Cooperative Compressive Wideband Power Spectrum Sensing

Dyonisius Dony Ariananda and Geert Leus
Faculty of EEMCS, Delft University of Technology
The Netherlands
{d.a.dyonisius, g.j.t.leus}@tudelft.nl.

***Abstract*—Compressive sampling is a popular approach to relax the rate requirement on the analog-to-digital converters and to perfectly reconstruct wideband sparse signals sampled below the Nyquist rate. However, there are some applications, such as spectrum sensing for cognitive radio, that demand only power spectrum recovery. For wide-sense stationary signals, power spectrum reconstruction based on samples produced by a sub-Nyquist rate sampling device is possible even without any sparsity constraints on the power spectrum. In this paper, we examine an extension of our proposed power spectrum reconstruction approach to the case when multiple sensors cooperatively sense the power spectrum of the received signals. In cognitive radio networks, this cooperation is advantageous in terms of the channel diversity gain as well as a possible sampling rate reduction per receiver. In this work, we mainly focus on how far this cooperative scheme promotes the sampling rate reduction at each sensor and assume that the channel state information is available. We concentrate on a centralized network where each sensor forwards the collected measurements to a fusion centre, which then computes the cross-spectra between the measurements obtained by different sensors. We can express these cross-spectra of the measurements as a linear function of the power spectrum of the original signal and attempt to solve it using a least-squares algorithm.**

I. Introduction

In the last few years, signal processing researchers have investigated the best possible way to conduct spectrum sensing in the wideband regime. An important application is a cognitive radio (CR) network where unlicensed users have to perform spectrum sensing over a wide frequency band in order to locate free licensed bands that can be exploited to set up secondary communication links. The conventional way is to divide the wide frequency band into several narrowband channels followed by channel-by-channel sequential sensing, which might introduce a significant amount of delay during the sensing process in a CR network. Another proposal yields a filter bank based multi-channel spectrum sensing, which is inefficient due to the large number of required bandpass filters. A third option consists of a direct wideband spectrum scanning using a high-rate analog-to-digital converter (ADC), which is, unfortunately, power hungry [1]. The sparsity in the spectrum of the licensed users (or their edge spectrum), however, can be exploited to alleviate the requirements on the ADC [2], [3], [4]. This allows for a sampling rate reduction below the Nyquist rate, while maintaining perfect signal reconstruction in a noiseless scenario. Several spectrum blind sampling (SBS) strategies are proposed to gain this sampling rate reduction. These methods aim to sample the received signal with unknown frequency support below the Nyquist rate and to perfectly reconstruct the spectrum of the original signal from the obtained sub-Nyquist samples. Such samplers include multi-coset sampling (see [3], [4]) and modulated wideband converters (MWCs) [5], which consist of paralel channels, each of which employs a different periodic mixing function followed by integrate-and-dump sampling. However, some applications such as spectrum sensing for CR, are only required to perfecty reconstruct the power spectrum, instead of the signal itself.

In [6], the fact that the covariance matrix of the frequency domain representation of a wide-sense stationary (WSS) signal is a diagonal matrix is exploited to reconstruct the power spectrum based on the obtained sub-Nyquist samples. A closely related approach is presented in [7], which reconstructs the auto-correlation sequence of the received WSS signal based on the correlation matrix of the compressive measurements. The major innovation in this work is the possibility to formulate the auto-correlation (or equivalently the power spectrum) recovery problem as an over-determined system, which can be solved using least-squares without putting any sparsity constraints on the power spectrum. The approach in [7] has been labeled as power spectrum blind sampling (PSBS) and extensions have been discussed in [8], [9], which take a further step by computing the correlation matrix of the compressive measurements for different lags and representing the linear relationship between these correlation values and all significant lags of the auto-correlation sequence of the original signal as an over-determined system. If the rank condition of the system matrix is satisfied, it is possible to reconstruct the auto-correlation or equivalently the power spectrum using least-squares.

Multiple sensors could cooperatively estimate the power spectrum of the received signal when the performance of a single sensor is not acceptable due to the existence of hidden terminals, shadowing, or fading. Note that, in addition to the resulting channel diversity gain, a cooperative power spectrum sensing scheme also offers a possible sampling rate reduction per individual sensor, which is critical in the context of wideband power spectrum sensing. In [10], two cooperative wideband spectrum sensing approaches for a CR network are proposed. The first approach jointly estimates the spectrum of the licensed users (LUs) based on the compressive measurements obtained by the individual CRs where channel state information (CSI) is assumed to be available. In the second method, each CR user individually recovers the spectrum of the received faded signal without the availability of CSI and makes a local decision on the frequency occupancy of the LU signal based on this spectrum estimate. Next, all CR users collaboratively make a global decision on the frequency occupancy by using a consensus algorithm based on one-hop communication. Some extensions of the methods in [10] are proposed in [11]. One notable extension is a joint estimation of the spectrum occupancy in the absence of CSI by exploiting the fact that the faded LU signals received by different CR users share the same non-zero support in the frequency domain. All CR users exploit this phenomenon and try to retrieve the joint sparse structure present in the different LU spectra.

