ALGORITHMS TO SEPARATE OVERLAPPING SECONDARY SURVEILLANCE RADAR REPLIES

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In this article, we investigate the separation of a linear mixture of Secondary Surveillance Radar (SSR) replies impinging on an M-element antenna array. At base-band, a received SSR signal consists of a binary sequence with alphabet $\{0, 1\}$, modulated by a complex exponential due to the residual carrier frequency. We present two algebraic algorithms to compute the separating beamformers by taking into account the particular modulation format of the received signal.

1. INTRODUCTION

Secondary Surveillance Radar (SSR) is essential for Air Traffic Control (ATC). Unlike the primary radar, it is a communication radar (transponder system) that informs the ATC about the identity and the altitude of the aircrafts in the line of sight. A base station sends a pulse at 1030 MHz to interrogate an aircraft, which answers with the SSR reply signal, a bursty pulse train modulated at a carrier frequency of 1090 MHz. The system was designed in the 1950s, but is currently limited by the fact that all replies use nominally the same carrier frequency, and may overlap in time. A new protocol (mode S, for Selective) will selectively address the aircrafts and permit short data communications between the station and the aircrafts. This new mode will also assist the Traffic Advisory and Collision Avoidance System (TCAS) by providing automated communication between the aircrafts. Nonetheless, also in this protocol overlaps may occur.

We assume that in the future base stations will be equipped with an antenna array [1, 2]. Our aim at the base station level is to separate the overlapping reply signals using beamforming, to detect the individual symbols, and to estimate the direction of arrival (DOA) and residual carrier frequency of each source.

Source separation can be done based on properties of the array response matrix or properties of the signals. The former has as disadvantage that a carefully calibrated array must be used, and that no multipath is tolerated. Therefore, we consider using the rich structure of the source signals.

We have noted before in [3] that it is impossible to use Higher-Order Statistical (HOS) methods because for SSR signals all cumulants of order 3, 4 and 5 have a large probability to be null. However, algebraic techniques are possible. Earlier results were reported in [4], and a full report is available in the PhD thesis [5]. In this article, we present two algorithms from [5] and compare these to the algorithms in [4] in simulations.

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2. DATA MODEL

2.1. Received data model

A Mode S reply frame contains either 56 or 112 binary symbols b_n. The bits are encoded in a “Manchester Encoding” scheme, which means that a bit $b_n = 0$ is coded as $\mathbf{b}_n = [0, 1]$, and a bit $b_n = 1$ as $\mathbf{b}_n = [1, 0]$. The emitted bit stream $\mathbf{b}$ consists of a preamble, $\mathbf{p} = [1, 0, 1, 0, 0, 0, 0, 1, 0, 1, 0]$, followed by the encoded data bits, i.e.,

$$\mathbf{b} = [\mathbf{p}, 0, 0, 0, 0, 0, 0, \mathbf{b}_1, \mathbf{b}_2, \cdots, \mathbf{b}_{56/112}]$$

of a total length $N = \{128, 240\}$. The preamble is aimed to facilitate the synchronization (detection of the start of a frame).

The Mode S reply signal emitted by the transponder is a pulse amplitude modulation of $\mathbf{b}$, and has the form

$$b(t) = \sum_{n=0}^{127/239} \mathbf{b}[n] p\,(t - nT) \qquad (1)$$

where $\mathbf{b}[n]$ is the n-th entry of $\mathbf{b}$, and $p(t)$ is a (nominally) rectangular pulse of width $T = 0.5\ \mu$s, for Mode S.

Before being emitted by the antenna, the signal is up-converted to the frequency band f_e. Nominally, the carrier frequency is $f_c = 1090$ MHz, but the tolerance permitted by the ICAO is ± 3 MHz, thus $f_e \neq f_c$. In future, this tolerance will be reduced to ± 1 MHz. Due to this carrier frequency mismatch, a residual frequency f remains after down-conversion by f_c to baseband. This residual carrier adds a significant phase rotation to the transmitted symbols : up to a complex gain, the received baseband signal $s[n] = s(nT)$ is described as

$$s[n] = b[n] \exp{(\jmath 2\pi n f T)} = b[n]\phi^n \qquad (2)$$

where $\phi = \exp{(\jmath 2\pi f T)}$ is the phase shift due to the carrier frequency shift over a sampling period.

