbank, "Space-time block codes from orthogonal designs," *IEEE Trans. Inform. Theory*, vol. 45, no. 5, pp. 1456-1467, Jul. 1999.

[5] S. Zhou and G. B. Giannakis, "Single-carrier space-time block coded transmissions over frequency-selective fading channels," *IEEE Trans. Inf. Theory,* vol. 49, no. 1, pp. 164-179, Jan. 2003.

[6] X. Ma, G. Leus and G. B. Giannakis, "Space-time-Doppler block coding for correlated time-selective fading channels," *IEEE Trans. Signal Process.,* vol. 53, no. 6, pp. 2167-2181, Jun. 2005.

[7] X. Ma and G. B. Giannakis, "Space-time coding for doubly selective channels," in *Proc. International Symposium on Circuits and Systems (ISCAS),* vol. 3, May 2002, pp. 647-650.

[8] G. Leus, S. Zhou, and G. B. Giannakis, "Orthogonal multiple access over time- and frequency-selective channels," *IEEE Trans. Inform. Theory,* vol. 49, no. 8, pp. 1942-1950, Aug. 2003.

[9] G. B. Giannakis and C. Tepedelenlioglu, "Basis Expansion Models and Diversity Techniques for Blind Identification and Equalization of Time-Varying Channels," *Proc. IEEE,* vol. 86, no. 10, pp. 1969-1986, Oct. 1998.

[10] T. Zemen and C. F. Mecklenbraüker, "Time-variant channel estimation using discrete prolate spheroidal sequences," *IEEE Trans. Signal Process.,* vol. 53, no. 9, pp. 3597-3607, Sep. 2005.

[11] X. Ma, and W. Zhang, "Fundamental limits of linear equalizers: diversity, capacity and complexity," *IEEE Trans. Inform. Theory,* to appear.

[12] K. Fang and G. Leus, "Space-Time Block Coding for Doubly-Selective Channels," *IEEE Trans. Signal Processing,* accepted.