SPATIAL FILTERING OF RF INTERFERENCE IN RADIO ASTRONOMY USING A REFERENCE ANTENNA

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Radio astronomical observations are increasingly contaminated by RF interference. Assuming an array of telescopes, we have previously considered spatial filtering techniques based on projecting out the interferer array signature vector. In this paper, we consider extending the astronomical array with a reference antenna (or array), and develop spatial filtering algorithms for this situation. The information from the reference antenna improves the quality of the interferer signature vector estimation, hence more of the interference can be projected out. The conditioning of the problem improves as well. The algorithms are tested both on simulated and experimental data.

1. INTRODUCTION

Radio astronomical observations are increasingly contaminated by man-made RF interference, and there is a growing need for interference cancellation techniques. Depending on the interference and the type of instrument, several kinds of RFI mitigation techniques are applicable [1, 2]. For continually present interference, an interesting option is to utilize a reference antenna which picks up only the interference, so that LMS-type adaptive cancellation techniques can be implemented [3–5].

With an array of p telescope dishes (an interferometer), spatial filtering techniques are applicable as well. The desired instrument outputs in this case are $p \times p$ correlation matrices, integrated to 10 s. Based on short-term correlation matrices (integration to e.g., 10 ms) and narrow subband processing, the array signature vector of an interferer can be estimated and subsequently projected out [6]—we describe this technique in section 3.2.

To improve the performance for weak or stationary interferers, we consider in this paper to extend the telescope array with one or more reference antennas. In general, a higher gain (interference-to-noise ratio) than that obtained from an omnidirectional antenna is needed to expect any benefits. Most flexibility is obtained by using a phased array which can adaptively be pointed towards the strongest interferers. We have actually built a demonstrator set-up along these lines, utilizing a wideband phased array of 64 elements (see section 5). The reference signal is correlated along with the telescope signals as if it was an additional telescope, and spatial filtering algorithms can be applied to the resulting short-term integrated covariance matrices. This set-up is shown in figure 1.

Spatial filtering on extended arrays was first considered by Briggs et al. [7] for a single dual-polarized telescope (two channels) and two reference antennas. With their technique a single interferer can be cancelled. Jeffs et al. [8, 9] propose spatial filtering algorithms along the lines of [6]; we will summarize their approach in section 3.2 and subsequently make extensions which may improve the performance.

*This research was supported in part by the STW under DEL77-4476.

2. PROBLEM STATEMENT

2.1. Data model

Assume we have a telescope array (primary array) with p_0 elements, and a reference array with p_1 elements.[1] The total number of elements is $p = p_0 + p_1$.

We consider the signals $x_i(t)$ received at the antennas $i = 1, \cdots, p$ in a sufficiently narrow subband. For the interference free case the primary array output vector $\mathbf{x}_0(t)$ is modeled in complex baseband form as

$$\mathbf{x}_0(t) = \mathbf{v}_0(t) + \mathbf{n}_0(t)$$

where $\mathbf{x}_0(t) = [x_1(t), \ldots, x_{p_0}(t)]^{\mathrm{T}}$ is the $p_0 \times 1$ vector of telescope signals at time t, $\mathbf{v}_0(t)$ is the received sky signal, assumed on the time scale of 10 s to be a stationary Gaussian vector with covariance matrix $\mathbf{R}_{v,0}$ (the astronomical 'visibilities'), and $\mathbf{n}_0(t)$ is the $p_0 \times 1$ noise vector with independent identically distributed Gaussian entries and covariance matrix $\sigma_0^2\mathbf{I}$. The astronomer is interested in $\mathbf{R}_{v,0}$.

If an interferer is present and the processing bandwidth is sufficiently narrow, then the primary array output is modeled as

$$\mathbf{x}_0(t) = \mathbf{v}_0(t) + \mathbf{a}_0(t)s(t) + \mathbf{n}_0(t)$$

where $s(t)$ is the interferer signal with spatial signature vector $\mathbf{a}_0(t)$ which is assumed stationary only over short time intervals. Without loss of generality, we can absorb the unknown amplitude of $s(t)$ into $\mathbf{a}_0(t)$ and thus set the power of $s(t)$ to 1.

