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Preface

Special Issue on Structured and Infinite Systems of Linear Equations

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Interest in 'structured and infinite systems of linear equations' has been there for a long time, ever since the 18th century. While nowadays we tend to study finite-dimensional structured linear equations almost in separation of what we would regard as more sophisticated (or even more demanding) infinite-dimensional structured systems, it is worth stressing that, in retrospect, both issues of structure and infinite dimensionality have manifested themselves jointly in the earliest works on such topics. This is clear from the contributions of a handful of distinguished mathematicians from the 18th, 19th and early 20th centuries, including Kronecker, Carathéodory, Toeplitz, Schur, Nevanlinna, Riesz, Szegö and others, in the areas of moment problems, interpolation theory, orthogonal polynomials and spaces of analytic functions. In most of these earlier investigations, the notions of structure and infinite dimensionality are handled jointly through the concepts of symbols or generating functions. Kronecker, for example, gave a characterization of Toeplitz matrices whose symbol is rational—namely that a related Hankel matrix should have finite rank. For such matrices, efficient computing schemes can be set up that exploit the 'system structure' of the matrix. Although not used much in the practice of handling finite Toeplitz matrices, the 'realization method' implicitly initiated by Kronecker turns out to be extremely valuable to study infinite Toeplitz matrices, and certainly leads to finite and efficient calculations for systems of equations induced by such matrices. Very deep insights in the properties of infinite Toeplitz systems were developed at the end of the 19th century and in the beginning of the 20th century, in what is now known as

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'Hardy space theory', and, of course, in Harmonic Analysis in general whose basis was laid in the same period. We need not dwell here on the practical relevance of these theories, just mentioning their importance for the analysis of stochastic processes is motivation enough for our statement.

In the second half of the 20th century, with the increasing reliance on efficient and reliable computational methods for circuit implementation and simulation on computing machines, emphasis has been placed on the use of linear algebraic methods in the characterization and exploitation of structure. However, a deep ravine still seems to exist between, on the one hand, the properties of infinite operators and their characterization through analytic function theory, and on the other, the algebraic properties needed to solve large systems of equations with structured matrices of various kinds efficiently. In this issue we attempt to put the two approaches back to back in a number of situations. We hope that correspondences, methodic similarities, and also essential discrepancies will become apparent in this way.

The papers presented in this issue roughly subdivide in two major categories depending on the kind of structure they handle. We distinguish:

- papers on the properties of systems with Hankel, Toeplitz or Cauchy structure and their combinations and extensions, e.g., to systems with finite displacement rank;
- papers on operators with other structured properties than the classical ones and their extensions, in particular structure derived from system or systematic properties.

We give next a brief run-down of the contributions (in alphabetic order per category). In the first category:

- Basor and Ehrhardt establish several relations between the determinants of Hankel, symmetric Toeplitz + Hankel, and Toeplitz matrices. They obtain Fisher–Hartwig type results on the asymptotics of certain skewsymmetric Toeplitz determinants and certain Hankel determinants.
- Bini, Gemignani and Meini relate polynomial computations with operations involving infinite band Toeplitz matrices and show applications to the numerical solution of Markov chains, of non-linear matrix equations, to spectral factorizations and to the solution of finite Toeplitz systems.
- Boros, Kailath and Olshevsky concentrate on the stability of fast algorithms for solving Cauchy linear equations.
- Böttcher, Embree and Sokolov study the change of the spectra of infinite Toeplitz and Laurent matrices under perturbations in a prescribed finite set of sites.
- Chen et al. discuss the "black-box" approach to linear algebra and apply it to derive new conditioners for Toeplitz-like matrices.
- Constantinescu, Sayed and Kailath study matrix extension problems in a context where structure is defined as 'displacement structure'.

- El Ghaoui studies a notion of 'structured condition number'. It takes into account possibly structured matrix perturbations. He uses semi-definite programming methods to compute bounds on the structured condition number for certain classes of perturbation structures.
- Heinig and Rost obtain the representation of inverses of symmetric and skewsymmetric Toeplitz matrices.
- Hendrickx and Van Barel present new fast direct methods for solving a large symmetric banded Toeplitz system with small bandwidth.
- Pan, Rami and Wang extend previous results on the use of Newton's iteration for the fast computation of inverses of structured matrices.
- Rovnyak and Sakhnovich use an approach by operator identities which is used to investigate some direct and inverse problems of spectral theory for canonical systems of difference equations in the indefinite case.
- Sakhnovich obtains explicit expressions for the well-known Szegö limits for the cases when the upper and lower triangular parts of the Toeplitz matrix are determined separately by the Taylor series of rational matrix functions and the exponential growth of the entries is permissible.
- Serra Capizzano extends his earlier result (jointly with Tyrtyshnikov) and shows that no superlinear matrix algebra preconditioner exists in the multilevel Toeplitz case.
- Strohmer studies the approximation of infinite (bi-infinite)-dimensional Toeplitz systems by finite-dimensional models, and stresses the role that is played by the decay behavior of the off-diagonal entries of such matrices and their inverses on accuracy of the results.
- Tyrtyshnikov and Zamarashkin show that the Szegö-like formulas hold true even for Toeplitz matrices generated by the so-called Radon measures.
- Van der Mee, Seatzu and Rodriguez show how to perform spectral factorization of Bi-infinite Multi-index Block Toeplitz matrices.
- Wimmer uses polynomial models to show the similarity of a block companion matrix over a field of characteristic 0 is similar to a unique block unit upper Hessenberg Toeplitz matrix.

In the second category, we have the following contributions:

- Alpay, Bolotnikov, Dewilde and Dijksma characterize the analogue of a Brune section for the non-stationary case.
- Eidelman and Gohberg study a class of block structured matrices admitting linear complexity solvers and discuss the algorithms related to those derived by Dewilde and van der Veen for finite and infinite matrices with a small Hankel rank.
- Iglesias extends the results on the Bode integral from the discrete-time time-varying systems to a continuous-time setting.
- Tabachnikova writes a brief note showing an application of the famous Gödel–Malcev theorem to the existence of solutions to infinite sets of equations with a finite number of non-zero elements in the solution vector.

As one can immediately infer from the list, the above plethora of results obtained by mathematicians, engineers and computer scientists fully illustrates different techniques and approaches to attack structured matrix problems. We hope that this issue will contribute to bringing together people from many courses of life: engineers because they face very large modeling and prediction problems, numerical analysts because of the hope to get more accuracy exploiting the structure, computer scientists because of their desire to study complexity issues, design engineers handling large modeling and simulation problems, and mathematicians because of the beauty of their solutions.