We extend this model to the reception of d independent source signals on an M-element antenna array, assuming the multipath is negligible. The baseband antenna signals are sampled at rate $1/T$ and stacked in vectors $\mathbf{x}[n]$ (size M). After collecting N samples, the observation model is

$$\mathbf{X} = \mathbf{A}\mathbf{S} + \mathbf{N} \qquad (3)$$

where $\mathbf{X} = [\mathbf{x}[1], \cdots, \mathbf{x}[N]]$ is the $M \times N$ received signal matrix, $\mathbf{A}$ is the $M \times d$ mixing matrix that contains the array signatures and the complex gains of the sources, and $\mathbf{S} = [\mathbf{s}[1], \cdots, \mathbf{s}[N]]$ is the $d \times N$ source matrix, where $\mathbf{s}[n] = [s_1[n]\ , \cdots,\ s_d[n]]^T$ is a stacking of the d source signals (superscript T denotes transpose). $\mathbf{N}$ is the $M \times N$ noise matrix. We assume that the noise is temporally and spatially white. We also assume that $M > d$ and that $\mathbf{A}$ has full column rank.

Our aim is to compute beamformers $\mathbf{w}_i$, $i = 1, \cdots, d$, such that $\mathbf{w}_i^H \mathbf{x}[n] = \hat{s}_i[n]$ is an estimate of the i-th SSR signal (superscript H denotes complex conjugate transpose). In this blind source separation context, we can only try to ensure that each $\hat{s}_i[n]$ looks like an SSR signal, and that the collection of signal estimates is independent.

2.2. Preprocessing

In our application, $\mathbf{A}$ is typically tall and full column rank, but not square. To simplify our algorithms, we assume that first a preprocessing is applied on $\mathbf{X}$ to reduces its row dimension from M to d. This is done by computing a singular value decomposition of $\mathbf{X}$,

$$\mathbf{X} = \mathbf{U\Sigma V}$$

where $\mathbf{U}$ and $\mathbf{V}$ are unitary and $\mathbf{\Sigma}$ is diagonal containing the singular values in decreasing order. The number of signals d is detected from $\mathbf{\Sigma}$. Let $\hat{\mathbf{U}}$ be a $M \times d$-dimensional matrix containing the corresponding left singular vectors, then

$$\mathbf{X}' = \hat{\mathbf{U}}^H \mathbf{X} = (\hat{\mathbf{U}}^H \mathbf{A})\mathbf{S} + (\hat{\mathbf{U}}^H \mathbf{N}) =: \mathbf{A}'\mathbf{S} + \mathbf{N}'$$

This is the same model as we had before, except that $\mathbf{X}'$ is $d \times N$ and $\mathbf{A}'$ is $d \times d$ and invertible. In the algorithms, we assume that this preprocessing has been done, and we drop the primes from the notation.

3. ALGORITHMS

3.1. Manchester encoding property

The Manchester encoding of the SSR signals gives rise to an interesting temporal correlation property which is deterministic and independent of the actual transmitted data. In case the receiver is synchronized to the beginning of a packet, the product of the data stream with a T-delayed version of itself will always be equal to zero. Otherwise, when unsynchronized, we can still multiply by an additional T-delayed version, so that we are sure that one of the three factors is zero. A similar property holds for a single baseband signal $s[n]$ at the receiver, independent of a fractional sampling offset.

Property 3.1 *Independent of the transmitted data, a mode S reply signal $b(t)$ obeys :*

$$b\,(t-T)\;\; b(t)\; b\,(t+T) = 0\,, \qquad \forall t \in \mathbb{R} \tag{4}$$

$$s[n-1]\; s[n]\; s[n+1] = 0\,, \qquad \forall n \in \mathbb{N} \tag{5}$$

This property can be used to design a receiver algorithm to separate multiple SSR signals.