Consider now that we also have a reference antenna array. The outputs of the p_1 reference antennas are stacked into a vector $\mathbf{x}_1(t)$, modeled as

$$\mathbf{x}_1(t) = \mathbf{a}_1(t)s(t) + \mathbf{n}_1(t).$$

It is assumed here that the contribution of the astronomical sources to the reference signals is negligible. The noise on the reference antennas is assumed to be i.i.d. Gaussian with covariance matrix $\sigma_1^2\mathbf{I}$. Stacking all antenna signals in a single vector $\mathbf{x}^T = [\mathbf{x}_0^T \ \mathbf{x}_1^T]^T$, we obtain

$$\mathbf{x}(t) = \mathbf{v}(t) + \mathbf{a}(t)s(t) + \mathbf{n}(t).$$

We make the following additional assumptions on this model:

(A1) The noise variances σ_0^2 and σ_1^2 are known from calibration.

(A2) $\mathbf{R}_{v,0} \ll \sigma_0^2\mathbf{I}$. This is reasonable as even the strongest sky sources are about 15 dB under the noise floor.

(A3) The interferer signature $\mathbf{a}(t)$ is stationary over short processing times (say 10 ms). It may or may not vary over longer periods.

[1]In subsequent notation, the subscript '0' will generally refer to the primary array and '1' to the reference array.

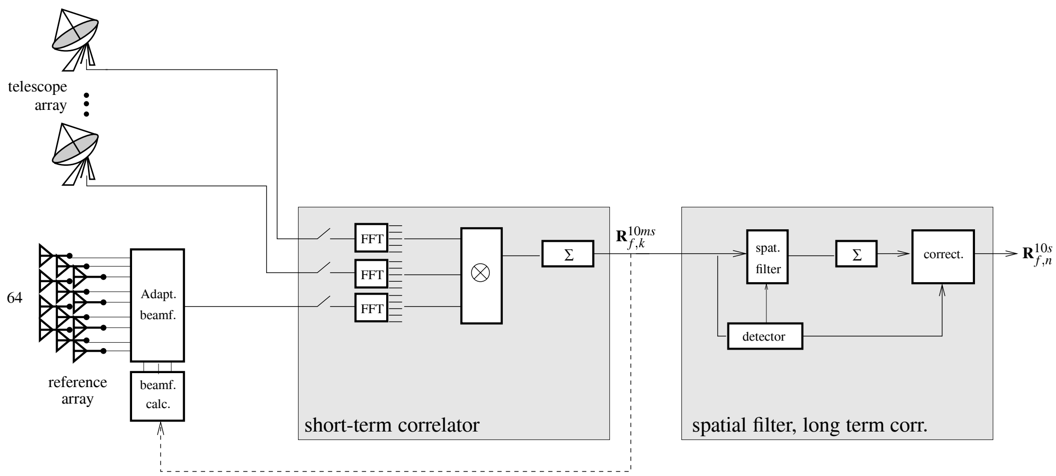

Figure 1. Telescope array augmented with a reference phased array

This was the model considered in [6]. The model is easily extended to multiple interfering sources, in which case we obtain

$$\mathbf{x}(t)=\mathbf{v}(t)+\mathbf{A}(t)\mathbf{s}(t)+\mathbf{n}(t) \Leftrightarrow \begin{cases} \mathbf{x}_0(t)=\mathbf{v}_0(t)+\mathbf{A}_0(t)\mathbf{s}(t)+\mathbf{n}_0(t) \\ \mathbf{x}_1(t)= \qquad\quad \mathbf{A}_1(t)\mathbf{s}(t)+\mathbf{n}_1(t) \end{cases}$$

where $\mathbf{A} : p \times q$ has q columns corresponding to q interferers, and $\mathbf{s}(t)$ is a vector with q entries.

2.2. Covariance model

Let be given observations $\mathbf{x}[n] := \mathbf{x}(nT_s)$, where T_s is the sampling period. We assume that $\mathbf{A}(t)$ is stationary at least over intervals of MT_s, and construct short-term covariance estimates $\hat{\mathbf{R}}_k$,

$$\hat{\mathbf{R}}_k = \frac{1}{M}\sum_{n=kM}^{(k+1)M} \mathbf{x}[n]\mathbf{x}[n]^{\mathrm{H}}$$

where M is the number of samples per short-term average. All interference filtering algorithms in this paper are based on applying operations to each $\hat{\mathbf{R}}_k$ to remove the interference, followed by further averaging over N resulting matrices to obtain a long-term average.

