Joint motion estimation and clock synchronization for a wireless network of mobile nodes

Raj Thilak Rajan[*,†], *Alle-Jan van der Veen*[†]

[*] ASTRON, Dwingeloo, The Netherlands
[†] TU Delft, Delft, The Netherlands

ABSTRACT

Localization and synchronization are critical challenges for a wireless network, which are conventionally solved independently. Recently, various estimators have been proposed to jointly synchronize and localize a node in a static network based on two way communication. In this paper, we present a novel and generic model based on two way communication between nodes, which are in relative motion with respect to each other. Furthermore, for the entire network we propose a closed form Extended Global Least Squares (EGLS) solution to solve for all the unknown clock skews, clock offsets, initial pairwise distances and relative radial velocities using a single clock reference within the network. A new Cramer Rao Bound (CRB) is derived and the proposed fusion center based Extended Global Least Squares (EGLS) solution is shown to be asymptotically optimal.

***Index Terms*—** joint estimation, clock synchronization, skew, offset, distance, wireless network, anchorless, global solution, range, range rate, relative, velocity

1. INTRODUCTION

Localization and synchronization are fundamental challenges for wireless networks for coherent data sampling, communication and processing. The locations of nodes in a network are estimated by measuring pairwise distances via ranging and later applying absolute localization ($e.g.$, TOA, TDOA) or relative localization algorithms ($e.g.$, MDS) [1]. On the other hand, network wide synchronization is achieved by first estimating unknown clock offsets and clock skews followed by correcting respective clocks aptly. Both these challenges are conventionally solved independently. However, recently various estimators have been proposed for joint localization and synchronization based on two way communication between nodes, during which the nodes exchange data packets alternatively. For a given node pair, assuming one as the clock reference, Noh $et\,al.$ [2] proposed a Gaussian Maximum Likelihood Estimator (GMLL) for estimating the clock offset and clock skew of the unknown node. Following this, a simplified least squares solution (LCLS) [3] was presented for estimating the clock parameters. More recently, the Pairwise Least Squares (PLS) [4] was presented to jointly estimate clock skew, offset and pairwise distance between the nodes. In addition, for the entire network, a Global Least Squares (GLS) [4] was proposed for network wide estimation of clock skews, offsets and pairwise distances using a single clock reference. Note that, all these estimators were presented for a static network where the positions of the nodes are fixed.

In this paper, we present a generic model for a network of mobile nodes, where the nodes are in relative motion with each other. Secondly, we propose a generic arbitrary two way communication between any node pair, in contrast to traditional alternating communication employed in [2, 3, 4]. Based on this new model, a novel Extended Global Least Squares (EGLS) algorithm is proposed for network wide estimation of clock skew, clock offset, range and range rate, using a single clock reference. The corresponding Cramer-Rao Bound is derived and the proposed estimators are shown to be asymptotically optimal.

This research was funded in part by the Dutch STW OLFAR project (Contract Number: 10556) within the ASSYS perspectief program.

Our motivation is OLFAR (Orbiting Low Frequency Array for Radio astronomy) [5], an anchorless network of ≥ 10 satellite nodes in space which is currently being designed. Each satellite in OLFAR has a light weight atomic clock and samples the sky at ultra low frequencies of 0.3-30 MHz, thus giving clock coherence up to 30 minutes. In comparison to the raw data exchange and the on board correlation in the satellites, the time stamp exchanges and proposed centralized algorithm are negligible, both in terms of communication and computational power. Hence, we assume a wireless network of nodes capable of two way communication with each other. Additionally, each node is equipped with a light atomic clock offering sufficient stability during the period of low frequency data collection. Furthermore we assume, every node is equipped with adequate processing and communication capabilities.

Notation: The element wise matrix Hadamard product is denoted by $\odot$, element wise Hadamard division by $\oslash$, $(\cdot)^{\odot N}$ denotes element-wise matrix exponent. The Kronecker product is indicated by $\otimes$ and the transpose operator by $(\cdot)^T$. $\mathbf{1}_N = [1, 1 \ldots, 1]^T, \mathbf{0}_N = [0, 0 \ldots, 0]^T \in \mathbb{R}^{N\times 1}$, are vectors of ones and zeros respectively. $\mathbf{I}_N$ is an $N \times N$ identity matrix and diag$(\cdot)$ represents a diagonal matrix.