Note that the aforementioned cooperative schemes focus on spectrum instead of power spectrum reconstruction and thus a sparsity constraint on the spectrum of the original signal is again required. This motivates us to extend the PSBS approach of [8], [9] into a cooperative power spectrum estimation scheme where multiple sensors collect the compressive measurements by sampling the received signals below the Nyquist rate. For simplicity, the CSI is assumed to be available and the whole scenario is discussed in the context of a CR network where the LU signals are assumed to be WSS. We focus on a centralized network where a fusion centre collects the measurements from all sensors and then computes the cross-spectra between the measurements collected by different sensors. Next, the computed cross-spectra are presented as a linear function of the power spectrum of the original signal. The power spectrum estimate can be reconstructed using least-squares (LS) if the rank condition of the system matrix is satisfied.

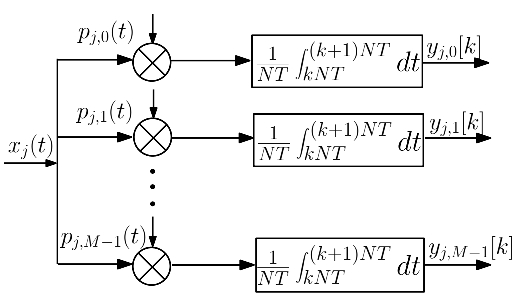


Fig. 1. Illustration of the sampling scheme at each CR, where the received analog signal is modulated with M different periodic waveforms followed by an integrate-and-dump process.

II. System Model

Consider a power spectrum sensing module at the j-th CR user that is receiving a complex wide-sense stationary signal $x_j(t)$. Let us assume that $x_j(t)$ is bandlimited with bandwidth $1/T$ and that the CR user employs a periodic sampling device as illustrated in Fig. 1. In general, the sampling device has M branches where the i-th branch modulates $x_j(t)$ with a periodic waveform $p_{j,i}(t)$ having a period of NT and then samples the output using an integrate-and-dump device with period NT. The output of the i-th branch at the k-th sampling index is then given by

$$\begin{aligned} y_{j,i}[k] &= \frac{1}{NT}\int_{kNT}^{(k+1)NT} p_{j,i}(t)x_j(t)dt \\ &= \frac{1}{T}\int_{kNT}^{(k+1)NT} c_{j,i}(t-kNT)x_j(t)dt \qquad (1) \end{aligned}$$

where $c_{j,i}(t)$ denotes a single period of $\frac{1}{N}p_{j,i}(t)$ i.e., $c_{j,i}(t) = \frac{1}{N}p_{j,i}(t)$ for $0 \leq t < NT$ and $c_{j,i}(t) = 0$ elsewhere. Let us assume that $c_{j,i}(t)$ is a piecewise constant function, which has a constant value in each interval of length T, i.e., $c_{j,i}(t) = c_{j,i}[-n]$ for $nT \leq t < (n+1)T$, where $n = 0, 1, \ldots, N-1$. This allows us to rewrite (1) as

$$y_{j,i}[k] = \sum_{n=0}^{N-1} c_{j,i}[-n]\frac{1}{T}\int_{(kN+n)T}^{(kN+n+1)T} x_j(t)dt =$$
$$\sum_{n=0}^{N-1} c_{j,i}[-n]x_j[kN+n] = \sum_{n=1-N}^{0} c_{j,i}[n]x_j[kN-n]. \quad (2)$$

Here, $x_j[n]$ can be perceived as the output of a virtual integrate-and-dump process operating at period T. It is clear from (1) and (2) that the sampling rate of this sampling device is equal to M/NT, where $1/T$ indicates the Nyquist rate. Note that we can also consider $y_{j,i}[k]$ in (2) as an N-fold downsampled version of $z_{j,i}[n]$, i.e., $y_{j,i}[k] = z_{j,i}[kN]$ where $z_{j,i}[n]$ is given by the following digital filtering operation

$$z_{j,i}[n] = c_{j,i}[n] \star x_j[n] = \sum_{m=1-N}^{0} c_{j,i}[m]x_j[n-m] \qquad (3)$$

with $\star$ representing the convolution operator.