3.2. MDA : Manchester Decoding Algorithm

Consider a beamformer $\mathbf{w}$. If the output of the beamformer, $\mathbf{w}^H\mathbf{x}[n]$, looks like an SSR signal, then using Property (3.1) we obtain that it satisfies

$$[\mathbf{x}[n+1] \otimes \mathbf{x}[n] \otimes \mathbf{x}[n-1]]^H\,(\mathbf{w} \otimes \mathbf{w} \otimes \mathbf{w}) = 0 \tag{6}$$

for $n = 2, \cdots, N-2$, where $\otimes$ is the Kronecker product. To collect these conditions, define the matrix $\mathbf{P} : N-2 \times d^3$ as the stack of rows $[\mathbf{x}[n+1] \otimes \mathbf{x}[n] \otimes \mathbf{x}[n-1]]^H$, so that

$$\mathbf{P}\mathbf{w}^\diamond = 0\,, \qquad \mathbf{w}^\diamond \overset{\text{def}}{=} \mathbf{w} \otimes \mathbf{w} \otimes \mathbf{w} \tag{7}$$

For d sources, there are d linearly independent separating beamformers $\mathbf{w}_i$, $i = 1, \cdots, d$. Thus we have d linearly independent vectors $\mathbf{w}_i^\diamond$ that belong to the kernel of $\mathbf{P}$. If the kernel is d-dimensional, then the subspace spanned by the $\mathbf{w}_i^\diamond$ is exactly equal to the kernel, and a basis for the kernel must be a linear combination of the $\mathbf{w}_i^\diamond$. Similar to [6], the algorithm will be to estimate an arbitrary basis for the kernel, then to find the linear combinations to map the basis to the structured vectors $\mathbf{w}_i^\diamond$, and subsequently to estimate the corresponding $\mathbf{w}_i$ for each vector.

In the next proposition, we state that as the number of samples increases and the sources are completely overlapping, then the kernel of $\mathbf{P}$ will be precisely of dimension d. This implies that there are no other solutions than $\mathbf{w}_i^\diamond$, $i = 1, \cdots, d$, so that the problem is identifiable.

Proposition 3.2 *Assume that $\mathbf{A}$ is square and invertible, the sources are temporally totally overlapping, and that there is no noise. Then for large number of samples N, the matrix $\mathbf{P}$ will almost surely have rank $(d^3 - d)$, equivalently its kernel will almost surely be of dimension d.*

A proof can be found in [5]. The last step of the algorithm consists in estimating a basis of the kernel of the matrix $\mathbf{P}$. From each vector of size d^3 of this basis, it is possible to create a tensor of size $d \times d \times d$. These d tensors can be jointly diagonalized, and the eigenvectors are the desired d beamformers. The joint diagonalization of a collection of cubes can be performed either by the method presented in [7] or by the technique from [6].

The cost of this algorithm is driven by the cost of estimating the kernel of $\mathbf{P}$ ($N - 2 \times d^3$), or its QR factorization, which is of order $Nd^6/2$ complex multiplications.

A limitation of the algorithm is that, for almost completely non-overlapping SSR replies, there are additional vectors in the kernel. This will break the assumption on which the algorithm is based (i.e., any vector in the kernel is a linear combination of the $\mathbf{w}_i^\diamond$, $i = 1, \cdots, d$), and without further corrections the algorithm will show poor performance in this situation.

3.3. MS-ZCMA : Multi-Shift Zero Constant Modulus Algorithm

The received signal model (2) states that for any integer τ, two non-zero samples with a distance of τ in time have a phase difference of ϕ^τ. So for the n-th sample, either the product is zero : $s[n]s^*[n-\tau] = 0$ or it is $s[n]s^*[n-\tau] = \phi^\tau$, where '$*$' denotes the complex conjugate. Combining the two conditions, we obtain the relation

$$s[n]s^*[n-\tau]\;\,(s[n]s^*[n-\tau] - \phi^\tau) = 0 \tag{8}$$

Let $\mathbf{w}$ be a d-dimensional beamforming vector to recover $s[n]$, $\mathbf{w}^H\mathbf{x}[n] = \hat{s}[n]$. Using properties of the Kronecker product, equation (8) becomes

$$\begin{aligned}(\mathbf{x}[n] \otimes \mathbf{x}^*[n-\tau] \otimes \mathbf{x}[n] \otimes \mathbf{x}^*[n-\tau])^T\,(\mathbf{w}^* \otimes \mathbf{w} \otimes \mathbf{w}^* \otimes \mathbf{w})\\ = \phi^\tau\,(\mathbf{x}[n] \otimes \mathbf{x}^*[n-\tau])^T\,(\mathbf{w}^* \otimes \mathbf{w})\end{aligned} \tag{9}$$