2. PROBLEM FORMULATION

2.1. Time

Consider a network of N nodes equipped with independent clock oscillators which, under ideal conditions, are synchronized to the global time. However, in reality, due to various oscillator imperfections and environment conditions the clocks vary independently. Let t_i be the local time at node i, then its divergence from the ideal *true* time t is to first order given by the affine clock model,

$$t_i = \omega_i t + \phi_i \quad \Leftrightarrow \quad t = \alpha_i t_i + \beta_i \tag{1}$$

where $\omega_i \in \mathbb{R}_+$ and $\phi_i \in \mathbb{R}$ are the clock skew and clock offset of node i. The clock skew and clock offset parameters for all N nodes are represented by $\boldsymbol{\omega} = [\omega_1, \omega_2, \ldots, \omega_N]^T \in \mathbb{R}_+{}^{N\times 1}$ and $\boldsymbol{\phi} = [\phi_1, \phi_2, \ldots, \phi_N]^T \in \mathbb{R}^{N\times 1}$ respectively. Alternatively, the translation from local time t_i to the global time t is written as a function of local time, where $[\alpha_i, \beta_i] \triangleq [\omega_i^{-1},\ -\phi_i\omega_i^{-1}]$ are the calibration parameters needed to correct the local clock at node i. Following immediately, for all N nodes in the network, we have $\boldsymbol{\alpha} \triangleq \mathbf{1}_N \oslash \boldsymbol{\omega} \in \mathbb{R}^{N\times 1}$ and $\boldsymbol{\beta} \triangleq -\boldsymbol{\phi} \oslash \boldsymbol{\omega} \in \mathbb{R}^{N\times 1}$.

2.2. Range

In addition to clock variations, the nodes are also in relative motion with each other which in reality, is non-linear. However we exploit the piecewise linearity of motion for smaller durations and hence pairwise distance can be written as a first order linear model. The pairwise distance between the node pair (i, j) is measured by the propagation delay $r_{ij} \equiv r_{ji}$ and is given by

$$r_{ij} = \nu_{ij} t + \tau_{ij} \tag{2}$$

where $\nu_{ij}, \tau_{ij} \in \mathbb{R}$ are the range rate delay and initial range delay between the node pair i, j respectively. The radial velocity is $c\nu_{ij} \equiv c\nu_{ji}$ and the pairwise distance at time $t = 0$ is $c\tau_{ij} \equiv c\tau_{ji}$, where c

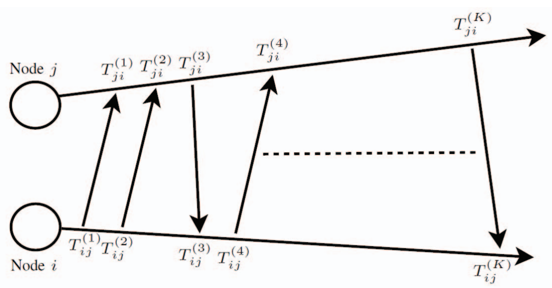

Fig. 1. Figure shows two way communication between a pair of mobile nodes where the nodes transmit and receive data packets, during which K time stamps are recorded at respective nodes. Unlike previous models [2, 3, 4] where the transmission and reception was alternating, the presented model puts no pre-requisite on the sequence of number of two way communications.

is the speed of the electromagnetic wave in the medium. All $M = \binom{N}{2}$ *unique* pairwise range delays between N nodes are stacked in vector $\boldsymbol{\tau} = \{\tau_{ij}\} \in \mathbb{R}^{M\times 1}\ \forall\ 1 \leq\ i \leq N-1$ and $i+1 \leq\ j \leq N$ and in similar lines the range rate delay $\boldsymbol{\nu} \in \mathbb{R}^{M\times 1}$. Now, the propagation delay r_{ij} can also be written as a function of local time t_i of a random Node i within the network. Thus from (1) we have,

$$r_{ij} = \gamma_{ji} t_i + \delta_{ji} \tag{3}$$

where $\gamma_{ji} = \nu_{ji}\alpha_i$ and $\delta_{ji} = \nu_{ji}\beta_i + \tau_{ji}$, which incorporates the clock discrepancy of Node i. Note that, although the system related parameters ($\gamma_{ij} \neq \gamma_{ji},\ \delta_{ij} \neq \delta_{ji}$) are dependent on the choice of clock reference i, the true range parameters ($\nu_{ij} \equiv \nu_{ji},\ \tau_{ij} \equiv \tau_{ji}$) remain unique to a given node pair. For the entire network we have $\boldsymbol{\gamma}, \boldsymbol{\delta} \in \mathbb{R}^{M\times 1}$ which are defined similar to $\boldsymbol{\tau}$ and $\boldsymbol{\nu}$.