Considering the $\mathbf{A}_k := \mathbf{A}(kMT_s)$ as deterministic, the expected value of each $\hat{\mathbf{R}}_k$ is denoted by $\mathbf{R}_k$, which can be written in block-partitioned form as

$$\mathbf{R}_k = \begin{bmatrix} \mathbf{R}_{00,k} & \mathbf{R}_{01,k} \\ \mathbf{R}_{10,k} & \mathbf{R}_{11,k} \end{bmatrix}.$$

According to the assumptions, $\mathbf{R}_k$ has model

$$\begin{aligned} \mathbf{R}_k &= \mathbf{\Psi} + \mathbf{A}_k\mathbf{A}_k^{\mathrm{H}} = \mathbf{R}_v + \mathbf{\Sigma} + \mathbf{A}_k\mathbf{A}_k^{\mathrm{H}} \\ &= \left[\begin{array}{c|c} \mathbf{R}_{v,0} + \mathbf{A}_{0,k}\mathbf{A}_{0,k}^{\mathrm{H}} + \sigma_0^2\mathbf{I} & \mathbf{A}_{0,k}\mathbf{A}_{1,k}^{\mathrm{H}} \\ \hline \mathbf{A}_{1,k}\mathbf{A}_{0,k}^{\mathrm{H}} & \mathbf{A}_{1,k}\mathbf{A}_{1,k}^{\mathrm{H}} + \sigma_1^2\mathbf{I} \end{array}\right] \end{aligned} \qquad (1)$$

where $\mathbf{\Psi}$ is the interference-free covariance matrix, and $\mathbf{\Sigma} := \mathrm{diag}[\sigma_0^2\mathbf{I}, \sigma_1^2\mathbf{I}]$ is the diagonal noise covariance matrix (assumed known). The objective is to estimate the interference-free covariance submatrix $\mathbf{\Psi}_{00} := \mathbf{R}_{v,0} + \sigma_0^2\mathbf{I}$.

3. ALGORITHMS

3.1. Traditional subtraction technique

In array signal processing, a classical technique for interference removal using a reference antenna is based on taking the covariance of the primary antennas, $\mathbf{R}_{00,k}$, and subtracting the estimated contribution of the interferers, $\mathbf{A}_{0,k}\mathbf{A}_{0,k}^{\mathrm{H}}$. In effect, the rank deficiency of the interference term

$$\mathbf{A}\mathbf{A}^{\mathrm{H}} = \begin{bmatrix} \mathbf{A}_0\mathbf{A}_0^{\mathrm{H}} & \mathbf{A}_0\mathbf{A}_1^{\mathrm{H}} \\ \mathbf{A}_1\mathbf{A}_0^{\mathrm{H}} & \mathbf{A}_1\mathbf{A}_1^{\mathrm{H}} \end{bmatrix}$$

is exploited: if $q \leq p_1$ and moreover $\mathbf{A}_1 : p_1 \times q$ has full column rank q, then the first p_0 columns must be linear combinations of the remaining p_1. Under these conditions,

$$\mathbf{A}_0\mathbf{A}_0^{\mathrm{H}} = \mathbf{A}_0\mathbf{A}_1^{\mathrm{H}}(\mathbf{A}_1\mathbf{A}_1^{\mathrm{H}})^{\dagger}\mathbf{A}_1\mathbf{A}_0^{\mathrm{H}}$$

where † indicates the pseudo-inverse, and hence a 'clean' instantaneous covariance estimate is

$$\hat{\mathbf{\Psi}}_{00,k} = \hat{\mathbf{R}}_{00,k} - \hat{\mathbf{R}}_{01,k}\hat{\mathbf{R}}_{11,k}^{\dagger}\hat{\mathbf{R}}_{10,k}$$

(ignoring the effect of the noise term $\sigma^2\mathbf{I}$). The final 'clean' covariance estimate is obtained by averaging over N such matrices to obtain a long-term estimate

$$\hat{\mathbf{\Psi}}_{00} = \frac{1}{N}\sum_{k=1}^{N}\hat{\mathbf{\Psi}}_{00,k}.$$

Briggs et al. [7] derive essentially this algorithm and several variants of it, for the special case of $q = 1$ and $p_1 = 2$. Jeffs et al. [8] describe the same technique as a generalization of the classical Multiple Sidelobe Canceller.

