Time Mark Estimators for MD-SiPM and Impact of System Parameters

E. Venialgo, S. Mandai and E. Charbon

***Abstract*—Positron emission tomography (PET) detector modules measure collinear gamma photons created by positron-electron annihilation. These gamma photons interact within a crystal scintillator and numerous light photons are emitted as response to the gamma energy absorption. PET detector modules measure energy and estimate the time of arrival of each gamma photon. The accuracy of the time of arrival is crucial for Time Of Flight (TOF) scanners to improve the quality of the reconstructed image. Since the introduction of multi-channel digital SiPM, it is possible to measure several time stamps that corresponds to the fastest light photons. Therefore, the time mark of the gamma photon can be estimated from the measured light photon time stamps. This work presents a complete simulation study of different weighted-average time stamp estimators. In addition, it was analyzed the impact of the PET detector module parameters, such as dark count rate (DCR), jitter, energy resolution and detected primary photons.**

I. MATERIALS AND METHODS

A. Simulation Model

A Monte Carlo code was implemented on Matlab to generate the time stamps. The model of this simulator is described throughout this section, in addition to all of the paramenter that were swept to evaluate the Coincidence Resolving Time (CRT).

The time of arrival statistics of scintillated light photons, which are produced within a crystal scintillator due to gamma interaction, was described in the early 1950s [2]. This model is based on the Poisson statistics and single exponential decay of the light photon emission rate. This approch was expanded to a double exponential model, and the concept of order statistics was introduced afterwards [3], [4]. We utilized a double exponential model to sample the time of arrival of the light photons (see Eq. 1).

$$F(t) = 1 - \frac{\tau_d \cdot exp[(t-\theta)/\tau_d] - \tau_r \cdot exp[-(t-\theta)/\tau_r]}{\tau_d - \tau_r} \quad (1)$$

The number of samples taken from the cumulative density function, which is the integral of Eq. 1, was sweept from 200 to 3000 to study the influence of number of detected primary photons. In addition, the number of samples was modified to study the effect of energy resolution. This last parameter was sweept between 10% to 20% Full Width Half Maximum (FWHM) following a Gaussian distribution. Furthermore, to the obtained time stamps that corresponds to the emitted light photons Gaussian jitter was added, as well as DCR. The jitter FWHM was sweeped from 50ps to 600ps and DCR frecuency from 1MHz to 20MHz. Fig. 1 shows the PDFs that corresponds to the time of arrival of the k^{th} light photon. As observed, the firsts light photons have the best time resolution; for this reason, the PET detector modules set the trigger value of the time mark electronics as low as possible [4].

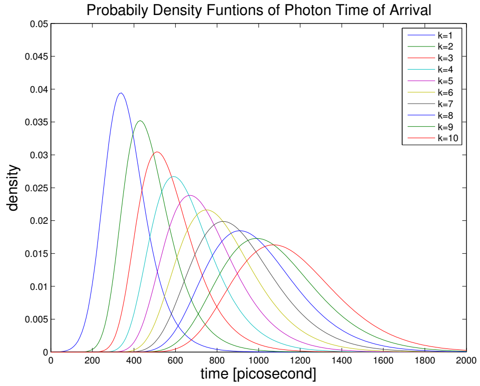

Fig. 1: Probability Density Functions for the time of arrival of the k^{th} light photon.

B. Real Time Time Mark Estimators

Since the introduction of MD-SiPM, it is possible to adquire individual time stamps that corresponds to the fastest emitted light photons [1]. Therefore, it is possible to use more information to estimate the time mark of the gamma photon. In this work, we focussed on weighted average estimators since it is possible to implement them on real-time processing devices, such as Digital Signal Processors (DSPs) and Field Programable Gate Arrays (FPGAs). These estimators were compared to a gold stantdar method, which is to estimate the gamma photon time mark based on a single light photon time stamp.

The first approach was to calculate the average value of a group of the first k_{th} photons. The number of photons that were utilized to calculate the mean value was varied from 2 to 48, since the MD-SiPM that was designed in our laboratory has 48 time to digital converters (TDCs) on chip [1]. In Eq. 2 T_k correspond to the time of arrival of the k^th light photon and TDCs varies from 2 to 48. We call this estimator as mean estimator.

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$$Time\hat{M}ark = \frac{1}{TDCs}\sum_{i=1}^{TDCs} T_k \tag{2}$$

The second method was a weighted-average estimator, in which the weights were calculated according to the variance of corresponding T_k. Furthermore, the weigths were normalized so their summation is equal to 1. This method was called variance-weighted estimator.

$$Time\hat{M}ark = \frac{1}{TDCs}\sum_{i=1}^{TDCs} T_k \times W_i \tag{3}$$

The third method was also a weighted-average estimator but we calculate the weights according to the covariance of the T_k time stamps following Eq. 4. Where D is a row vector filled with ones and its length is equal to the number of time stamps, C is the covariance matrix of the time stamps, N is the weigth's normalization constant and W is the weight vector. This method follows the GaussMarkov theorem to obtain a weighted average with minimim variance.

$$W = D \times C^{-1} \tag{4}$$

$$N = D \times C^{-1} \times D^T \tag{5}$$

The last estimator was a linear Artificial Neural Network (ANN), which is also an weighted-average method that has a bias compensation. In this case, the training set was generated with the Matlab simulation tool by shifting the beginning of the gamma event. The utilized training algorithm was LevenbergMarquardt backpropagation [5], [6].

