CHANNEL ESTIMATION FOR LONG-CODE WCDMA

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ABSTRACT

A new blind and semiblind channel estimation for long code wideband CDMA is proposed based on the structure of RAKE receiver. Using a decorrelating matched filter, the received signal is projected into subspaces from which channel parameters and data symbols can be identified blindly via rank-one decompositions. A new identifiability condition is established. The mean square error of the estimated channel is compared with the Cramér-Rao bound, and a bit error rate (BER) expression for the proposed algorithm is presented.

1. INTRODUCTION

WCDMA features coherent detection in the up-link. To estimate the channel, a variable number of pilot symbols are provided through the control channel. For high rate data options, however, the processing gain is relatively low, and multiaccess/multipath interference may be strong and the conventional matched filter-based single user channel estimation and detection inadequate. A challenge of using more sophisticated channel estimation technique, especially those based on subspace methods (see [1] and references therein), is that aperiodic spreading codes–also referred to as long codes—are used in WCDMA. The aperiodic spreading makes the signal model time varying, and many existing techniques are either not applicable or prohibitively complex.

For long code CDMA, several attempts have been made in obtaining channel estimates blindly. Iterative techniques based on maximizing the likelihood function [2, 3] and least squares [4] have been proposed. These are high performance techniques but also have well known drawbacks in ill convergence and high complexity. They are best complemented by good initialization techniques such as the algorithm developed in this paper. Also in the literature are second order moment techniques based on the i.i.d. assumption of the spreading code or the symbols [5–9]. These techniques rely on the convergence of time averages to statistical averages, which often requires a hundred to thousands of symbols. The work of Weiss and Freidlander [10] is perhaps the closest to our approach although they focused on down link applications. By assuming the multipath delays are limited to a small fraction of a bit interval and some samples can be dropped to eliminate intersymbol interference, the decorrelation in the code domain is also performed in [10] followed by a different subspace algorithm and an iterative likelihood maximization. The algorithm by Weiss and Freidlander is not applicable to systems with asynchronous users and long multipath delays.

In this paper, we focus on techniques that can obtain channel estimates quickly using only few symbols, which allows us to deal with rapid fading. The receiver estimates the channel parameters and performs detection based on the estimated channel. In the absence of pilot symbols, the channel estimation will be blind. If pilot symbols are present, they will be used in semiblind estimation to enhance performance.

The main contribution of this work is twofold. First, we present a new blind channel estimation algorithm based on the use of pre-computed decorrelating matched filter constructed from users' spreading codes. By projecting the received chip-rate signal into a bank of symbol rate vector sequences, each corresponds to the outer product of the channel and the symbol sequence of one user. Channel estimation is made using a simple least squares fitting. The algorithm imposes no condition on the channel, and it needs data from only a few symbols (one symbol would have been sufficient in theory). Second, we present a new identifiability condition that depends only on the spreading codes used in the system but not on channel parameters. Implied by this identifiability condition is that the use of long code enhances channel identifiability; channels not identifiable in short-code CDMA are almost surely identifiable in a long code system.

2. THE MODEL

We consider a system with K users with aperiodic spreading sequences of spreading gain G. Suppose that the channel of a particular user is modeled by complex path gains separated by multiples of chip intervals. Specifically, the continuous-time channel impulse response of user i has the form

$$h_i(t) = \sum_{l=1}^{L_i} h_{li}\delta(t - lT_c - d_iT_c),$$

where $\{h_{li}\}$ are path gains, $T_c = 1/G$ the chip interval, and d_i the delay of user i relative to the reference. We will assume that d_i and L_i are known[1]. Sometimes, the channel is sparse, and it is more efficient to model the channel as separate clusters of fingers. In that case, we assume that the

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[1] If d_i is unknown, we may set it to 0 and model all paths. L_i is a modeling parameter and its choice is often left to algorithm designers.

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approximate locations of these clusters are known. Since any path gain can be zero, one can over-parameterize the channel to accommodate channel uncertainties.