The mentioned conditions on $\mathbf{A}_1$ entail that this technique can be used for at most p_1 interferers, and only if the reference antennas are sufficiently independent so that they receive independent linear combinations of the interferers. Unlike some of the techniques to be discussed in later sections, the technique does not rely on the variation of $\mathbf{A}_k$: in principle, $\mathbf{A}_k$ can be stationary. Also, no detection of the number of interferers is done, nor of any noise powers. This simplifies the algorithm but might also limit its performance.

3.2. Spatial filtering using projections

In [6], a spatial filtering algorithm based on projections was introduced. Although that algorithm did not assume the presence of reference antennas, it can also be used in our current situation.

Suppose that an orthogonal basis $\mathbf{U}_k$ of the subspace spanned by interferer spatial signatures $\mathrm{span}(\mathbf{A}_k)$ is known. We can then form a spatial projection matrix $\mathbf{P}_k := \mathbf{I} - \mathbf{U}_k\mathbf{U}_k^{\mathrm{H}}$ which is such that $\mathbf{P}_k\mathbf{A}_k = 0$. When this spatial filter is applied to the data covariance matrix all the energy due to the interferer will be nulled: let

$$\hat{\mathbf{Q}}_k := \mathbf{P}_k\hat{\mathbf{R}}_k\mathbf{P}_k$$

then

$$\mathrm{E}[\hat{\mathbf{Q}}_k] = \mathbf{P}_k\mathbf{\Psi}\mathbf{P}_k$$

When we subsequently average the modified covariance matrices $\hat{\mathbf{Q}}_k$, we obtain a long-term estimate

$$\hat{\mathbf{Q}} := \frac{1}{N}\sum_{k=1}^{N}\hat{\mathbf{Q}}_k = \frac{1}{N}\sum_{k=1}^{N}\mathbf{P}_k\hat{\mathbf{R}}_k\mathbf{P}_k\,. \quad (2)$$

$\hat{\mathbf{Q}}$ is an estimate of $\boldsymbol{\Psi}$, but it is biased due to the projection. To correct for this we first write the two-sided multiplication as a single-sided multiplication employing the matrix identity $\text{vec}(\mathbf{ABC}) = (\mathbf{C}^{\mathrm{T}}\otimes\mathbf{A})\text{vec}(\mathbf{B})$, This gives

$$\text{vec}(\hat{\mathbf{Q}}) = \frac{1}{N}\sum_{k=1}^{N}\mathbf{C}_k\text{vec}(\hat{\mathbf{R}}_k) \quad \text{where } \mathbf{C}_k := \mathbf{P}_k^{\mathrm{T}}\otimes\mathbf{P}_k\,. \quad (3)$$

If the interference was completely removed then

$$\mathrm{E}[\text{vec}(\hat{\mathbf{Q}})] = \frac{1}{N}\sum_{k=1}^{N}\mathbf{C}_k\text{vec}(\boldsymbol{\Psi}) = \mathbf{C}\text{vec}(\boldsymbol{\Psi})\,; \quad \mathbf{C} := \frac{1}{N}\sum_{k=1}^{N}\mathbf{C}_k\,. \quad (4)$$

In view of this, we can apply a correction $\mathbf{C}^{-1}$ to $\hat{\mathbf{Q}}$ and define

$$\hat{\boldsymbol{\Psi}} := \text{unvec}(\,\mathbf{C}^{-1}\text{vec}(\hat{\mathbf{Q}}))\,.$$

If the interference was completely projected out then $\hat{\boldsymbol{\Psi}}$ is an unbiased estimate of the covariance matrix without interference. This was the algorithm introduced in [6].

The reconstructed covariance matrix is size $p \times p$. In the present case, we are only interested in the submatrix corresponding to the primary antennas. Hence, the estimate produced by the algorithm is the $p_0 \times p_0$ submatrix in the top-left corner, $\hat{\boldsymbol{\Psi}}_{00}$. This is one of the algorithms introduced in [8, 9].

The spatial signature of the interferer is generally unknown, but it can be estimated from an eigen-analysis of the sample covariance matrices $\hat{\mathbf{R}}_k$. To do this, recall that the noise powers on the two antenna arrays are not necessarily the same, and first they have to be made equal. This noise whitening is done by working with $\boldsymbol{\Sigma}^{-1/2}\hat{\mathbf{R}}_k\boldsymbol{\Sigma}^{-1/2}$. Without interference and assuming $\mathbf{R}_v$ is negligible compared to $\boldsymbol{\Sigma}$, all eigenvalues of this matrix are expected to be close to 1. With q interferers, q eigenvalue become larger, and the eigenvectors corresponding to these eigenvalues are an estimate of $\text{span}(\mathbf{A}_k)$.