Denote the total number of CR users by J and assume that each of them has the same number of branches M in their periodic samplers. Let us then compute the cross-correlation function of $y_{j,i}[k]$ with $y_{j',i'}[k]$ for all $i, i' = 0, 1, \ldots, M-1$ and $j, j' = 0, 1, \ldots, J-1$. First, it is clear from (3) that

$$\begin{aligned} r_{z_{j,i},z_{j',i'}}[n] &= E\left(z_{j,i}[m]z^*_{j',i'}[m-n]\right) \\ &= r_{c_{j,i},c_{j',i'}}[n] \star r_{x_j,x_{j'}}[n] \\ &= \sum_{m=-N+1}^{N-1} r_{c_{j,i},c_{j',i'}}[m]r_{x_j,x_{j'}}[n-m] \quad (4) \end{aligned}$$

where $r_{c_{j,i},c_{j',i'}}[n]$ is the deterministic correlation function of $c_{j,i}[n]$ with $c_{j',i'}[n]$, i.e., $r_{c_{j,i},c_{j',i'}}[n] = \sum_{m=1-N}^{0} c_{j,i}[m]c^*_{j',i'}[m-n]$ and $r_{x_j,x_{j'}}[n]$ is the cross-correlation function of the Nyquist-rate samples received by the j-th CR user with that received by the j'-th CR user, i.e., $r_{x_j,x_{j'}}[n] = E\left(x_j[m]x^*_{j'}[m-n]\right)$. As a result, the cross-correlation function between $y_{j,i}[k]$ and $y_{j',i'}[k]$ is given by

$$\begin{aligned} r_{y_{j,i},y_{j',i'}}[k] &= E\left(y_{j,i}[l]y^*_{j',i'}[l-k]\right) = r_{z_{j,i},z_{j',i'}}[kN] \\ &= \sum_{m=-N+1}^{N-1} r_{c_{j,i},c_{j',i'}}[m]r_{x_j,x_{j'}}[kN-m]. \qquad (5) \end{aligned}$$

We can also apply the Fourier transform on (4) in order to express the cross-spectrum of $z_{j,i}[n]$ with $z_{j',i'}[n]$ as

$$P_{z_{j,i},z_{j',i'}}(\omega) = P_{c_{j,i},c_{j',i'}}(\omega)P_{x_j,x_{j'}}(\omega), \; 0 \leq \omega < 2\pi$$

where $P_{c_{j,i},c_{j',i'}}(\omega)$ is the deterministic cross-spectrum of $c_{j,i}[n]$ with $c_{j',i'}[n]$, i.e., $P_{c_{j,i},c_{j',i'}}(\omega) = C_{j,i}(\omega)C^*_{j',i'}(\omega)$, and $P_{x_j,x_{j'}}(\omega)$ is the cross-spectrum between $x_j[n]$ with $x_{j'}[n]$, i.e. $P_{x_j,x_{j'}}(\omega) = \sum_{n=-\infty}^{\infty} r_{x_j,x_{j'}}[n]e^{-jn\omega}$. Correspondingly, the cross-spectrum between $y_{j,i}[n]$ and $y_{j',i'}[n]$ can be written as an N-fold aliased version of $P_{z_{j,i},z_{j',i'}}(\omega)$:

$$\begin{aligned} P_{y_{j,i},y_{j',i'}}(\omega) &= \frac{1}{N}\sum_{n=0}^{N-1} P_{z_{j,i},z_{j',i'}}(\tfrac{\omega}{N} + 2\pi\tfrac{n}{N}) \\ &= \frac{1}{N}\sum_{n=0}^{N-1} P_{c_{j,i},c_{j',i'}}(\tfrac{\omega}{N} + 2\pi\tfrac{n}{N}) P_{x_j,x_{j'}}(\tfrac{\omega}{N} + 2\pi\tfrac{n}{N}) \\ &= \mathbf{p}^T_{c_{j,i},c_{j',i'}}(\omega)\mathbf{p}_{x_j,x_{j'}}(\omega),\ 0 \le \omega < 2\pi \end{aligned} \quad (6)$$

where $\mathbf{p}_{x_j,x_{j'}}(\omega) = [P_{x_j,x_{j'}}(\frac{\omega}{N}), \dots, P_{x_j,x_{j'}}(\frac{\omega}{N} + 2\pi\frac{N-1}{N})]^T$ and $\mathbf{p}_{c_{j,i},c_{j',i'}}(\omega) = \frac{1}{N}[P_{c_{j,i},c_{j',i'}}(\frac{\omega}{N}), \dots, P_{c_{j,i},c_{j',i'}}(\frac{\omega}{N} + 2\pi\frac{N-1}{N})]^T$. For given values of j and j', we stack M^2 different cross-spectra $P_{y_{j,i},y_{j',i'}}(\omega)$ in the $M^2 \times 1$ vector $\mathbf{p}_{y_j,y_{j'}}(\omega) = [\dots, P_{y_{j,i},y_{j',i'}}(\omega), \dots]^T$, for $i, i' = 0, 1 \dots, M-1$ in order to obtain