We let the received signal $y(t)$ pass through a chip-matched filter and sample its output at the chip rate. The received noiseless signal vector $\mathbf{y}_{ik}$ that corresponds to s_{ik} is given by

$$\mathbf{y}_{ik} = \mathcal{T}_k(\mathcal{C}_i)\mathbf{h}_i s_{ik}, \tag{1}$$

where $\mathcal{T}_k(\mathcal{C}_i)$ is the Toeplitz matrix whose first column is made of $kG+d_i$ zeros followed by the code vector $\mathbf{c}_{ki}$—the kth segment of G chips of the spreading code of user i—and additional zeros that make the size of $\mathbf{y}_{ik}$ the total number of chips of the entire M-symbol frame. For user i, the total received noiseless signal is given by

$$\mathbf{y}_i = \sum_{k=1}^{M} \mathcal{T}_k(\mathcal{C}_i)\mathbf{h}_i s_{ik} = \mathcal{T}(\mathcal{C}_i)(\mathbf{I}_M \otimes \mathbf{h}_i)\mathbf{s}_i, \tag{2}$$

where $\mathbf{s}_i$ contains all symbols from user i, and

$$\mathcal{T}(\mathcal{C}_i) \triangleq [\mathcal{T}_1(\mathcal{C}_i), \cdots, \mathcal{T}_M(\mathcal{C}_i)] \tag{3}$$

is the code matrix of user i, and it has a special block shifting structure. Specifically, $\mathcal{T}_m(\mathcal{C}_i)$ is the shift of $\mathcal{T}_{m-1}(\mathcal{C}_i)$ by G positions. Including all users, we have the complete matrix model

$$\begin{aligned} \mathbf{y} &= [\mathcal{T}_1(\mathcal{C}_1)\cdots\mathcal{T}_1(\mathcal{C}_K)]diag(\mathbf{I}_M \otimes \mathbf{h}_1, \cdots, \mathbf{I}_M \otimes \mathbf{h}_K)\mathbf{s} + \mathbf{w} \\ &= \mathcal{T}(\mathcal{C})\mathcal{D}(\mathbf{h})\mathbf{s} + \mathbf{w}, \end{aligned} \tag{4}$$

where the overall code matrix $\mathcal{T}(\mathcal{C})$ contains all code matrices, $\mathbf{h}$ channels coefficients from all users, and $\mathbf{s}$ all symbols. Matrix $\mathcal{D}(\mathbf{h})$ is block diagonal with $\mathbf{I} \otimes \mathbf{h}_i$ as the ith block. The additive Gaussian noise is put in $\mathbf{w}$.

We will impose the following assumptions.

- A1: The code matrix $\mathcal{T}(\mathcal{C})$ is known.
- A1': The code matrix $\mathcal{T}(\mathcal{C})$ has full column rank.
- A2: The noise vector is complex Gaussian $\mathbf{w} \sim \mathcal{N}(\mathbf{0}, \sigma^2\mathbf{I})$ with possibly unknown σ^2.

Assumption (A1) implies that the receiver knows codes from all users. This assumption is usually valid for the uplink. For the down link, the relative delays d_i and the number of multipaths L_i are all the same. The channelization codes are often known. These information are sufficient for the proposed scheme.

Assumption (A1') is sufficient but not necessary for the channel to be identifiable and for the proposed algorithm to produce good estimates. This condition can of course be verified off line and is easy to satisfy for a relative large spreading gain. When (A1') fails, the channel may still be identifiable, and the proposed identification algorithm can be applied with simple modifications. See Sec. 3.

3. BLIND AND SEMIBLIND CHANNEL ESTIMATOR

The receiver structure of the proposed Decorrelating RAKE Receiver (DRR) requires only slight modifications of the conventional RAKE Receiver (RR) where a bank of matched filters are used to project the received signal into the code domains of individual users. The key difference is that the matched filter in DRR eliminates multiuser interference, and there are extra processing steps at the receiver for each user.