3.3. Improved spatial filter with projections

We now derive an improved algorithm. Compute the projections and long-term average of the projected estimates $\hat{\mathbf{Q}}$ as before in (2). Then (4) applies:

$$\mathrm{E}[\text{vec}(\hat{\mathbf{Q}})] = \mathbf{C}\text{vec}(\boldsymbol{\Psi})\,.$$

Based on this, we previously set $\text{vec}(\hat{\boldsymbol{\Psi}}) = \mathbf{C}^{-1}\text{vec}(\hat{\mathbf{Q}})$, which is the solution in Least Squares sense of the covariance model error minimization problem, $\|\text{vec}(\hat{\mathbf{Q}}) - \mathbf{C}\text{vec}(\hat{\boldsymbol{\Psi}})\|^2$. Now, instead of this, partition $\boldsymbol{\Psi}$ as in (1) into 4 submatrices. Since we are only interested in recovering $\boldsymbol{\Psi}_{00}$, the other submatrices in $\hat{\boldsymbol{\Psi}}$ are replaced by their expected values, respectively $\boldsymbol{\Psi}_{01} = \mathbf{0}$, $\boldsymbol{\Psi}_{10} = \mathbf{0}$, $\boldsymbol{\Psi}_{11} = \sigma_1^2\mathbf{I}$. This corresponds to solving the reduced-size covariance model error minimization problem,

$$\hat{\boldsymbol{\Psi}}_{00} = \underset{\boldsymbol{\Psi}_{00}}{\arg\min}\ \|\,\text{vec}(\hat{\mathbf{Q}}) - \mathbf{C}\text{vec}\Big(\begin{bmatrix}\boldsymbol{\Psi}_{00} & \mathbf{0}\\ \mathbf{0} & \sigma_1^2\mathbf{I}\end{bmatrix}\Big)\,\|^2\,.$$

The solution of this problem reduces to a standard LS problem after separating the knowns from the unknowns. Thus, rearrange the entries of $\text{vec}(\boldsymbol{\Psi})$ into

$$\begin{bmatrix}\text{vec}(\boldsymbol{\Psi}_{00})\\ \sigma_1^2\mathbf{1}\\ \mathbf{0}\end{bmatrix}$$

where $\mathbf{1}$ indicates a vector with all entries equal to 1, and repartition $\mathbf{C}$ accordingly, to obtain the equivalent problem

$$\begin{aligned}\text{vec}(\hat{\boldsymbol{\Psi}}_{00}) &= \underset{\boldsymbol{\Psi}_{00}}{\arg\min}\ \|\,\text{vec}(\hat{\mathbf{Q}}) - [\mathbf{C}_1\ \ \mathbf{C}_2\ \ \mathbf{C}_3]\begin{bmatrix}\text{vec}(\boldsymbol{\Psi}_{00})\\ \sigma_1^2\mathbf{1}\\ \mathbf{0}\end{bmatrix}\|^2\\ &= \underset{\boldsymbol{\Psi}_{00}}{\arg\min}\ \|\,(\text{vec}(\hat{\mathbf{Q}}) - \sigma_1^2\mathbf{C}_2\mathbf{1}) - \mathbf{C}_1\text{vec}(\boldsymbol{\Psi}_{00})\,\|^2\\ &= \mathbf{C}_1^{\dagger}(\text{vec}(\hat{\mathbf{Q}}) - \sigma_1^2\mathbf{C}_2\mathbf{1})\,.\end{aligned}$$

The advantage compared to the preceding algorithm is that $\mathbf{C}_1$ is a tall matrix, and better conditioned than $\mathbf{C}$. This improves the performance of the algorithm in cases where $\mathbf{C}$ is ill-conditioned, e.g., for stationary interferers, or an interferer entering on only a single telescope. Asymptotically for large INR of the reference array, the algorithm is seen to behave similar to the traditional subtraction technique.