In this paper, we intend to estimate the system parameters $(\boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\gamma}, \boldsymbol{\delta})$, given an arbitrary clock reference and communication between all nodes. The absolute clock skews ($\boldsymbol{\omega}$), clock offsets ($\boldsymbol{\phi}$) and range delays ($\boldsymbol{\tau}$) and the range rate delays ($\boldsymbol{\nu}$) of the network can be obtained from the system parameters without any ambiguities via back substitution.

3. NETWORK SYNCHRONIZATION AND RANGING

Prior to investigating the entire network, we consider a single pair of nodes. Consider a pair of mobile nodes (i, j) such that $\{i, j\} \leq N$ and $i < j$, which are capable of two way communication with each other as shown in Figure 1. The two nodes communicate messages back and forth, and the transmission and reception time stamps are registered independently at respective nodes in respective local time coordinates. The k th time stamp recorded at node i when communicating with node j is denoted by $T_{ij}^{(k)}$ and similarly at node j the time stamp is $T_{ji}^{(k)}$. For the sake of generality, we do not presume any specific sequence of data packet exchange or number of transmissions/receptions between these nodes. In all there are K time stamps recorded at each node, during which the propagation delay between the two nodes is governed by the linear range model given by (3). Under ideal circumstances, when the nodes are completely synchronized, the noise free k th communication time markers are related as

$$T_{ji}^{(k)} = \begin{cases} T_{ij}^{(k)} + r_{ij} & \text{for } i \rightarrow j \\ T_{ij}^{(k)} - r_{ij} & \text{for } i \leftarrow j \end{cases}$$

which can be combined as

$$T_{ji}^{(k)} \quad = \quad T_{ij}^{(k)} + E_{ij}^{(k)} r_{ij} \qquad \text{for } i \leftrightarrow j \tag{4}$$

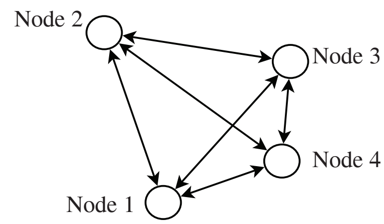

Fig. 2. An illustration of a network with $N = 4$ nodes, each capable of two way communication. The clock skews and clock offsets of node 2, 3 and 4 are unknown and are to be estimated, in addition to all the pairwise distances.

where $E_{ij}^{(k)} = +1$ for transmission from node i to node j and $E_{ij}^{(k)} = -1$ for transmission from node j to node i. Note that $E_{ij}^{(k)} \neq E_{ji}^{(k)}$ represents the direction information of the data packet. However in reality, due to clock uncertainties modeled in (1), we have (5), where $\{q_i^{(k)}, q_j^{(k)}\} \sim \mathcal{N}(0, 0.5\sigma^2)$ are Gaussian i.i.d noise variables plaguing the timing measurements at respective nodes. Without loss of generality, we assume the same noise variance on both transmission and reception markers. Now, incorporating the range model for r_{ij} from (3) as a function of local time at node j we have (6). Expanding (6) and rearranging the terms, we obtain (7). For all K communications, a generalized model for a pair of nodes is

$$[\mathbf{t}_{ij} \quad -\mathbf{t}_{ji} \quad \mathbf{1}_K \quad -\mathbf{1}_K \quad \mathbf{e}_{ij} \odot \mathbf{t}_{ji} \quad \mathbf{e}_{ij}] \begin{bmatrix} \alpha_i \\ \alpha_j \\ \beta_i \\ \beta_j \\ \gamma_{ij} \\ \delta_{ij} \end{bmatrix} = \mathbf{q}_{ij} \tag{8}$$

where the measurements are

$$\mathbf{t}_{ij} \quad = \quad [T_{ij}^{(1)}, T_{ij}^{(2)}, \ldots, T_{ij}^{(K)}]^T \in \mathbb{R}^{K\times 1} \tag{9}$$
$$\mathbf{e}_{ij} \quad = \quad [E_{ij}^{(1)}, E_{ij}^{(2)}, \ldots, E_{ij}^{(K)}]^T \in \mathbb{R}^{K\times 1} \tag{10}$$