$$\mathbf{p}_{y_j,y_{j'}}(\omega) = \mathbf{P}_{c_j,c_{j'}}(\omega)\mathbf{p}_{x_j,x_{j'}}(\omega),\ 0 \le \omega < 2\pi \quad (7)$$

where $\mathbf{P}_{c_j,c_{j'}}(\omega)$ is the $M^2 \times N$ matrix given by $\mathbf{P}_{c_j,c_{j'}}(\omega) = [\dots, \mathbf{p}_{c_{j,i},c_{j',i'}}(\omega), \dots]^T$, for $i, i' = 0, 1, \dots, M-1$.

III. Cooperative Power Spectrum Reconstruction

Consider P active LUs whose signal is assumed to be a zero mean wide-sense stationary process. The discrete representation of the signal received by the j-th CR user corresponding to these P LUs can then be written as

$$x_j[n] = \sum_{p=0}^{P-1} h_{p,j}[n] \star s_p[n] + n_j[n] \quad (8)$$

where $s_p[n]$ is the discrete representation of the actual signal transmitted by the p-th LU, $h_{p,j}[n]$ is the impulse response of the wireless channel between the p-th LU and the j-th CR user, and $n_j[n]$ is the additive white Gaussian noise at the j-th CR receiver having a zero mean and variance of σ_n^2. We assume that the signals coming from different LUs are uncorrelated to each other and they are also uncorrelated to the noise $n_j[n]$. In the next analysis, we consider the noiseless case in order to simplify the writing. Denote the power spectrum of the LU signal $s_p[n]$ by $P_{s_p}(\omega)$ and the frequency domain representation for $h_{p,j}[n]$ by $H_{p,j}(\omega)$. Based on (8) and the assumption that signals from different LUs are uncorrelated to each other, it is straightforward to write the cross-spectrum of $x_j[n]$ with $x_{j'}[n]$ in (6) as

$$\begin{aligned} P_{x_j,x_{j'}}(\omega) &= \sum_{p=0}^{P-1} H_{p,j}(\omega)H^*_{p,j'}(\omega)P_{s_p}(\omega) \\ &= \tilde{\mathbf{h}}'^T_{j,j'}(\omega)\tilde{\mathbf{p}}'_s(\omega),\ 0 \le \omega < 2\pi \end{aligned} \quad (9)$$

where $\tilde{\mathbf{p}}'_s(\omega) = \left[P_{s_0}(\omega), P_{s_1}(\omega), \dots, P_{s_{P-1}}(\omega)\right]^T$ and $\tilde{\mathbf{h}}'_{j,j'}(\omega) = \left[H_{0,j}(\omega)H^*_{0,j'}(\omega), \dots, H_{P-1,j}(\omega)H^*_{P-1,j'}(\omega)\right]^T$. By using (9), we can now express $\mathbf{p}_{y_j,y_{j'}}(\omega)$ in (7) as a function of the power spectrum of the LU signals $\left\{P_{s_p}(\omega)\right\}_{p=0}^{P-1}$:

$$\mathbf{p}_{y_j,y_{j'}}(\omega) = \mathbf{P}_{c_j,c_{j'}}(\omega)\tilde{\mathbf{H}}_{j,j'}(\omega)\tilde{\mathbf{p}}_s(\omega),\ 0 \le \omega < 2\pi \quad (10)$$

where the $PN \times 1$ vector $\tilde{\mathbf{p}}_s(\omega)$ and the $N \times PN$ block diagonal matrix $\tilde{\mathbf{H}}_{j,j'}(\omega)$ are given by

$$\begin{aligned} \tilde{\mathbf{p}}_s(\omega) &= [\tilde{\mathbf{p}}'^T_s(\tfrac{\omega}{N}), \tilde{\mathbf{p}}'^T_s(\tfrac{\omega}{N} + 2\pi\tfrac{1}{N}), \dots, \\ &\quad \tilde{\mathbf{p}}'^T_s(\tfrac{\omega}{N} + 2\pi\tfrac{N-1}{N})]^T \\ \tilde{\mathbf{H}}_{j,j'}(\omega) &= \text{diag}\left\{\tilde{\mathbf{h}}'^T_{j,j'}(\tfrac{\omega}{N}), \tilde{\mathbf{h}}'^T_{j,j'}(\tfrac{\omega}{N} + 2\pi\tfrac{1}{N}), \dots, \right. \\ &\quad \left. \tilde{\mathbf{h}}'^T_{j,j'}(\tfrac{\omega}{N} + 2\pi\tfrac{N-1}{N})\right\}. \end{aligned}$$