4. SIMULATIONS

We first test the performance of the algorithms in a simulation setup. We use $p = 6$ antennas, with $p_0 = 5$ primary antennas (telescopes) and $p_1 = 1$ reference antenna. For simplicity, the array is a uniform linear array with half-wavelength spacing and the same noise power on all antennas. The astronomical source is simulated by a source with a constant direction-of-arrival of 10° with respect to array broadside. The source has $\text{SNR}_0 = -20$ dB with respect to each primary array element, and $\text{SNR}_1 = -40$ dB for the reference antenna. The interferer is simulated by a source with a randomly generated and varying complex $\mathbf{a}_k$, and varying INRs as explained in the simulations. This corresponds to a Rayleigh fading interferer.

The following algorithms are compared:

- the subtraction method in section 3.1 denoted ‘traditional’,
- the spatial filtering algorithm using projections and eigenvalue computations, section 3.2, denoted ‘eig filt’,
- the spatial filtering algorithm with reduced-size covariance reconstruction, section 3.3, denoted ‘eig filt (red corr)’,
- for comparison, the spatial filtering technique without reference antenna, denoted ‘eig filt (no ref)’, the covariance estimate without RFI (‘RFI free’), and the estimate obtained without any filtering (‘no filtering’).

Figure 2(*a*)-(*b*) shows the mean-squared-error (MSE) of the primary filtered covariance estimate compared to the theoretical value $\mathbf{R}_{v,0} + \sigma_0^2\mathbf{I}$, for varying interferer power INR_0 and interferer array gain $\text{INR}_1 - \text{INR}_0$ respectively. Here, we took $M = 400$ short-term samples and $N = 2$ long-term averages, which is unrealistically small but serves to illustrate the effect of limited variability of $\mathbf{a}_k$ (only two different vectors).

It is seen that the new algorithm has a great advantage over the spatial filtering algorithm without reference antena in case the $\mathbf{a}_k$-vector is not sufficiently varying. The MSE performance is flat for varying INR and INR difference, which is very desirable. Moreover, it is very close to the RFI-free case. The new algorithm is also often better than the subtraction technique.

Additional simulations (not shown here) indicate that if the interferer enters only on one telescope and on the reference antenna, then the algorithm without a reference antenna is performing poorly: it cannot reconstruct the contaminated dimension. The algorithm with reference antennas performs fine.

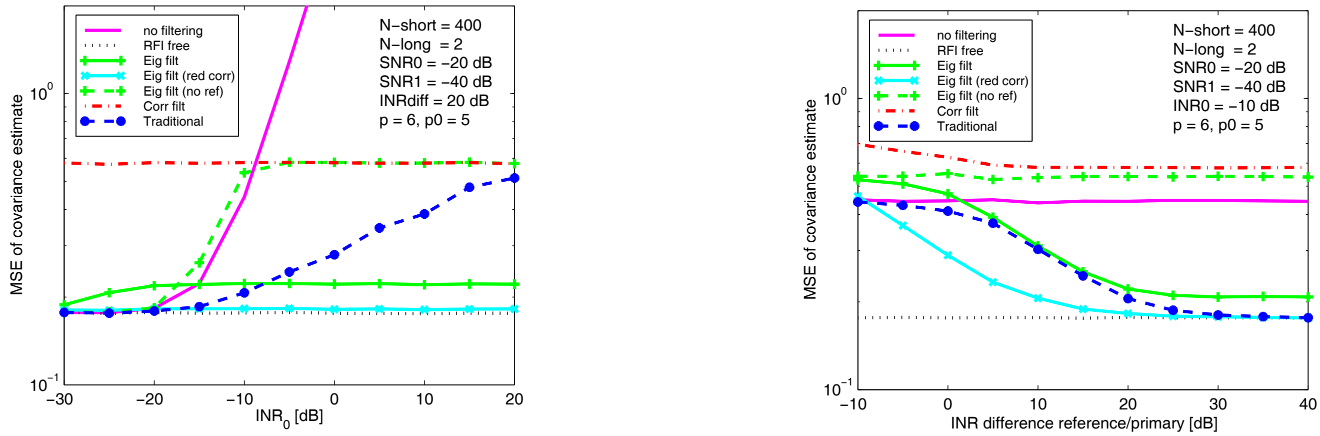

Figure 2. MSE (a) as function of interferer power at the reference antenna, (b) as function of the interferer power difference between the reference antenna and the primary array elements.