Let $\mathbf{w}^{\oslash 4}$ be an $(d(d+1)/2)^2$-dimensional vector that contains only the non-redundant elements of the Kronecker product $(\mathbf{w}^* \otimes \mathbf{w} \otimes \mathbf{w}^* \otimes \mathbf{w})$. We define by $\mathbf{J}'$ the $d^4 \times (d(d+1)/2)^2$ matrix such that $(\mathbf{w}^* \otimes \mathbf{w} \otimes \mathbf{w}^* \otimes \mathbf{w}) = \mathbf{J}'\mathbf{w}^{\oslash 4}$. We also define

$$\begin{aligned}\mathbf{w}^{\oslash 2} &= \mathbf{w}^* \otimes \mathbf{w}\\ \mathbf{p}_{\tau,n}^{(1)} &= (\mathbf{x}[n] \otimes \mathbf{x}^*[n-\tau] \otimes \mathbf{x}[n] \otimes \mathbf{x}^*[n-\tau])^T\,\mathbf{J}'\\ \mathbf{p}_{\tau,n}^{(2)} &= (\mathbf{x}[n] \otimes \mathbf{x}^*[n-\tau])^T\end{aligned}$$

Then (9) can be written as

$$\mathbf{p}_{\tau,n}^{(1)}\mathbf{w}^{\oslash 4} = \phi^{\tau}\mathbf{p}_{\tau,n}^{(2)}\mathbf{w}^{\oslash 2} \tag{10}$$

Stacking the rows $\mathbf{p}_{\tau,n}^{(1)}$ and $\mathbf{p}_{\tau,n}^{(2)}$, $n = \tau, \cdots, N$, into matrices $\mathbf{P}_{\tau}^{(1)}$ and $\mathbf{P}_{\tau}^{(2)}$, respectively, we obtain

$$\mathbf{P}_{\tau}^{(1)}\mathbf{w}^{\oslash 4} = \phi^{\tau}\mathbf{P}_{\tau}^{(2)}\mathbf{w}^{\oslash 2} \tag{11}$$

where $\mathbf{P}_{\tau}^{(1)}$ is $(N-\tau)\times L$, and $\mathbf{P}_{\tau}^{(2)}$ is $(N-\tau)\times d^2$. This equation holds for all $\tau \in \mathbb{N}$, and is somewhat similar to an matrix pencil problem (eigenvalue equation for non-square matrices).

To solve this equation, we first need to find the common column span of $\mathbf{P}_{\tau}^{(1)}$ and $\mathbf{P}_{\tau}^{(2)}$. Let $\mathbf{U}_{\tau}$ be a matrix whose columns form an orthonormal basis of this subspace, and let $\mathbf{U}_{\tau}^{(1)}$ and $\mathbf{U}_{\tau}^{(2)}$ be the orthogonal complements of $\mathbf{U}_{\tau}$ over $\mathbf{P}_{\tau}^{(1)}$ and $\mathbf{P}_{\tau}^{(2)}$. Then we can compute the decomposition into 'common' and 'not common' subspaces as

$$\mathbf{P}_{\tau}^{(1)} = \begin{bmatrix}\mathbf{U}_{\tau} & \mathbf{U}_{\tau}^{(1)}\end{bmatrix}\begin{bmatrix}\mathbf{A}_{\tau}^{(1)} \\ \mathbf{B}_{\tau}^{(1)}\end{bmatrix}$$

$$\mathbf{P}_{\tau}^{(2)} = \begin{bmatrix}\mathbf{U}_{\tau} & \mathbf{U}_{\tau}^{(2)}\end{bmatrix}\begin{bmatrix}\mathbf{A}_{\tau}^{(2)} \\ \mathbf{B}_{\tau}^{(2)}\end{bmatrix}$$

where the $\mathbf{B}_{\tau}^{(i)}$, $i \in \{1,2\}$ are of full row rank. Inserting these two equations into (11), we obtain

$$\begin{bmatrix}\mathbf{U} & \mathbf{U}_{\tau}^{(1)} & \mathbf{U}_{\tau}^{(2)}\end{bmatrix}\begin{bmatrix}\mathbf{A}_{\tau}^{(1)}\mathbf{w}^{\oslash 4} - \phi^{\tau}\mathbf{A}_{\tau}^{(2)}\mathbf{w}^{\oslash 2} \\ \hline \mathbf{B}_{\tau}^{(1)}\mathbf{w}^{\oslash 4} \\ \hline -\phi^{\tau}\mathbf{B}_{\tau}^{(2)}\mathbf{w}^{\oslash 2}\end{bmatrix} = \mathbf{0} \tag{12}$$