Now, let us consider $\mathbf{p}_{y_j,y_{j'}}(\omega)$ in (10) only for a finite number of frequency bins ω by taking into account that $r_{y_{j,i},y_{j',i'}}[k]$ in (5) has only a limited support $-L \le k \le L$. Note that in practice, this simplification has to be considered since it is not possible to collect $r_{y_{j,i},y_{j',i'}}[k]$ for infinite support k. If we also collect all M^2 different cross-correlation functions $r_{y_{j,i},y_{j',i'}}[k]$ in the $M^2 \times 1$ vector $\mathbf{r}_{y_j,y_{j'}}[k] = [\dots, r_{y_{j,i},y_{j',i'}}[k], \dots]^T$ for $i, i' = 0, 1 \dots, M-1$, we can relate $\{\mathbf{r}_{y_j,y_{j'}}[k]\}_{k=-L}^{L}$ and $\mathbf{p}_{y_j,y_{j'}}(\omega)$, for $\omega = 0, \varphi, \dots, 2L\varphi$, and $\varphi = \frac{2\pi}{2L+1}$, using a simple discrete Fourier transform (DFT) operation:

$$\mathbf{p}_{y_j,y_{j'}} = (\mathbf{F}_{2L+1} \otimes \mathbf{I}_{M^2})\mathbf{r}_{y_j,y_{j'}} \quad (11)$$

where $\otimes$ represents the Kronecker product operation, $\mathbf{I}_{M^2}$ is an $M^2 \times M^2$ identity matrix, $\mathbf{F}_{2L+1}$ is the $(2L+1) \times (2L+1)$ DFT matrix, and the $(2L+1)M^2 \times 1$ vectors $\mathbf{p}_{y_j,y_{j'}}$ and $\mathbf{r}_{y_j,y_{j'}}$ are given by $\mathbf{p}_{y_j,y_{j'}} = [\mathbf{p}^T_{y_j,y_{j'}}(0), \mathbf{p}^T_{y_j,y_{j'}}(\varphi), \dots, \mathbf{p}^T_{y_j,y_{j'}}(2L\varphi)]^T$ and $\mathbf{r}_{y_j,y_{j'}} = [\mathbf{r}^T_{y_j,y_{j'}}[0], \dots, \mathbf{r}^T_{y_j,y_{j'}}[L], \mathbf{r}^T_{y_j,y_{j'}}[-L], \dots, \mathbf{r}^T_{y_j,y_{j'}}[-1]]^T$, respectively. It is now possible to rewrite (10) using $2L+1$ matrix equations:

$$\mathbf{p}_{y_j,y_{j'}}(\varphi l) = \mathbf{P}_{c_j,c_{j'}}(\varphi l)\tilde{\mathbf{H}}_{j,j'}(\varphi l)\tilde{\mathbf{p}}_s(\varphi l),\ l = 0, 1, \dots, 2L. \quad (12)$$

Let us make a further simplification by assuming that the frequency bands licensed by the different LUs are non-overlapping and thus at most, only one LU can be active in a particular frequency bin. This assumption simplifies (12) into

$$\mathbf{p}_{y_j,y_{j'}}(\varphi l) = \mathbf{P}_{c_j,c_{j'}}(\varphi l)\mathbf{H}_{j,j'}(\varphi l)\mathbf{p}_s(\varphi l),\ l = 0, 1, \dots, 2L \quad (13)$$

where $\mathbf{p}_s(\varphi l)$ is the $N \times 1$ vector given by

$$\mathbf{p}_s(\varphi l) = [P_s(\tfrac{\varphi l}{N}), P_s(\tfrac{\varphi l}{N} + 2\pi\tfrac{1}{N}), \dots, P_s(\tfrac{\varphi l}{N} + 2\pi\tfrac{N-1}{N})]^T \quad (14)$$

with $P_s(\frac{\varphi l}{N} + 2\pi\frac{n}{N}) = \sum_{p=0}^{P-1} P_{s_p}(\frac{\varphi l}{N} + 2\pi\frac{n}{N})$ for $n = 0, 1, \dots, N-1$ and $\mathbf{H}_{j,j'}(\varphi l)$ is an $N \times N$ diagonal matrix given by