$\mathbf{t}_{ij}$ and $\mathbf{t}_{ji}$ contains the time markers recorded at node i and node j respectively while communicating with each other and $\mathbf{e}_{ij}$ is a known vector indicating the transmission direction for each data packet. $\mathbf{q}_{ij}$ is the i.i.d noise vector, which is modeled as $\mathbf{q}_{ij} \sim \mathcal{N}(0, 0.5\sigma^2(\alpha_i^2 + (\alpha_j + \gamma_{ij})^2)) \in \mathbb{R}^{K\times 1}$. In reality, the clock skews ω_i, ω_j are very close to 1 and the errors are of the order of 10^{-4}. Hence $\alpha_i^2 + \alpha_j^2 \approx 2$, which is satisfactory and is implicitly employed in various literature such as [2, 3]. In addition, γ_{ij}^2 is by definition, negligibly small and therefore the noise vector is approximated as

$$\mathbf{q}_{ij} \sim \mathcal{N}(0, \sigma^2) \in \mathbb{R}^{K\times 1} \tag{11}$$

Now, by asserting one of the two nodes as the reference node, say node i with $[\alpha_i, \beta_i] = [1, 0]$, equation (8) is simplified to

$$\mathbf{A}_{ji}\boldsymbol{\theta}_j \quad = \quad -\mathbf{t}_{ij} + \mathbf{q}_{ij} \tag{12}$$

where

$$\mathbf{A}_{ji} = [-\mathbf{t}_{ji} \quad -\mathbf{1}_K \quad \mathbf{e} \odot \mathbf{t}_{ji} \quad \mathbf{e}] \in \mathbb{R}^{K\times 4}$$
$$\boldsymbol{\theta}_j = [\alpha_j \quad \beta_j \quad \gamma_{ij} \quad \delta_{ij}]^T \in \mathbb{R}^{4\times 1}$$

A least Squares solution is obtained by minimizing the least squares norm, *i.e.*,

$$\hat{\boldsymbol{\theta}}_j = \arg\min_{\boldsymbol{\theta}_j} \quad \|\mathbf{A}_{ji}\boldsymbol{\theta}_j + \mathbf{t}_{ij}\|_2^2 = (\mathbf{A}_{ji}^T\mathbf{A}_{ji})^{-1}\mathbf{A}_{ji}^T\mathbf{t}_{ij} \tag{13}$$

which has a unique solution provided (a) the number of communications $K \geq 4$, (b) $\mathbf{e}_{ij} \neq -\mathbf{1}_K$ and (c) $\mathbf{e}_{ij} \neq +\mathbf{1}_K$. Thus among the $K \geq 4$ data exchanges between the two nodes, there must be at least one transmission from i to j and j to i respectively. The least squares solution is a trivial extension of PLS [4] where the range rate delay was not estimated. We now extend this pairwise model to the entire network and wish to find a global optimal solution for joint motion estimation and synchronization. Aggregating (8), for all *unique* pairwise links in the network, we have a linear global model of the form

$$\overbrace{\begin{bmatrix}\mathbf{T}_1 & \mathbf{E}_1 & \mathbf{E}_2 \odot \mathbf{T}_2 & \mathbf{E}_2\end{bmatrix}}^{\mathbf{A}} \overbrace{\begin{bmatrix}\boldsymbol{\alpha} \\ \boldsymbol{\beta} \\ \boldsymbol{\gamma} \\ \boldsymbol{\delta}\end{bmatrix}}^{\boldsymbol{\theta}} = \mathbf{q} \tag{14}$$

where matrices $\mathbf{T}_1, \mathbf{T}_2 \in \mathbb{R}^{KM \times N}$ contain timing vectors recorded at all N nodes, $\mathbf{E}_1 \in \mathbb{R}^{KM \times N}$ is matrix of $\mathbf{1}_K$ and $\mathbf{0}_K$, $\mathbf{E}_2 \in \mathbb{R}^{KM \times M}$. $\mathbf{q} = [\mathbf{q}_{12}, \mathbf{q}_{13}, \ldots, \mathbf{q}_{(N-1)(N)}] \in \mathbb{R}^{KM \times 1}$ is the noise vector where each $\mathbf{q}_{ij}$ is given by (11). Equation (14) does not have a unique solution, unless we impose some linear constraints on the system. For instance, assigning one node as the clock reference with known clock offset and skew. More generally, the unknown vector $\boldsymbol{\theta} \in \mathbb{R}^{2(N+M) \times 1}$ can be estimated by minimizing the cost function