where the first matrix compound has full column rank by definition. Thus, we have

$$\begin{aligned}\mathbf{A}_{\tau}^{(1)}\mathbf{w}^{\oslash 4} - \phi^{\tau}\mathbf{A}_{\tau}^{(2)}\mathbf{w}^{\oslash 2} &= \mathbf{0} \\ \mathbf{B}_{\tau}^{(1)}\mathbf{w}^{\oslash 4} &= \mathbf{0} \\ \mathbf{B}_{\tau}^{(2)}\mathbf{w}^{\oslash 2} &= \mathbf{0}\end{aligned} \tag{13}$$

Since it is hard to work with the three equations simultaneously, we propose in our algorithm to use only equation (13). This equation holds for any τ, and we can obtain several similar conditions by taking a range of L different $\tau \in \mathbb{N}$. Stacking the matrices $\mathbf{B}_{\tau}^{(2)}$ in a single matrix $\mathbf{Z}$, we obtain

$$\mathbf{Z}\mathbf{w}^{\oslash 2} := \begin{bmatrix}\mathbf{B}_0^{(2)} \\ \mathbf{B}_1^{(2)} \\ \vdots \\ \mathbf{B}_L^{(2)}\end{bmatrix}\mathbf{w}^{\oslash 2} = \mathbf{0} \tag{14}$$

For d SSR sources, there are d linearly independent beamformers $\mathbf{w}_i$, $i = 1, \cdots, d$, and these correspond to d independent solutions : nonzero vectors in the kernel of $\mathbf{Z}$. Note that $\mathbf{Z}$ has d^2 columns. We assume that, for a sufficient number of time-lags L, the matrix is very tall and does not have other vectors in the kernel.

Thus, the algorithm continues by estimating an orthogonal basis $\{\mathbf{y}_i\}_{i=1}^{d}$ for the kernel of the matrix $\mathbf{Z}$. Each vector $\mathbf{y}_i$ of size d^2 of this basis is a linear combination of the solutions, or $\mathbf{y}_i = \sum_{j=1}^{d}\lambda_{ij}\mathbf{w}_j^{\oslash 2}$. Reshaping the $\mathbf{y}_i$ into $d \times d$ matrices $\mathbf{Y}_i$, we obtain $\mathbf{Y}_i = \mathbf{W}\mathbf{\Lambda}_i\mathbf{W}^H$, where $\mathbf{W} = [\mathbf{w}_1, \cdots, \mathbf{w}_d]$, and the $\mathbf{\Lambda}_i$ are diagonal matrices containing the coefficients λ_{ij}. This is a joint diagonalization problem which can be solved as in [6] to obtain the beamformer matrix $\mathbf{W}$.

The computational cost is dominated by the decomposition (12) for each τ. This corresponds to the cost of a QR factorization of $[\mathbf{P}_{\tau}^{(1)} \;\; \mathbf{P}_{\tau}^{(2)}]$, which is of the order $Nd^8/8$.

4. SIMULATIONS

To demonstrate the effectiveness of the proposed algorithms (MDA and MS-ZCMA), we compare them to JADE [8] (a HOS method), to AFZA, AZCMA, and AZCMA0 proposed in [4], to ESPRIT [9], and to the relevant Cramer-Rao Bound [10]. The algorithms compute beamformers, but for simplicity of presentation, we convert this into direction-of-arrival estimates.

For the simulations, we have considered an array of four elements, with an inter-element distance of a half wavelength (to be able to use ESPRIT). We consider two equal powered sources with a SNR of 30 dB, $N = 100$ samples taken at rate $1/T = 2$ MS/s (corresponding to 50 SSR symbols), DOAs of [70, 110] degree, and frequency shifts of $[-5 \cdot 10^4, 5 \cdot 10^4]$ Hz. There are 1000 independent Monte-Carlo runs.

In the implementation of our algorithms, the joint diagonalization algorithm is the Jacobi angle method in [6]. To recover the DOAs, we use the beamformers to estimate $\mathbf{S}$, then estimate the matrix $\mathbf{A}$ using $\widehat{\mathbf{S}}$ in equation (3), and apply ESPRIT to each column of $\widehat{\mathbf{A}}$.