The decorrelating matched filter implements $\mathcal{T}^\dagger(\mathcal{C})$ can be pre-computed. Furthermore, there is an efficient state-space realization available that significantly reduced the complexity and storage requirement of the decorrelating receiver by exploiting the structure of the code matrix [11]. The output of the decorrelating receiver can be written in the vector form as

$$\mathbf{z} = \mathcal{T}^\dagger(\mathcal{C})\mathbf{y} = diag(\mathbf{I} \otimes \mathbf{h}_1, \cdots, \mathbf{I} \otimes \mathbf{h}_K)\mathbf{s} + \mathbf{n}, \tag{5}$$

where $\mathbf{n} = \mathcal{T}^\dagger(\mathcal{C})\mathbf{w}$ is now colored. Picking out the data corresponding to user i and symbol k from $\mathbf{z}$, we have

$$\mathbf{z}_{ik} = \mathbf{h}_i s_{ik} + \mathbf{n}_{ik}, \quad k = 1, \cdots, M. \tag{6}$$

where $\mathbf{z}_{ik}$ is the $((i-1)M+k)$th G-dimensional subvector of $\mathbf{z}$. Collecting all data for user i gives

$$\mathbf{Z}_i = [\mathbf{z}_{i1}, \cdots, \mathbf{z}_{iM}] = \mathbf{h}_i\mathbf{s}_i^T + \mathbf{N}_i. \tag{7}$$

Treating $\mathbf{h}_i$ and $\mathbf{s}_i$ as deterministic parameters, the least squares estimator of the outer product $\mathbf{X}_i \triangleq \mathbf{h}_i\mathbf{s}_i^T$ is given by

$$\min_{\mathbf{h},\mathbf{s}_i} ||\mathbf{Z}_i - \mathbf{h}_i\mathbf{s}_i^T||_F^2$$

which produces estimates of $\mathbf{h}_i$ and $\mathbf{s}_i$ (with an unknown scaling factor) from the rank-one decomposition of $\mathbf{Z}_i$. In other words, denoting

$$\mathbf{R}_i \triangleq \sum_k \mathbf{z}_{ik}\mathbf{z}_{ik}^H, \tag{8}$$

we obtain the least squares estimator

$$\hat{\mathbf{h}}_i = \arg\max_{||\mathbf{g}||=1} \mathbf{g}^H\mathbf{R}_{\mathbf{z}_{ik}}\mathbf{g}, \quad \hat{s}_{ik} = \hat{\mathbf{h}}_i^T\mathbf{z}_{ik}^*. \tag{9}$$

The scaling ambiguity in the above estimate must be removed by either incorporating prior knowledge of the symbol, using pilot symbols, or employing differential encoding of s_{ik}. Because $\mathbf{n}_{ik}$ is colored, a bias is introduced, which can be removed. See [11].

If arbitrarily placed pilot symbols exist in $\mathbf{s}_i$, the above least squares can be amended. Let $\mathbf{s}_i$ be partitioned into two subvectors, $\mathbf{s}_{ip}$ contains the pilot and $\mathbf{s}_{id}$ the data. We partition $\mathbf{Z}_i$ accordingly to $\mathbf{Z}_{ip}$ and $\mathbf{Z}_{id}$. The least squares estimator is given by

$$\hat{\mathbf{h}}_i = \arg\min_{\mathbf{h},\mathbf{s}} ||\mathbf{Z}_{ip} - \mathbf{h}\mathbf{s}_{ip}^T||_F^2 + ||\mathbf{Z}_{id} - \mathbf{h}\mathbf{s}^T||_F^2.$$

The above optimization does not have a closed-form solution. Simple iterative schemes can be applied. We note that for a fixed $\hat{\mathbf{h}}$, the optimal choice of $\mathbf{s}$ is $\hat{\mathbf{s}} = \frac{\hat{\mathbf{h}}^H\mathbf{Z}_{id}}{||\hat{\mathbf{h}}||^2}$. This leads to the following iteration given $\hat{\mathbf{h}}_k$.

$$\hat{\mathbf{h}}_{k+1} = \arg\min_{\mathbf{h}} ||\mathbf{Z}_{ip} - \mathbf{h}\mathbf{s}_{ip}^T||_F^2 + ||\mathbf{Z}_{id} - \frac{1}{||\hat{\mathbf{h}}||^2}\mathbf{h}\hat{\mathbf{h}}_k^H\mathbf{Z}_{id}||_F^2$$

which is equivalent to treating $\hat{\mathbf{h}}_k^H\mathbf{Z}_{id}$ as known symbols. One can of course make hard decisions on $\hat{\mathbf{h}}_k^H\mathbf{Z}_{id}$ for further enhancement. Other iterative techniques can also be applied.