$$\begin{aligned} \min_{\boldsymbol{\theta}} \quad & \| \mathbf{A}\boldsymbol{\theta} \|^2 \\ \text{s.t.} \quad & \mathbf{C}\boldsymbol{\theta} = \mathbf{d} \end{aligned} \tag{15}$$

where $\mathbf{C} \in \mathbb{R}^{P \times N}$, the known constraint matrix and $\mathbf{d} \in \mathbb{R}^{P \times 1}$ form the primal feasibility condition, enforcing P linearly independent constraints on $\boldsymbol{\theta}$. Assuming the constraints are selected such that $\begin{bmatrix}\mathbf{A} \\ \mathbf{C}\end{bmatrix}$ is non singular and $\mathbf{d} \neq \mathbf{0}_P$, the solution to (15) is obtained by solving the *Karush-Kuhn-Tucker* equations [6] and is given by

$$\begin{bmatrix}\boldsymbol{\theta} \\ \boldsymbol{\lambda}\end{bmatrix} = \begin{bmatrix}2\mathbf{A}^T\mathbf{A} & \mathbf{C}^T \\ \mathbf{C} & \mathbf{0}_P\mathbf{0}_P^T\end{bmatrix}^{-1} \begin{bmatrix}\mathbf{0}_{2(N+M)} \\ \mathbf{d}\end{bmatrix} \tag{16}$$

where $\boldsymbol{\lambda} \in \mathbb{R}^{P \times 1}$ is the Lagrange vector. As an illustration, Figure 2 shows a network consisting of $N = 4$ nodes with $M = 6$ *unique* two way communication links. For $N = 4$, $\mathbf{T}_1, \mathbf{E}_1$ are of the form

$$\begin{aligned}
\mathbf{T}_1 &= \begin{bmatrix} \mathbf{t}_{12} & -\mathbf{t}_{21} & & \\ \mathbf{t}_{13} & & -\mathbf{t}_{31} & \\ \mathbf{t}_{14} & & & -\mathbf{t}_{41} \\ & \mathbf{t}_{23} & -\mathbf{t}_{32} & \\ & \mathbf{t}_{24} & & -\mathbf{t}_{42} \\ & & \mathbf{t}_{34} & -\mathbf{t}_{43} \end{bmatrix} \\
\mathbf{E}_1 &= \begin{bmatrix} +\mathbf{1}_K & -\mathbf{1}_K & & \\ +\mathbf{1}_K & & -\mathbf{1}_K & \\ +\mathbf{1}_K & & & -\mathbf{1}_K \\ & +\mathbf{1}_K & -\mathbf{1}_K & \\ & +\mathbf{1}_K & & -\mathbf{1}_K \\ & & +\mathbf{1}_K & -\mathbf{1}_K \end{bmatrix} \\
\mathbf{T}_2 &= \text{diag}(\mathbf{t}_{21}, \mathbf{t}_{31}, \mathbf{t}_{41}, \mathbf{t}_{32}, \mathbf{t}_{42}, \mathbf{t}_{43}) \\
\mathbf{E}_2 &= \text{diag}(\mathbf{e}_{12}, \mathbf{e}_{13}, \mathbf{e}_{14}, \mathbf{e}_{23}, \mathbf{e}_{24}, \mathbf{e}_{34})
\end{aligned}$$

A similar structure can be generalized for $N \geq 4$. The vectors $\mathbf{t}_{ij}$ are the time stamps recorded at the i the node when communicating with the j node in the network and is defined in (9). Similarly, vector $\mathbf{e}_{ij}$ contains the direction information of the corresponding pairwise communication and is defined in (10). Matrix $\mathbf{A}$ is rank deficient by 2 which expects the number of constraints $P \geq 2$. In addition, since $\mathbf{T}_2$ and $\mathbf{E}_2$ are diagonal matrices and hence full rank, the P constraints must be on the clock parameters of the system. A simple constraint would be to choose a random node, say Node 4 as the clock reference *i.e.*, $[\alpha_4, \beta_4] = [1, 0]$, which yields

$$\mathbf{C} = \left[\begin{array}{cccc|cccc|c|c} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & \mathbf{0}_M & \mathbf{0}_M \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & \mathbf{0}_M & \mathbf{0}_M \end{array}\right] \quad \mathbf{d} = \begin{bmatrix}1 \\ 0\end{bmatrix}$$