For the MS-ZCMA, the set of time delays $\{\tau_1, \cdots, \tau_L\}$ can be chosen arbitrarily, as long as the matrix $\mathbf{Z}$ is expected to achieve its maximal rank. In our implementation, the following set has given satisfactory results : $\{0,\ 1,\ -1,\ 2,\ -2,\ \frac{N}{10},\ \frac{-N}{10},\ \frac{N}{4},\ \frac{-N}{4},\ \frac{N}{2},\ \frac{-N}{2}\}$. In order to save some computational cost, the algorithm implementation worked in an iterative fashion : the τ were taken one by one until the estimate of the kernel of $\mathbf{Z}$ was declared stable, i.e. the subspaces at the iteration $k+1$ and k were similar.

We show the results of two simulations. In the first simulation, the source packets are totally overlapping, and we varied the angle separation between the two sources. In the second simulation, we inserted a varying time offset between the arrival of the two packets. We show the failure rate, where a failure is declared if we recover the same source twice, rather than two independent sources. For the cases without failure, we show the root mean squared error (RMSE) of the DOA estimates, because they show a better discrimination between the algorithms. In these figures, we included Wiener estimates, which were obtained from the known symbols. The DOA's are estimated by an ESPRIT on each columns of the estimated mixing matrix.

For varying angle separations, figure 1 shows that the algorithms JADE, AZCMA, and AZCMA0 are not trustworthy. We also note that if the angle separation is below 4 degree, ESPRIT algorithm begin to break down, and only MDA, MS-ZCMA and AZFA can handle closely spaced sources. Figure 2 demonstrates that only MDA and MS-ZCMA can attain the CRB, along with ESPRIT.

For varying delays between the two SSR data bursts (non-overlapping packets), figure 3 presents clearly the shortcoming of MDA : it cannot resolve well in case of non-overlapping sources. Note that as soon as the sources are not completely overlapping, JADE performs well. This effect is explained in [3]. Figure 4 demonstrates that ESPRIT and MS-ZCMA are the only algorithms that have an acceptable performance over the range of time delay offset.

5. CONCLUSIONS

We presented two algorithms (MDA and MS-ZCMA) to separate SSR replies impinging on an antenna array. Numerous si-

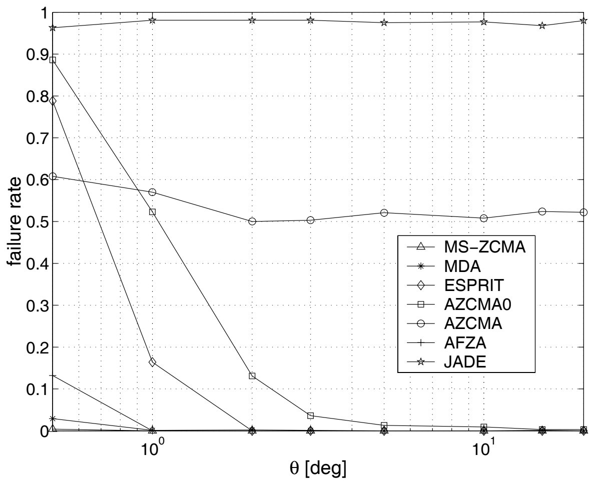

Fig. 1. Failure rate of the algorithms as a function of the angle separation.

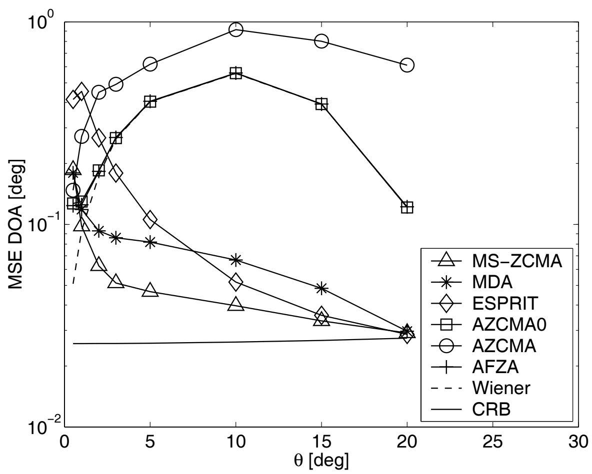

Fig. 2. DOA RMSE as a function of the angle separation.