$$\begin{aligned} \mathbf{H}_{j,j'}(\varphi l) = \text{diag}&\left\{H_{(l,0),j}(\tfrac{\varphi l}{N})H^*_{(l,0),j'}(\tfrac{\varphi l}{N}),\right. \\ &H_{(l,1),j}(\tfrac{\varphi l}{N} + 2\pi\tfrac{1}{N})H^*_{(l,1),j'}(\tfrac{\varphi l}{N} + 2\pi\tfrac{1}{N}), \dots, \\ &\left. H_{(l,N-1),j}(\tfrac{\varphi l}{N} + 2\pi\tfrac{N-1}{N})H^*_{(l,N-1),j'}(\tfrac{\varphi l}{N} + 2\pi\tfrac{N-1}{N})\right\} \end{aligned} \quad (15)$$

with $H_{(l,n),j}(\omega)$ representing the frequency response at frequency ω of the channel between the j-th CR user and the LU who has a license for the frequency bin $\frac{\varphi l}{N} + 2\pi\frac{n}{N}$. Based on (13) and (14), our goal is to reconstruct the power spectrum

$P_s(\omega)$ at $(2L+1)N$ frequency bins ω. Note also that a particular LU might have a license over multiple frequency bins ω.

Next, we can finally stack $\mathbf{p}_{y_j,y_{j'}}(\varphi l)$ in (13) for all $j, j' = 0, 1, \ldots, J-1$ into a $J^2M^2 \times 1$ vector $\mathbf{p}_y(\varphi l)$ given by $\mathbf{p}_y(\varphi l) = \left[\mathbf{p}^T_{y_0,y_0}(\varphi l), \mathbf{p}^T_{y_0,y_1}(\varphi l), \ldots, \mathbf{p}^T_{y_{J-1},y_{J-1}}(\varphi l)\right]^T$ and write $\mathbf{p}_y(\varphi l)$ as

$$\mathbf{p}_y(\varphi l) = \mathbf{\Phi}(\varphi l)\mathbf{p}_s(\varphi l), \; l = 0, 1, \ldots, 2L \qquad (16)$$

where the $J^2M^2 \times N$ matrix $\mathbf{\Phi}(\varphi l)$ is given by

$$\mathbf{\Phi}(\varphi l) = \Big[\left(\mathbf{P}_{c_0,c_0}(\varphi l)\mathbf{H}_{0,0}(\varphi l)\right)^T, \left(\mathbf{P}_{c_0,c_1}(\varphi l)\mathbf{H}_{0,1}(\varphi l)\right)^T, \ldots, \left(\mathbf{P}_{c_{J-1},c_{J-1}}(\varphi l)\mathbf{H}_{J-1,J-1}(\varphi l)\right)^T\Big]^T \qquad (17)$$

for $l = 0, 1, \ldots, 2L$. Note that if $J^2M^2 \geq N$ and all matrices $\{\mathbf{\Phi}(\varphi l)\}_{l=0}^{2L}$ in (16) have full column rank, we can solve (16) for $l = 0, 1, \ldots, 2L$ using LS. Once we reconstruct $\{\mathbf{p}_s(\varphi l)\}_{l=0}^{2L}$ in (16), we can organize the resulting $P_s(\omega)$ for $\omega = 0, \frac{\varphi}{N}, \frac{2\varphi}{N}, \ldots, 2\pi - \frac{\varphi}{N}$ into a $(2L+1)N \times 1$ vector $\mathbf{s}_s = \left[P_s(0), P_s(\frac{\varphi}{N}), P_s(\frac{2\varphi}{N}), \ldots, P_s(2\pi - \frac{\varphi}{N})\right]^T$, which is given by

$$\mathbf{s}_s = \mathbf{\Gamma}_{(2L+1)N}\mathbf{p}_s \qquad (18)$$

where $\mathbf{p}_s = \left[\mathbf{p}_s^T(0), \mathbf{p}_s^T(\varphi), \mathbf{p}_s^T(2\varphi), \ldots, \mathbf{p}_s^T(2L\varphi)\right]^T$ and $\mathbf{\Gamma}_{(2L+1)N}$ is a $(2L+1)N \times (2L+1)N$ permutation matrix that performs a $(2L+1) \times N$ interleaving process (read column-wise and write row-wise).

