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**MS165**

### Multiscale Model Reduction for Wave Equations

It is important to develop fast yet accurate numerical methods for seismic wave propagation to characterize complex geological structures and oil and gas reservoirs. However, the computational cost of conventional numerical modeling methods, such as finite-difference method and finite-element method, becomes prohibitively expensive when applied to very large models. We propose a Generalized Multiscale Generalized Multiscale Finite-Element Method (GMsFEM) for elastic wave propagation in heterogeneous, anisotropic media, where we construct basis functions from multiple local problems for both boundaries and the interior of a coarse node support or coarse element. The application of multiscale basis functions can capture the fine scale medium property variations, and allows us to greatly reduce the degrees of freedom that are required to implement the modeling compared with conventional finite-element method for wave equation, while restricting the error to low values. The research is partially supported by the Hong Kong RGC General Research Fund (Project: 14304217) and CUHK Direct Grant for Research 2017-18.

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**MS165**

### Hyperbolic Homogenization

There has been significant work on numerical homogenization methods for problems governed by the wave equation, even for cases without scale separation. However, these methods typically derive from the study of elliptic problems, exploiting the concept of G-convergence to produce approximate wave equations. The problems which arise to define the approximate system are inherently global but approximated via localization on patch neighborhoods. In this talk we propose methods specifically designed for hyperbolic systems, which in particular exploit the fundamental features of local domain-of-dependence and causality. The mathematical framework is that of reduced order modeling (ROM) and the tools are essentially algebraic. Specifically, we consider the direct approximation of a fine grid solution operator,  $S_h$ , by a computationally-discovered coarse grid subspace, spanned by  $V_H$ , leading to a reduced evolution matrix  $V_H^T S_h V_H$ .

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**MS165**

### Efficient Density Estimation in Noisy PDEs and

### Uncertainty Propagation

The effect of model uncertainties and noise on a model output is often best described by its probability density function (PDF). Although density estimation is a common uncertainty-quantification (UQ) task, the adequacy of approximation methods (surrogates) for density estimation has scarcely been analyzed before. We first show that standard spectral methods in uncertainty propagation, such as generalized polynomial chaos (gPC), sometimes fail to approximate the PDF even in the case of one-dimensional noise. Therefore, we developed a novel spline-based algorithm for this task. Our method offers significant advantages over existing methods for density estimation, primarily a guaranteed convergence rate which is polynomial in the sampling resolution. This convergence rate is better than that of standard statistical density-estimation methods (such as histograms and kernel density estimators) given an input noise of moderate dimension. Furthermore, our spline-based approximation outperforms spectral methods when the sample size is small and the quantity of interest has sharp-gradients regions. Finally, we present applications of our algorithm for problems in nonlinear optics and fluid dynamics.

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**MS165**

### Rational Krylov Subspaces and Phase-preconditioning for Model Reduction of Wave Equations

In many applications like approximation of matrix functions or dynamical systems, rational Krylov subspaces show superior approximation qualities over polynomial Krylov subspaces. Especially, if the spectrum of the function and the domain where the function is approximated are well separated from each other, rational approximants are known to converge fast. This, however, is generally not the case when approximating discretizations of wave equations with a reduced order model. Nevertheless, reduced order models based on rational Krylov subspaces still perform well for applications with very lossy or resonant structures. In case the considered application is dominated by waves traveling over long distances the order of the rational function needed to approximate a response grows proportional to the travel time in accordance with the Nyquist sampling limit. In this talk we combine rational Krylov subspaces with asymptotic methods to obtain a projection-based model order reduction method that can handle strong scatterers as well as long travel times. The resulting method shifts the computational cost from

the poorly scalable forward solvers to the evaluation of large inner products which are embarrassingly parallelizable. We show that incorporating asymptotic methods into a projection-based framework relaxes the spatial discretization required and allows sub-Nyquist sampling in the frequency domain.

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#### MS166

##### Ultra High Resolution Topology Optimization: Brute Force or Smart Discretizations?

The optimal topology of large structural systems has until now been concerned with the design of individual parts and not that of complete assemblies. Following recent advances in numerical algorithms tailored for large scale structural optimization, this limitation has now be circumvented, mainly by utilizing powerfull HPC system [Aage, N., et.al., Topology optimization using PETSc: An easy-to-use, fully parallel, open source topology optimization framework. SMO, 51:565-572, 2015.]. The design approach has been demonstrated on both aircraft, bridge and ship design problems, resulting in noticeable performance enhancement. However, the increase in design resolution comes at a great cost in terms of the needed computational power. Therefore, it is interesting, if not paramount, to pursue alternative design representation schemes that allows for ultra high resolution optimal designs without the need for large computing resources. This talk will present both the brute force topology optimization approach, in which the governing PDEs are solved by classical methods, i.e. Krylov methods and multigrid preconditioning [Aage, N., et.al. Giga-voxel computational morphogenesis for structural design. Nature, 52:84-86, 2017.], as well as novel homogenization based projection schemes. The latter allows for the solution of the design problem at low resolution, and by parameter extraction, one can represent the design using orders of magnitude finer design representations.

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#### MS166

##### Topology Optimization of Structures Subject to

##### Snapping Behaviour

In this talk we discuss methods which is intended to be used for of design structures that are expected to snap. One example of such structures are multi-stable micro-flexures that buckle to perform digital computations. Since such structures inevitably function under finite strains we model the material by general finite strain elasticity. The balance equations are solved using the finite element method in a total Lagrangian setting along with Newton-Raphson iterations. To trace the load path and to be able to pass singular points we make use of path following technique. To find the optimal material layout we associate one design variable with each element such that material and void can be represented. The optimization problem is to find a material distribution such that the objective is minimized while fulfilling the constraints. In our application the objective is to find a layout that gives a stable state in terms of energy level while having distinct states, i.e. the difference in deformation between the stable states should exceed a given threshold. We also impose a constrains on the available mass of the device. To solve the optimization problem we use mathematical programming and in particular we use the Method of Moving Asymptotes. The gradients required to form the convex approximations are established via the adjoint sensitivity approach. To form a well-posed problem we regularize the optimization problem via the use of a PDE filter.

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#### MS166

##### A Density Gradient Approach to Topology Optimization under Design-dependent Boundary Loading

The paper proposes a density gradient based approach to topology optimization under design-dependent boundary loading. In the density-based topology optimization method, we impose the design dependent loads through spatial gradient of the density. We transform design-dependent boundary loads into a volume form through volume integral of density gradient. In many applications where loadings only need to be exerted on partial boundary, we introduce an auxiliary loading density to keep track of the loading boundary. During the optimization, the loading density is updated by tracking the changes of the physical density in the vicinity of the loading boundary at previous iteration. The proposed approach is easy to implement and computationally efficient. In addition, by adding more auxiliary density fields, the proposed approach is applicable to multiple design-dependent loads. To prevent the intersection of different loading boundaries, a Heaviside projection based integral constraint is developed. Both heat conduction problems under convection loading and elastic problems under hydrostatic pressure loading are