4. CRAMER RAO LOWER BOUND

Since the error vector $\mathbf{q}$ in (14) is Gaussian by assumption, the Cramer Rao Bound (CRB) on the error variance for an unbiased estimator for the constrained case is given by [7]

$$\mathcal{E}\left\{(\hat{\tilde{\boldsymbol{\theta}}} - \tilde{\boldsymbol{\theta}})(\hat{\tilde{\boldsymbol{\theta}}} - \tilde{\boldsymbol{\theta}})^T\right\} \geq \mathbf{U}(\mathbf{U}^T\mathbf{F}\mathbf{U})^{-1}\mathbf{U}^T \tag{17}$$

where $\tilde{\boldsymbol{\theta}} = [\boldsymbol{\omega},\ \boldsymbol{\phi},\ \boldsymbol{\nu},\ \boldsymbol{\tau}]^T$, $\mathbf{U} \in \mathbb{R}^{2(N+M) \times (2(N+M)-P)}$ is an orthonormal basis for the null space of the constraint matrix $\mathbf{C}$ and $\mathbf{F} = \sigma^{-2}\mathbf{J}^T\mathbf{J} \in \mathbb{R}^{2(N+M) \times 2(N+M)}$ is the Fisher Information Matrix [8] of (14). For jointly estimating the clock parameters $(\boldsymbol{\omega}, \boldsymbol{\phi})$ and the range parameters $(\boldsymbol{\nu}, \boldsymbol{\tau})$ we have

$$\mathbf{J} = \left[\frac{\partial \mathbf{A}\boldsymbol{\theta}}{\partial \tilde{\boldsymbol{\theta}}^T}\right] \triangleq \begin{bmatrix}\mathbf{J}_{\boldsymbol{\omega}} & \mathbf{J}_{\boldsymbol{\phi}} & \mathbf{J}_{\boldsymbol{\nu}} & \mathbf{J}_{\boldsymbol{\tau}}\end{bmatrix} \quad \in \mathbb{R}^{2KM \times 2(N+M)} \tag{18}$$

where the independent components can be shown as

$$\begin{aligned}
\mathbf{J}_{\boldsymbol{\omega}} &= -(\mathbf{T}_1 - \mathbf{E}_1 \odot \boldsymbol{\Phi} + \tilde{\boldsymbol{\Gamma}} \odot (\tilde{\mathbf{T}}_1 - \boldsymbol{\Phi})) \oslash \boldsymbol{\Omega}^{\odot 2} \\
\mathbf{J}_{\boldsymbol{\phi}} &= -(\tilde{\boldsymbol{\Gamma}} + \mathbf{E}_1) \oslash \boldsymbol{\Omega} \\
\mathbf{J}_{\boldsymbol{\nu}} &= \mathbf{E}_2 \\
\mathbf{J}_{\boldsymbol{\tau}} &= \mathbf{E}_2 \odot \tilde{\mathbf{T}}_2
\end{aligned}$$

and $\boldsymbol{\Omega} = \mathbf{1}_{KM}\boldsymbol{\omega}^T$ and $\boldsymbol{\Phi} = \mathbf{1}_{KM}\boldsymbol{\phi}^T$. For $N = 4$, $\tilde{\mathbf{T}}_2 \in \mathbb{R}^{KM \times M} = \text{diag}(\alpha_2\mathbf{t}_{21} + \beta_2,\ \alpha_3\mathbf{t}_{31} - \beta_3, \alpha_4\mathbf{t}_{41} + \beta_4,\ \alpha_3\mathbf{t}_{32} - \beta_3, \alpha_4\mathbf{t}_{42} + \beta_4,\ \alpha_4\mathbf{t}_{43} + \beta_4)$ and $\tilde{\boldsymbol{\Gamma}}, \tilde{\mathbf{T}}_1 \in \mathbb{R}^{KM \times N}$ are of the form

$$\begin{aligned}
\tilde{\boldsymbol{\Gamma}} &= \left[\begin{array}{c|ccc} & \mathbf{e}\nu_{21} & & \\ & & \mathbf{e}\nu_{31} & \\ \mathbf{0}_{KM} & & & \mathbf{e}\nu_{41} \\ & & \mathbf{e}\nu_{32} & \\ & & & \mathbf{e}\nu_{42} \\ & & & \mathbf{e}\nu_{43} \end{array}\right] \\
\tilde{\mathbf{T}}_1 &= \left[\begin{array}{c|ccc} & \mathbf{t}_{21} & & \\ & & \mathbf{t}_{31} & \\ \mathbf{0}_{KM} & & & \mathbf{t}_{41} \\ & & \mathbf{t}_{32} & \\ & & & \mathbf{t}_{42} \\ & & & \mathbf{t}_{43} \end{array}\right]
\end{aligned}$$