There are many possible implementations for the sampler coefficients $c_{j,i}[n]$ in (2) such as binary, complex Gaussian, or multi-coset sampling. Our interest is to select $\{c_{j,i}[n]\}_{i=0}^{M-1}$ for all $j = 0, 1, \ldots, J-1$ to ensure the full column rank condition of $\{\mathbf{\Phi}(\varphi l)\}_{l=0}^{2L}$ in (16). First, we consider the case when there is no fading, which means that the matrices $\{\mathbf{H}_{j,j'}(\varphi l)\}_{j,j'=0}^{J-1}$ in (17) are identity matrices and the matrices $\{\mathbf{\Phi}(\varphi l)\}_{l=0}^{2L}$ in (16) are given by $\mathbf{\Phi}(\varphi l) = \left[\mathbf{P}^T_{c_0,c_0}(\varphi l), \mathbf{P}^T_{c_0,c_1}(\varphi l), \ldots, \mathbf{P}^T_{c_{J-1},c_{J-1}}(\varphi l)\right]^T$. In this case, one possible option is to implement multi-coset sampling where the sampler coefficients $\{c_{j,i}[n]\}_{i=0}^{M-1}$ for all $j = 0, 1, \ldots, J-1$ are constructed by selecting JM rows of the $N \times N$ identity matrix $\mathbf{I}_N$. Here, we have two possible cases depending on the value of M and J. When $JM \leq N$, we select JM rows of $\mathbf{I}_N$ based on the procedure discussed in [9], which can be explained as follows. First, we select K rows of $\mathbf{I}_N$, where the value of K and the rows to be selected are governed by the so-called minimal length-$\lfloor N/2 \rfloor$ sparse ruler problem. Next, we randomly select the additional $JM - K$ rows from the remaining $N - K$ rows of $\mathbf{I}_N$. If $\left\lfloor \frac{JM}{N} \right\rfloor = b > 0$, the JM sampler coefficients $c_{j,i}[n]$ are given by all rows of b identity matrices $\mathbf{I}_N$ plus $JM - bN$ additional randomly selected rows from a $(b+1)$-th identity matrix $\mathbf{I}_N$. For both cases, we can then exploit the derivation in [9] to show that the generated sampler coefficients $\{c_{j,i}[n]\}_{i=0}^{M-1}$ for all $j = 0, 1, \ldots, J-1$ will lead to full column rank matrices $\{\mathbf{\Phi}(\varphi l)\}_{l=0}^{2L}$, i.e., $\mathbf{\Phi}(\varphi l)$ is a product of a matrix having Vandermonde structure and two diagonal matrices.

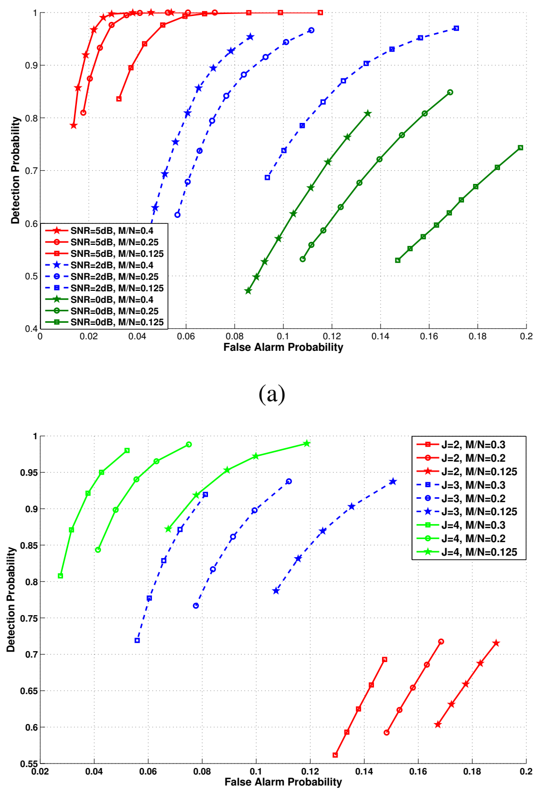
When wireless fading exists, however, the condition that ensures the full column rank of $\{\mathbf{\Phi}(\varphi l)\}_{l=0}^{2L}$ becomes unclear due to the existence of $\{\mathbf{H}_{j,j'}(\varphi l)\}_{j,j'=0}^{J-1}$ in (17). However, these diagonal matrices are generally random because the channel taps $h_{p,j}[n]$ in (8) are also random (e.g., having a complex Gaussian distribution for a Rayleigh fading channel). Therefore, constructing multi-coset sampler coefficients $c_{j,i}[n]$ based on the procedure elaborated in the previous paragraph still leads to full column rank matrices $\{\mathbf{\Phi}(\varphi l)\}_{l=0}^{2L}$ with a very high probability.

Note that in our scenario, the j-th CR user forwards the CSI, the sampler coefficients $\{c_{j,i}[n]\}_{i=0}^{M-1}$, and the measurements $\{y_{j,i}[k]\}_{i=0}^{M-1}$ to the fusion centre, which then calculates the cross-spectra vectors $\{\mathbf{p}_y(\varphi l)\}_{l=0}^{2L}$ and the system matrices $\{\mathbf{\Phi}(\varphi l)\}_{l=0}^{2L}$ in (16). Eventually, the fusion centre recovers the power spectrum vector $\mathbf{s}_s$ by using (16) and (18).

