Radio pulsar navigation. An alternative to GPS?

ET4386

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- Time-of-arrival based navigation (GPS)
- PulsarPlane project
- Detection of radio pulsars
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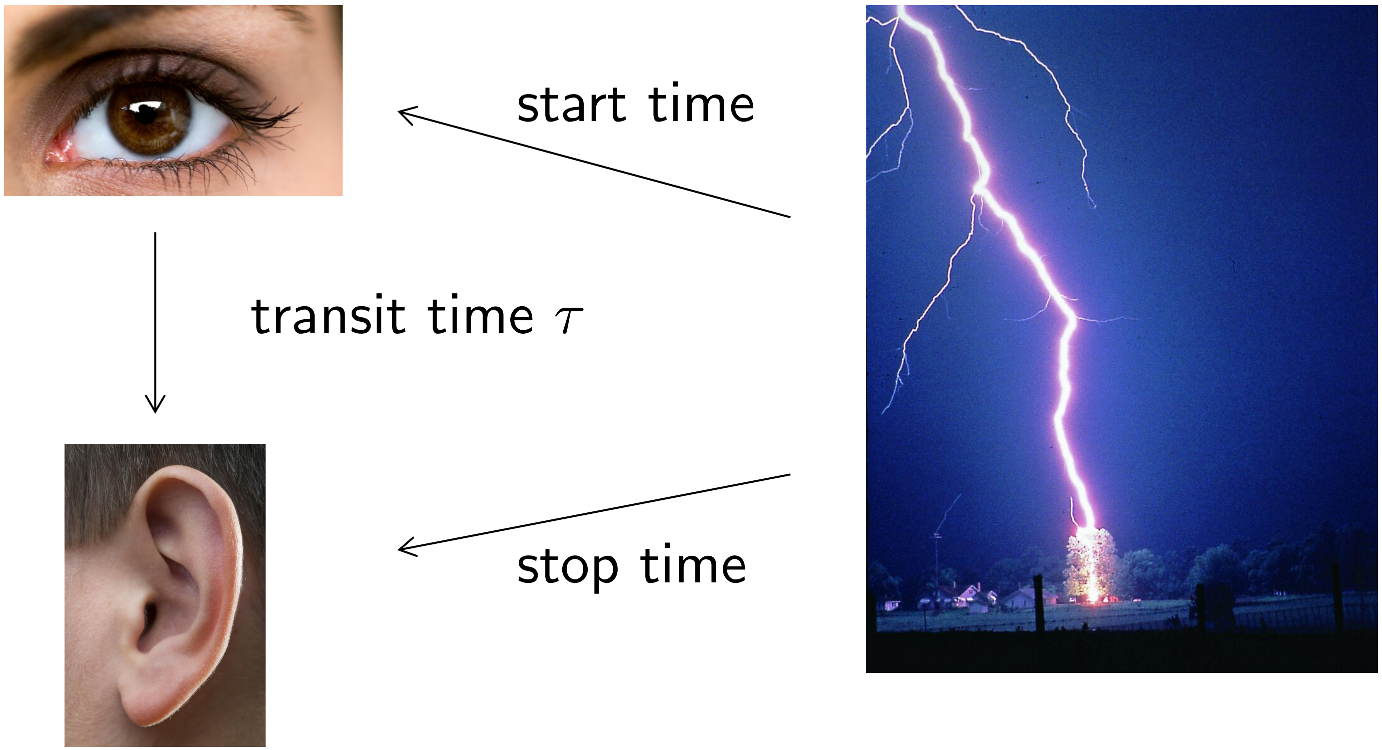
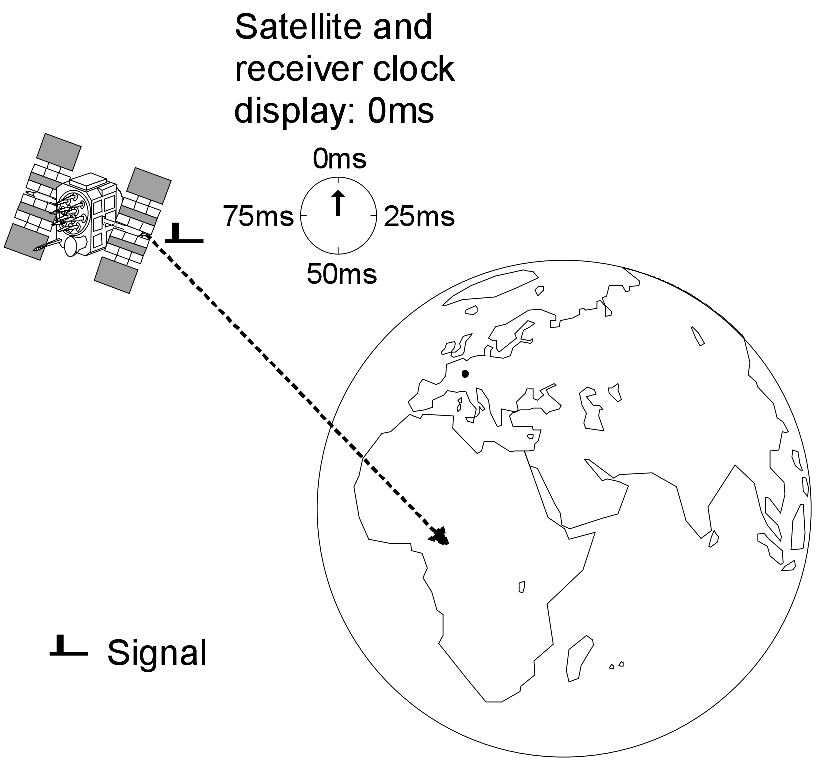
Time-of-arrival based navigation (GPS)