$$\alpha_i(T_{ij}^{(k)} + q_i^{(k)}) + \beta_i + E_{ij}^{(k)} r_{ij} = \alpha_j(T_{ji}^{(k)} + q_j^{(k)}) + \beta_j \tag{5}$$

$$\alpha_i(T_{ij}^{(k)} + q_i^{(k)}) + \beta_i + E_{ij}^{(k)}(\gamma_{ij}(T_{ji}^{(k)} + q_j^{(k)}) + \delta_{ij}) = \alpha_j(T_{ji}^{(k)} + q_j^{(k)}) + \beta_j \tag{6}$$

$$\alpha_i T_{ij}^{(k)} - \alpha_j T_{ji}^{(k)} + \beta_i - \beta_j + E_{ij}^{(k)}(\gamma_{ij} T_{ji}^{(k)} + \delta_{ij}) = \alpha_j q_j^{(k)} - \alpha_i q_i^{(k)} + E_{ij}^{(k)} \gamma_{ij} q_i^{(k)} \tag{7}$$

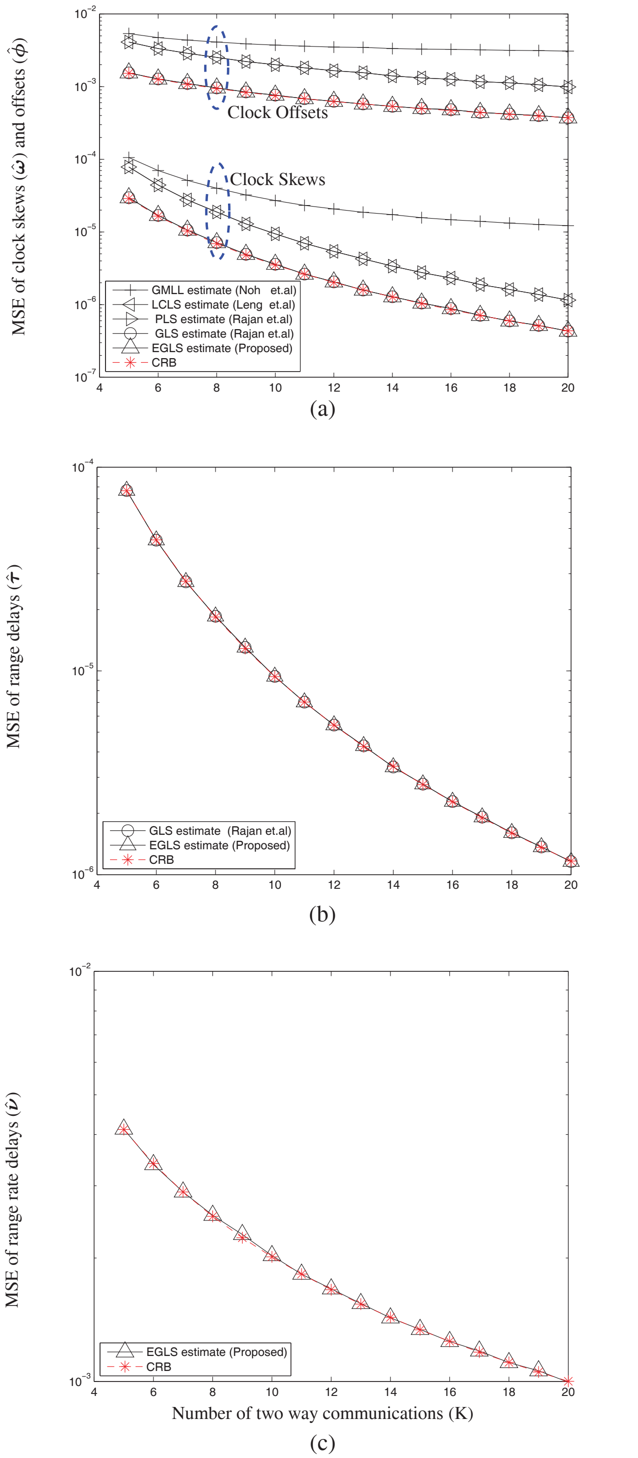

Fig. 3. Mean Square Error (MSE) plot of estimated (a) Clock skews ($\hat{\omega}$), (a) Clock offsets ($\hat{\phi}$), (b) Range delay ($\hat{\tau}$) and (c) Range rate delay ($\hat{\nu}$) for a network of $N = 4$ nodes, where the noise is Gaussian with $\sigma = 0.1$