3.1. Identifiability

We have so far assumed that the code matrix $\mathcal{T}(\mathcal{C})$ has full column rank and therefore invertible from the left. This assumption is usually valid for systems with large spreading gains. Under this assumption, it is clear that each user's channel is identifiable up to a scaling factor. A single pilot symbol will be sufficient to remove the scaler ambiguity.

Singularity does occur when the spreading gain is small and the system is heavily loaded. Even if the case of having singular $\mathcal{T}(\mathcal{C})$ is rare, it remains a theoretical interest to investigate whether the channel is still identifiable, and if not, how many known symbols are necessary and how to place these known symbols.

We now present an identifiability result that appears to be more general than existing conditions. The condition shown next is independent of the channel parameters, and can be checked easily off-line, and appropriate measures taken. More significant, perhaps, is that it decouples the identifiability of a particular user from that of others; one user's channel may be identifiable even when those of others are not.

Theorem 1 *Let $\mathcal{T}_k(\mathcal{C}_i)$ be the code matrix of user i for symbol k, and $\check{\mathcal{T}}_k(\mathcal{C}_i)$ the submatrix of $\mathcal{T}(\mathcal{C})$ after removing $\mathcal{T}_k(\mathcal{C}_i)$. The channel $\mathbf{h}_i$ of user i is identifiable if there exists a k such that*

$$\mathcal{R}\{\mathcal{T}_k(\mathcal{C}_i)\}\bigcap\mathcal{R}\{\check{\mathcal{T}}_k(\mathcal{C}_i)\}=\{\mathbf{0}\}. \qquad (10)$$

Note that since the identifiability condition can be checked off line, one can device remedies when a particular channel is not identifiable. This can be accomplished by introducing pilot symbols at specific locations. The proof of the above theorem gives the algorithm that identifies the channel when the identifiability condition holds [11].

3.2. Multirate, Multicode and Multiple Antennas

To facilitate multimedia applications, multirate and multicode CDMA have been proposed in the third generation wireless systems. Here we consider generalizations of the proposed algorithms for multicode and multirate transmissions.

The multicode system assigns multiple codes to the same user. This is equivalent to the unicode case with multiple users having the same multipath channel. Suppose that a set of codes $\bar{\mathcal{C}}=\{\mathcal{C}_i\}$ are allocate to a particular user with channel $\mathbf{h}$. Then the channel and symbol estimation problem reduces to

$$\hat{\mathbf{h}}=\arg\max_{\mathbf{h},||\mathbf{h}||=1}\mathbf{h}^H(\sum_{k,i,\mathcal{C}_i\in\bar{\mathcal{C}}}\mathbf{z}_{ik}\mathbf{z}_{ik}^H)\mathbf{h}. \qquad (11)$$

This effectively increases the number of samples available for the estimation of the channel. This is also the same situation for estimating down link channels if the mobile user has the knowledge of multiple spreading codes. In WCDMA, for example, the spreading codes for the in-phase and quadrature part are known, and therefore can be used in the channel estimation.

Multirate transmission can be accomplished in several ways by using multiple spreading gains or multiple chip rates. In both cases, only the decorrelating matched filter needs to be modified, and the channel estimation and symbol detection algorithm applies directly. The problem of channel estimation in multirate systems has been considered in [12] where it is assumed that training symbols are available from all users simultaneously.

Only minor changes are necessary when the proposed algorithm is applied to multiple receiving antenna systems. Indeed, the general model, obtained by Nyquist sampling as shown in the Appendix, for a single receiver system is equivalent to one with two diversity reception channels. When multiple antennas are used, the reception at the ith antenna is given by

$$\mathbf{y}^{(i)}=\mathcal{T}(\mathcal{C})\mathcal{D}(\mathbf{h}^{(i)})\mathbf{s}+\mathbf{w}^{(i)}.$$

A simple approach is to apply the same decorrelating matched filter at each antenna element. The channel $\mathbf{h}^{(i)}$ can then be estimated either separately at individual antenna element or jointly by exploiting the fact that all have the same symbol sequence. The same rank-one decomposition is used in both cases. One can of course apply decorrelating matched filter jointly to all receiving antenna elements by stacking all $\mathbf{y}^{(i)}$. A supper decorrelating matched filter needs to be computed for the stacked code matrix.