IV. Simulation Study

In this section, some numerical results from a simulation study are presented. Let us consider two LUs that are active in the frequency bands $[0.4\pi, 0.5\pi]$ and $[0.5\pi, 0.6\pi]$, respectively. We generate the complex baseband signals corresponding to these two LUs by filtering a circular complex zero-mean Gaussian i.i.d. signal with variance σ^2, which defines the signal power in the active band. Rayleigh fading channels are assumed by generating the channel taps according to a complex zero-mean Gaussian distribution having unit variance normalized by the channel length. We set the number of fading channel taps, L, and N to 11, $L = 1$, and $N = 80$, respectively and employ a multi-coset sampling implementation for the sampler cofficients $\{c_{j,i}[n]\}_{i=0}^{M-1}$ for all $j = 0, \ldots, J-1$. The selection of the JM rows of the identity matrix $\mathbf{I}_{80}$ to construct the JM sampler cofficients $c_{j,i}[n]$ follows the procedure presented in Section III.

In the first scenario, we keep the number of CR users fixed to $J = 3$ while varying the compression rate (M/N) at each CR user from 0.125 to 0.4. The signal-to-noise ratio (SNR) is measured as the ratio between the received signal power and the noise power only at the occupied bands. Here, the SNR is varied from 0 dB to 5 dB. The detection threshold is varied manually while the resulting detection events are evaluated in the occupied bands. The false alarm events are examined in a frequency band that is significantly far from the occupied bands, i.e., at $[-0.6\pi, -0.4\pi]$. It is important to note that the shape of the active bands is not perfectly rectangular due to the existence of two transition bands at the edges of the occupied bands. Therefore, we decide to leave guard bands at the edges of each occupied band and examine the detection probability at the frequency bands $[0.41\pi, 0.49\pi]$ and $[0.51\pi, 0.59\pi]$. Fig. 2.a shows the receiver operating characteristic (ROC) for this first scenario. Observe how the performance of the proposed approach is acceptable for reasonable SNR and how it improves as M/N is getting larger, which is to be expected. Also observe that for SNR = 5 dB, a compression rate of 0.125 per CR still results in a satisfactory performance. However, the performance of the



(a)

(b)

Fig. 2. The detection performance of the proposed approach for 150000 received Nyquist-rate samples; (a) The number of CRs is fixed to $J = 3$; (b) The SNR is set to 2 dB.

proposed cooperative spectrum sensing approach for SNR = 0 dB is unacceptable for CR applications.

In the second scenario, the SNR is fixed to 2 dB while the number of cooperating CR users is varied from $J = 2$ to $J = 4$. In addition, we vary the compression rate M/N between 0.125 and 0.3 and evaluate the detection and false alarm probability in the same bands as in the first scenario. Fig. 2.b illustrates the ROC for this second scenario and shows how the performance improves as we have more cooperating CR users and the M/N per CR user increases. This observation is to be expected. It is interesting, however, to compare the performance between the case when $J = 4$ and $M/N = 0.2$, which leads to a matrix $\mathbf{\Phi}(\varphi l)$ in (16) of size 4096×80, to the case when $J = 3$ and $M/N = 0.3$, which leads to a matrix $\mathbf{\Phi}(\varphi l)$ of size 5184×80. Observe how the first case offers a better performance although it results in a matrix $\mathbf{\Phi}(\varphi l)$ having less rows compared to the second case. This might be because the condition number of the matrix $\mathbf{\Phi}(\varphi l)$ for $J = 4$ and $M/N = 0.2$ is more likely to be better than the one for $J = 3$ and $M/N = 0.3$ due to the fact that the matrix $\mathbf{\Phi}(\varphi l)$ for $J = 4$ and $M/N = 0.2$ contains contributions from more fading channels.

V. Conclusion

In this paper, an extension of our proposed compressive power spectrum estimation approach into a cooperative scenario is investigated by assuming the primary user signals are WSS processes. Since we mainly concentrate on how this cooperative scheme might lead to a possible reduction of the sampling cost at each cooperating sensor, CSI is assumed to be present. Our focus is on the centralized approach where each sensor collects sub-Nyquist rate samples and forwards them to a fusion centre together with the CSI and the sampler coefficients. Then, the cross-spectra between all measuremens are calculated by the fusion centre, who later recovers the power spectrum of the received signals by exploiting the WSS property of the primary user signals. The simulation study shows how the proposed approach performs satisfactory for a reasonable SNR and a sufficient number of cooperating sensors. Future research will focus on how to extend the proposed approach to the unknown CSI case.

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