5. SIMULATIONS

The example in section 3 is simulated to test the performance of the estimator. We assume the nodes are located within 150 Km of each other and consequently $\boldsymbol{\tau}$ is a random vector in the range [0,150km]/c. The relative velocities of the nodes are assumed to be within $\pm$ 1 m/s and hence the range rate delay vector $\boldsymbol{\nu}$ is a random vector in the range [-1m, +1m]/c. The clock skews ($\boldsymbol{\omega}$) and clock offsets ($\boldsymbol{\phi}$) are uniform randomly distributed in the range $[0.998, 1.002]$ and $[-1, 1]$ seconds respectively, which is typical for a Rubidium clock. The transmission time markers $\mathbf{t}_{ij}$ are linearly distributed between 1 to 100 seconds, for a number of two way communication links K from 5 to 20. The noise variance on the timing markers is $\sigma = 0.1$ and all results presented are averaged over 10,000 independent Monte Carlo runs.

In Figure 3, the Mean Square Errors (MSEs) of clock skews ($\boldsymbol{\omega}$), clock offsets ($\boldsymbol{\phi}$), range delays ($\boldsymbol{\tau}$) and range rate delays ($\boldsymbol{\nu}$) are plotted against the number of two way communications K. The Low Complexity Least Squares (LCLS) [3], the Maximum Likelihood GMLL [2] algorithms are independently applied, pairwise from node 1 to every other node, to estimate all the unknown skews $\boldsymbol{\omega}$ and offsets $\boldsymbol{\phi}$. In Figure 3(a) and 3(b), not surprisingly, the Extended Global Least Squares (EGLS) solution matches the Global Least Squares (GLS) solution and achieves the Cramer Rao Bound for clock offsets, clock skews and range rates. In addition, the relative velocities are also estimated in terms of range rates ($\boldsymbol{\nu}$) using (EGLS). Figure 3(c) shows that the proposed Extended Global Least Squares (EGLS) solution for $\boldsymbol{\nu}$, which achieves the Cramer Rao Lower Bound and is thus optimal.

6. REFERENCES

[1] N. Patwari, J. Ash, S. Kyperountas, I. Hero, A.O., R. Moses, and N. Correal, “Locating the nodes: cooperative localization in wireless sensor networks,” *IEEE Signal Processing Magazine*, vol. 22, no. 4, pp. 54 – 69, 2005.

[2] K.-L. Noh, Q. Chaudhari, E. Serpedin, and B. Suter, “Novel clock phase offset and skew estimation using two-way timing message exchanges for wireless sensor networks,” *IEEE Transactions on Communications*, vol. 55, no. 4, pp. 766 –777, 2007.

[3] M. Leng and Y.-C. Wu, “On clock synchronization algorithms for wireless sensor networks under unknown delay,” *IEEE Transactions on Vehicular Technology*, vol. 59, no. 1, pp. 182 –190, 2010.

[4] R. T. Rajan and A. J. van der Veen, “Joint ranging and clock synchronization for a wireless network,” in *4th IEEE International Workshop on Computational Advances in Multi-Sensor Adaptive Processing (CAMSAP2011)*, San Juan, Puerto Rico, Dec. 2011.

[5] R. T. Rajan, S. Engelen, M. Bentum, and C. Verhoeven, “Orbiting low frequency array for radio astronomy,” in *IEEE Aerospace Conference*, March 2011, pp. 1 –11.

[6] S. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge University Press, Mar. 2004. [Online]. Available: http://www.amazon.com/exec/obidos/redirect?tag=citeulike07-20&path=ASIN/0521833787

[7] P. Stoica and B. C. Ng, “On the Cramer-Rao Bound under parametric constraints,” *IEEE Signal Process. Lett.*, vol. 5, no. 7, pp. 177–179, 1998.

[8] S. M. Kay, *Fundamentals of statistical signal processing: estimation theory*. Upper Saddle River, NJ, USA: Prentice-Hall, Inc., 1993.