4. SIMULATION RESULTS

In this section, we present some simulation results. For channel estimation, MSE is used and our estimator is compared with the CRB using Monte Carlo runs. For symbol detection, BER is used as the performance measure, and Monte Carlo runs are used to estimate the BER.

We considered a case with two asynchronous users with equal power and randomly generated spreading codes with spreading gain $G=16$. The channel for each user had three fingers with considerable delay spread. The relative delays of the three multipath fingers were 1, 4, 9 chips for user 1 and 8, 12, 18chips for user 2. The slot size was $M=32$ and the same number of pilot symbols are included at the beginning of the slot of each user. This pilot symbols were used to remove the scaling ambiguity of the blind estimator. The signal-to-noise ratio (SNR) is defined by E_b/σ^2 where E_b is the bit energy and σ^2 is the chip noise variance (or the noise power spectrum density).

We compared the proposed channel estimator using the decorrelating RAKE structure with the Cramér-Rao Bound and the conventional training based algorithm based on standard matched filter RAKE. For the training based RAKE receiver, pilot symbols were used to obtain the least squares channel estimate using the output of the (non-decorrelating) matched filter. Fig. 1 shows the MSE performance for the case when a single pilot is used to remove the scaling factor. We observed that the conventional RAKE Receiver (RR) had a significant performance floor due to multiaccess interference. The Decorrelating RAKE Receiver (DRR), on the other hand, tracked CRB closely. We must note, however, that the gap of DRR to CRB increases as the number of user increases. This indicates that the proposed algorithm, while providing good channel estimate at high SNR, is not statistically efficient and may be improved using more sophisticated iterative schemes.

We performed BER evaluation for the Decorrelating RAKE Receiver (DRR) and for the conventional RAKE Receiver (RR), using true channel as the performance bound. Fig. 2 shows

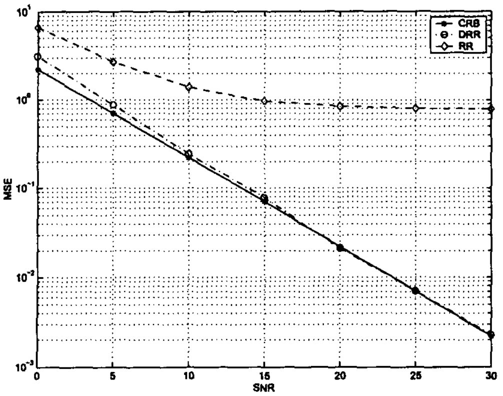

Fig. 1. Channel MSE vs. SNR. Both users have three fingers with relative delays of 1, 4, 9 chips for user 1 and 8, 12, 18 for user 2. Five hundred Monte Carlo runs. One pilot symbol per user is used.

the BER performance for one user[2]. We observed a loss of 2dB l2dB loss at BER of 10^{-2} due channel estimation errors. At BER $= 10^{-3}$, the loss is less than 1dB, and at BER below 10^{-5}, the loss is around $0.2dB$. It was clear that when SNR is above 10dB, the performance is close to that of the coherent detection with known channel parameters. The well known flooring effect of the conventional RR is due to multiaccess interference.

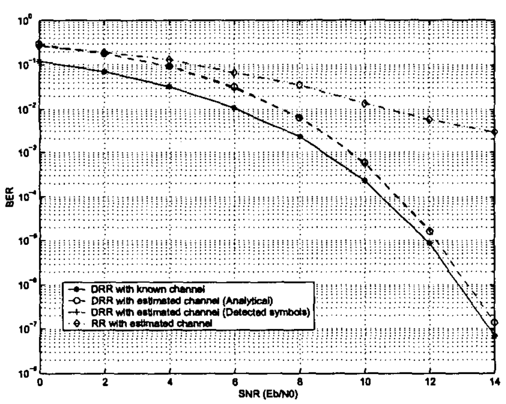

Fig. 2. BER vs. SNR. Five hundred Monte Carlo runs. One pilot symbol per user is used.

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[2]Because the two users have the same power, similar performance is observed for the other user.