

Accelerated SAR Computations by Exploiting Sparsity in the Anatomical Domain

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Synopsis

Accelerated SAR computations can improve awareness on actual RF exposure levels, which are known to depend heavily on subject anatomy. In this work, we accelerate SAR computations by reformulating the underlying electromagnetic field equations in terms of anatomical differences rather than the entire anatomy itself. This yields a “sparse” representation of the target anatomy, reducing problem complexity without compromising accuracy.

Purpose

RF exposure is well-known to vary significantly among subjects, depending on anatomical parameters such as tissue distribution, body habitus and positioning within the RF coil.¹ The main bottleneck in the assessment of specific absorption rate (SAR), or RF heating in general, is in the numerical computation of RF fields as a function of anatomy. Many attempts have been made on accelerating this type of problem, for example by using boundary integral methods or surface integral equations, which lend itself more for modeling homogeneous regions or RF coils.^{2,3}

In this work, we present a numerical approach to simplify SAR calculations by reformulating the underlying electromagnetic field equations in terms of anatomical differences with respect to a database model, rather than solving for the entire anatomy itself. This yields a “sparse” representation of the anatomy, reducing the number of unknowns without compromising accuracy. Furthermore, by exploiting a threshold operation we can further reduce the number of unknowns, effectively compressing the problem.

Methods

Theory

Building on electromagnetic scattering theory, we express the total electric field E for a given body model as a field perturbation with respect to a “background” body model, viz.

$$E = (\mathbb{I} - \mathbb{G}\chi)^{-1}E^b.$$

Here, E^b denotes the background electric field and $\chi = \varepsilon - \varepsilon^b$ denotes the electric susceptibility of the target body model with respect to the background model. \mathbb{G} denotes the Green's tensor which accounts for the response of the heterogeneous background and is characterized in advance during an offline stage of the algorithm.

Configuration and Implementation

As a proof-of-concept, a 7T neuroimaging scenario was modeled on a very coarse 10 mm isotropic grid in order to test the approach within available computational resources. A 30-cm diameter 16-rung high-pass birdcage was loaded with the head models “Duke” and “Ella”.⁴ Incident fields in the empty coil were calculated using XFDTD (v7.4, Remcom, State College, PA) and fed into a custom iterative solver implemented using GMRES in Matlab (r2016a, Mathworks inc., MA, USA) to compute the total fields in both models. 10-g averaging of the SAR distributions was obtained using an iterative kernel growing method.⁵

“Ella” was taken to be the target model for which SAR is to be computed, while assigning “Duke” as the background model. Its response was evaluated by computing field-responses for each of the edges within the background model. This resulted in a total of ~14,000 edges, which took approximately 12 hours to evaluate using a -30 dB convergence criterion on a single workstation.

Exploiting sparsity

By thresholding $|\chi|$, we can reduce the number of unknowns further, while preserving the dominant perturbations in the RF field. By removing the corresponding entries in \mathbb{G} , we can substantially reduce the size of the system matrix, leading to faster computations.

Results

Figure 1 shows the database body model “Duke”, target model “Ella” and their anatomical difference in terms of their electric susceptibility.

Figure 2 shows the SAR₁₀ maps obtained in these models, where results in the proposed approach were obtained approximately four times faster, without compromising accuracy. A very small error of ~3% can be observed, which is due to error accumulation from the offline calculations. This can be improved by increasing their convergence threshold. The improved convergence rate of the iterative solver is shown in Figure 3.

Finally, the thresholding procedure enables further acceleration of the method of up to an order of magnitude, at the cost of a ~15% error in the maximum SAR₁₀ values.

Discussion/Conclusion

A novel approach on SAR computations was presented which casts the computation of RF field in terms of anatomical differences with respect to a pre-characterized database model. This allows for reducing the number of unknowns, improving the convergence rate of numerical solvers. This approach can potentially become useful in online SAR assessments, where a database body model can serve as a background model.

The current benchmark of the method was performed on a male and female body model, which are quite different from each other. Having multiple body models in a pre-characterized database would increase the chances of obtaining a better starting point, further accelerating this approach. An important challenge is the scalability of the method. Where the current 10 mm assessment resulted in a matrix G of 1.5 GB in single precision, in a corresponding 5 mm model the number of edges would increase by a factor of ~8, and memory requirements by a factor of ~64. Future work should therefore include model compression to improve scalability of this approach within realistic computational resources.