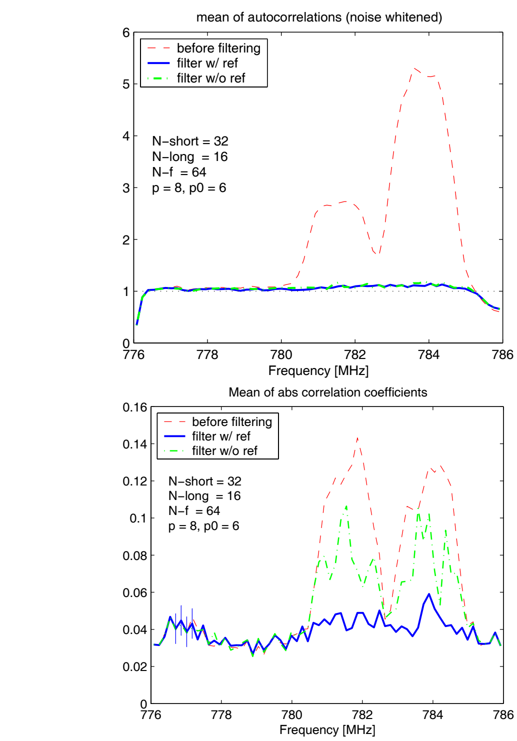

Figure 3. (a) Averaged autocorrelation spectrum before and after filtering, (b) Averaged cross-correlation spectrum

5. EXPERIMENT

To test the algorithm on actual data, we have made a short observation of the strong astronomical source 3C48 contaminated by Afristar satellite signals. The set-up follows figure 1. The primary array consists of $p_0 = 6$ of the 14 telescope dishes of the Westerbork Synthesis Radio Telescope (WSRT), located in The Netherlands. As reference we use $p_1 = 2$ beamforming outputs of a wideband 64-element phased array constructed by ASTRON. One beam was pointed approximately to the satellite, the other was used for scanning. We recorded 65 kSamples at 20 MS/s, and processed these offline. After a short-term windowed Fourier transforms, the data was split into 64 frequency bins, correlated, and averaged over 32 samples to obtain 16 short-term covariance matrices.

The resulting auto- and crosscorrelation spectra after filtering are shown in figure 3. The autocorrelation spectra are almost flat, and close to 1 (the whitened noise power). The cross-correlation spectra show that the spatial filtering with reference antenna has done much better to remove the interference than the case without reference antenna. The residual correlation of about 4% is known to be the SNR of the astronomical source. The lines are noisy due to the finite sample effect; the predicted standard deviation (based on number of samples averaged) are indicated for a few frequencies.

References

[1] A. Leshem, A-J. van der Veen, and A-J. Boonstra, “Multichannel interference mitigation techniques in radio astronomy,” *The Astrophysical Journal Supplement Series*, vol. 131, pp. 355–373, Nov. 2000.

[2] P.A. Fridman and W.A. Baan, “RFI mitigation methods in radio astronomy,” *Astronomy and Astrophysics*, vol. 378, pp. 327–344, 2001.

[3] C. Barnbaum and R.F. Bradley, “A new approach to interference excision in radio astronomy: Real time adaptive filtering,” *The Astronomical Journal*, vol. 115, pp. 2598–2614, 1998.

[4] S.W. Ellingson, J.D. Bunton, and J.F. Bell, “Cancellation of glonass signals from radio astronomy data,” in *Proc. SPIE, Vol. 4015, Radio Telescopes* (Harvey R. Butcher, ed.), vol. 4015, pp. 400–407, 2000.

[5] Lisha Li and B.D. Jeffs, “Analysis of adaptive array algorithm performance for satellite interference cancellation in radio astronomy,” *URSI General Assembly*, Aug. 2002.

[6] J. Raza, A.J. Boonstra, and A.J. van der Veen, “Spatial filtering of RF interference in radio astronomy,” *IEEE Signal Processing Letters*, vol. 9, Mar. 2002.

[7] F.H. Briggs, J.F. Bell, and M.J. Kesteven, “Removing radio interference from contaminated astronomical spectra using an independent reference signal and closure relations,” *The Astronomical Journal*, vol. 120, pp. 3351–3361, 2000.

[8] B.D. Jeffs, K. Warnick, and Lisha Li, “Improved interference cancellation in synthesis array radio astronomy using auxiliary antennas,” in *IEEE ICASSP*, (Hong Kong), Apr. 2003.

[9] B.D. Jeffs and K. Warnick, “Auxiliary antenna assisted interference cancellation for radio astronomy imaging arrays,” *Preliminary draft*, Aug. 2002.