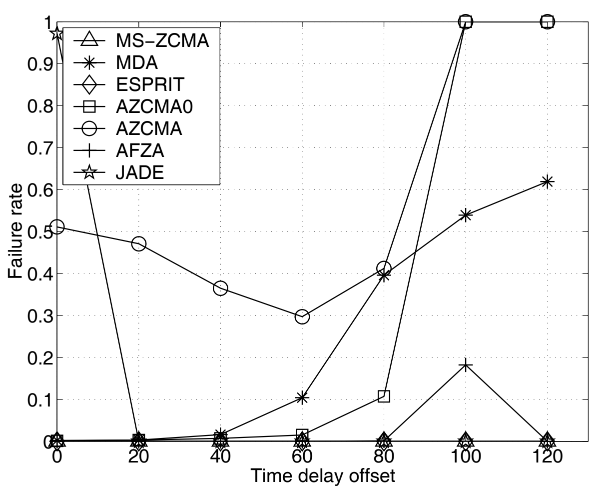

Fig. 3. Failure rate as a function of the time delay offset.

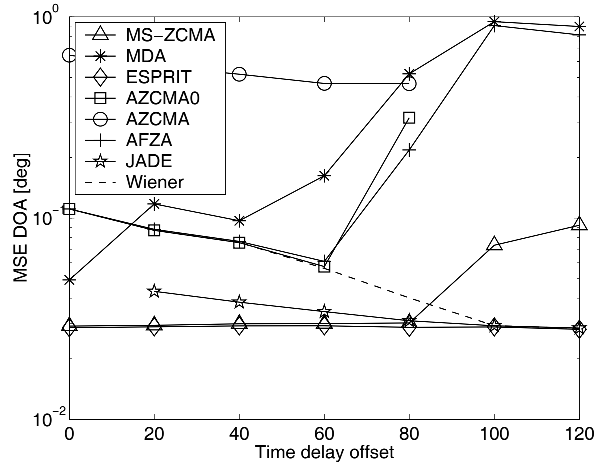

Fig. 4. DOA RMSE as a function of the time delay offset.

mulations not presented here for reasons of space have shown that both of them behave reliably. The proposed algorithms use properties of the sources, hence can work with uncalibrated or non-linear arrays, which is an advantage over algorithms based on the array manifold structure, such as ESPRIT.

6. REFERENCES

[1] N. Petrochilos and P. van Genderen, "A new approach to handle ssr replies," in *Conférence Radar*, Brest, France, 17-21 May 1999.

[2] P. Bezousek, "A passive radar surveillance system VERA for ATC," in *IRS'98*, Munich, Germany, 1998.

[3] N. Petrochilos and P. Comon, "Separation de signaux ZCM : application en radar SSR," in *Proc. of GRETSI 2003*, Paris, France, 8-11 September 2003.

[4] A.J. van der Veen and J. Tol, "Separation of zero/constant modulus signals," in *Proc. of IEEE ICASSP*, Munich (FRG), April 1997, pp. 3445–3448.

[5] N. Petrochilos, "Algorithms for Separation of Secondary Surveillance Radar Replies," Phd thesis, Universiy of Nice-Sophia-Antipolis and TU Delft, Nice, France, July 2002, ISBN 90-407-2371-0, www.i3s.unice/˜ petro, in english.

[6] A.J. van der Veen and A. Paulraj, "An analytical constant modulus algorithm," *IEEE Trans. Signal Processing*, vol. 44, no. 5, pp. 1136–1155, May 1996.

[7] Lieven De Lathauwer, "Signal Processing based on Multilinear Algebra," Phd thesis, K.U. Leuven, Belgium, Sept. 1997.

[8] Jean-François Cardoso and Antoine Souloumiac, "Blind beamforming for non-Gaussian signals," *IEE Proceedings-F*, vol. 140, no. 6, pp. 362–370, Dec. 1993.

[9] R. Roy and T. Kailath, "ESPRIT estimation of signal parameters via rotational invariance techniques," *IEEE Trans. on acoustics, speech, and Signal Processing*, vol. 37, no. 7, pp. 984–995, July 1989.

[10] N. Petrochilos and P. Comon, "Ml estimation of SSR signals, identifiability, and cramer-rao bounds," in *Proc. of EUSIPCO 2000*, Tampere, Finlande, 5-8 Sept 2000.