Acknowledgements

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References

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Figures

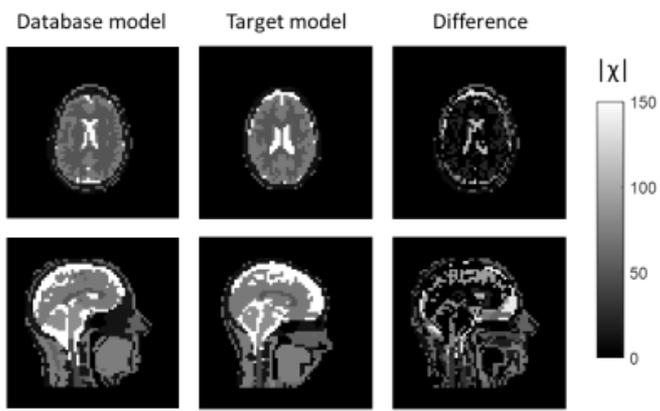


Figure 1. Anatomical models cast in terms of the "sparse" anatomical difference with respect to a database model. Head models shown here at 5 mm spatial resolution for illustrative purposes.

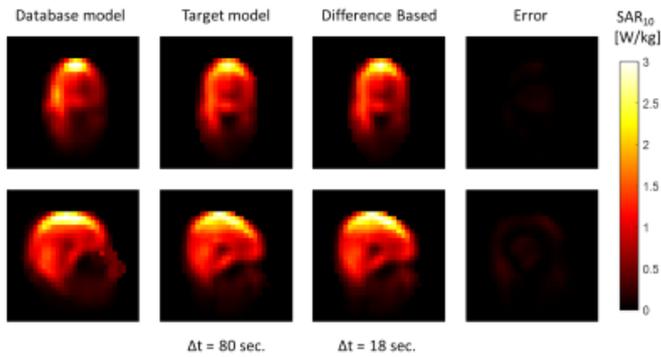


Figure 2. Simulation results showing 10g-averaged SAR derived from the anatomical difference between the target subject model and the database model. The results are essentially the same, while reducing the computation time by a factor of four.

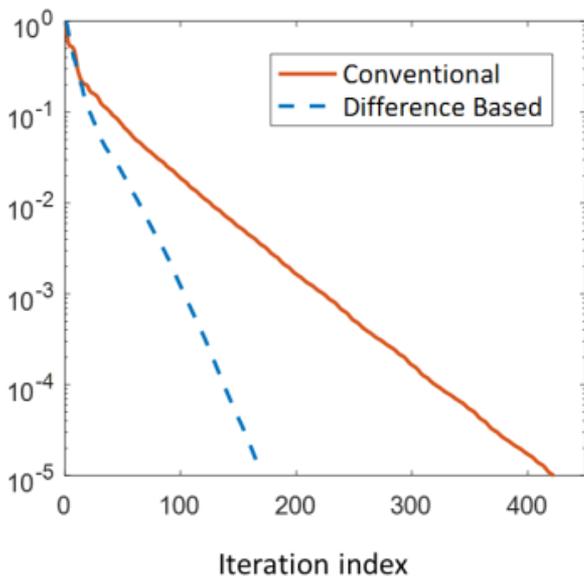


Figure 3. Convergence of GMRES when based on the conventional free space background, or the proposed. The convergence rate has approximately tripled when casting the problem in terms of anatomical differences.

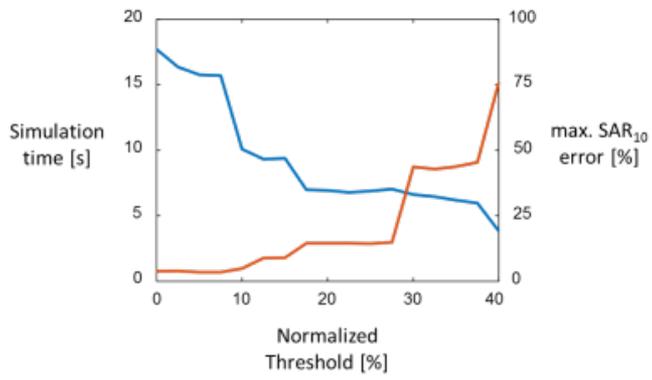


Figure 4. Trade-off between additional speed gains and accuracy obtained using a threshold operation on the electric susceptibility. Over an order of magnitude of acceleration can be reached with respect to the conventional solver based on a free space background, for instance at a normalized threshold of 20%, however at the cost of a ~15% error in SAR.