II. RESUTLS

The estimators showed robustness respecto to energy resolution and DCR variations, but CRT depends significantly on the number of detected primary photons and the total jitter of the system. Fig. 2 shows the CRT of all of the estimators for two different amounts of detected primary photons. In all of this simulations the energy resoltion was kept to 14%, DCR to 9MHz and jitter FWHM to 200ps. As observed, the GaussMarkov and linear ANN estimators are equivalent, and the weights are correct since the CRT remains constant as the number of TDCs increases. All of these results are consistent to recent measured data [7].

Fig. 3 shows the same results but jitter was modified from 200ps to 600ps . The energy resoltion was kept to 14%, DCR to 9MHz and the number of primary photons was 2600.

III. CURRENT WORK

Currently, we are utilizing GATE/Geant4 to study the spatial distribution of the light photons over the MD-SiPM sensitive area [5]. The objective of this simulations is to analyze the spatial saturation effect of TDCs; in other words, when more than three primary photons hits the same column since our MD-SiPM has three TDCs per column [1]. In addition, we are implementing maximum-likelihood based estimators to calculate the optimum CRT for each simulation condition although this method cannot be implemented in real time [7].

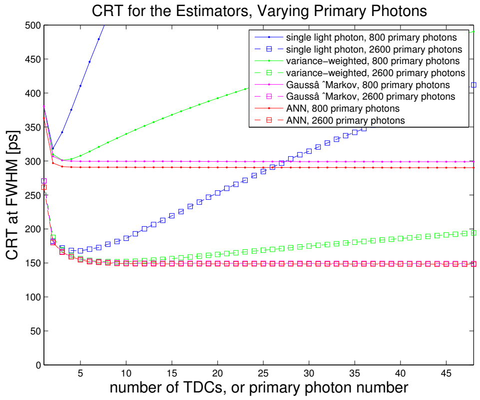

Fig. 2: CRT for the all of the estimators, sweeping the number of detected primary photons

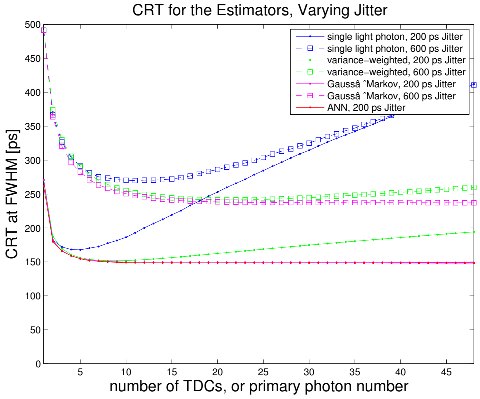

Fig. 3: CRT for the all of the estimators, sweeping the jitter level.

REFERENCES

[1] S. Mandai, V. Jain, and E. Charbon, "A fully-integrated 700x800μm^{2}multi-digital silicon photomultiplier with column-parallel time-to-digital converter," in *ESSCIRC (ESSCIRC), 2012 Proceedings of the*, 2012, pp. 89–92.

[2] R. F. Post and L. I. Schiff, "Statistical limitations on the resolving time of a scintillation counter," *Phys. Rev.*, vol. 80, pp. 1113–1113, Dec 1950. [Online]. Available: http://link.aps.org/doi/10.1103/PhysRev.80.1113

[3] M. Fishburn and E. Charbon, "System tradeoffs in gamma-ray detection utilizing spad arrays and scintillators," *Nuclear Science, IEEE Transactions on*, vol. 57, no. 5, pp. 2549–2557, 2010.

[4] H. T. v. D. Stefan Seifert and D. R. Schaart, "The lower bound on the timing resolution of scintillation detectors," *Phys. Med. Biol.*, vol. 57, no. 7, p. 1797, 2012. [Online]. Available: http://stacks.iop.org/0031-9155/57/i=7/a=1797

[5] E. Venialgo, C. Verrastro, D. Estryk, M. Belzunce, A. Carimatto, E. da Ponte, L. Garbino, and J. Alarcon, "Pet calibration method of nonlinear position estimation algorithms for continuous nai(tl) crystals," in *Nuclear Science Symposium and Medical Imaging Conference (NSS/MIC), 2011 IEEE*, 2011, pp. 3359–3364.

[6] ——, "Pet calibration method of nonlinear position estimation algorithms for continuous nai(tl) crystals," in *Nuclear Science Symposium and Medical Imaging Conference (NSS/MIC), 2011 IEEE*, 2011, pp. 3359–3364.

[7] S. Seifert, H. T. van Dam, and D. R. Schaart, "The lower bound on the timing resolution of scintillation detectors," *Physics in Medicine and Biology*, vol. 57, no. 7, p. 1797, 2012. [Online]. Available: http://stacks.iop.org/0031-9155/57/i=7/a=1797