Distance measurement

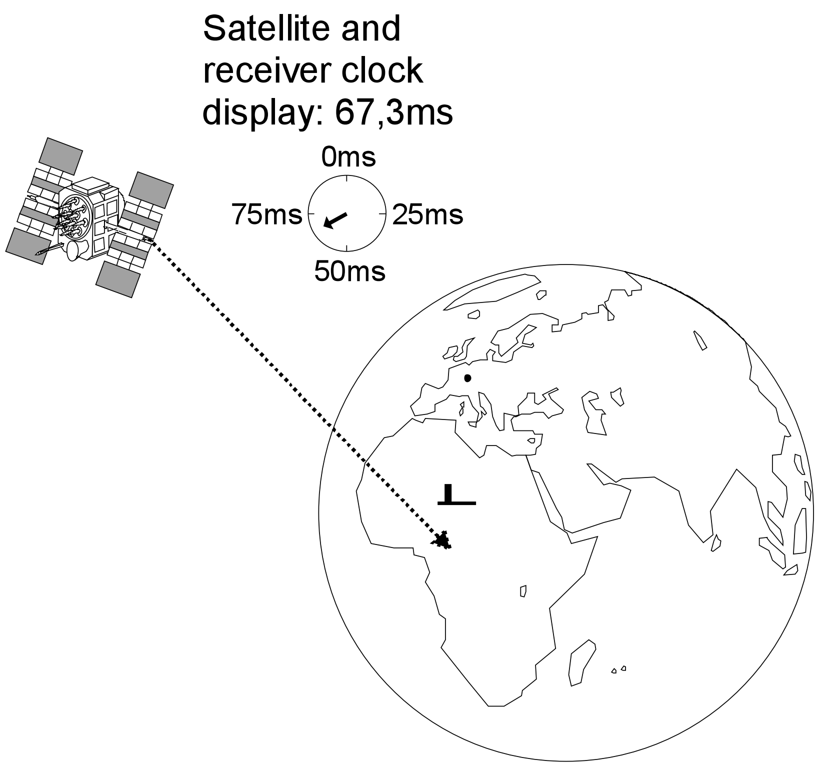


distance $= \tau \cdot c$ $\quad (c = 343$ m/s or 1235 km/h$)$

Transit time

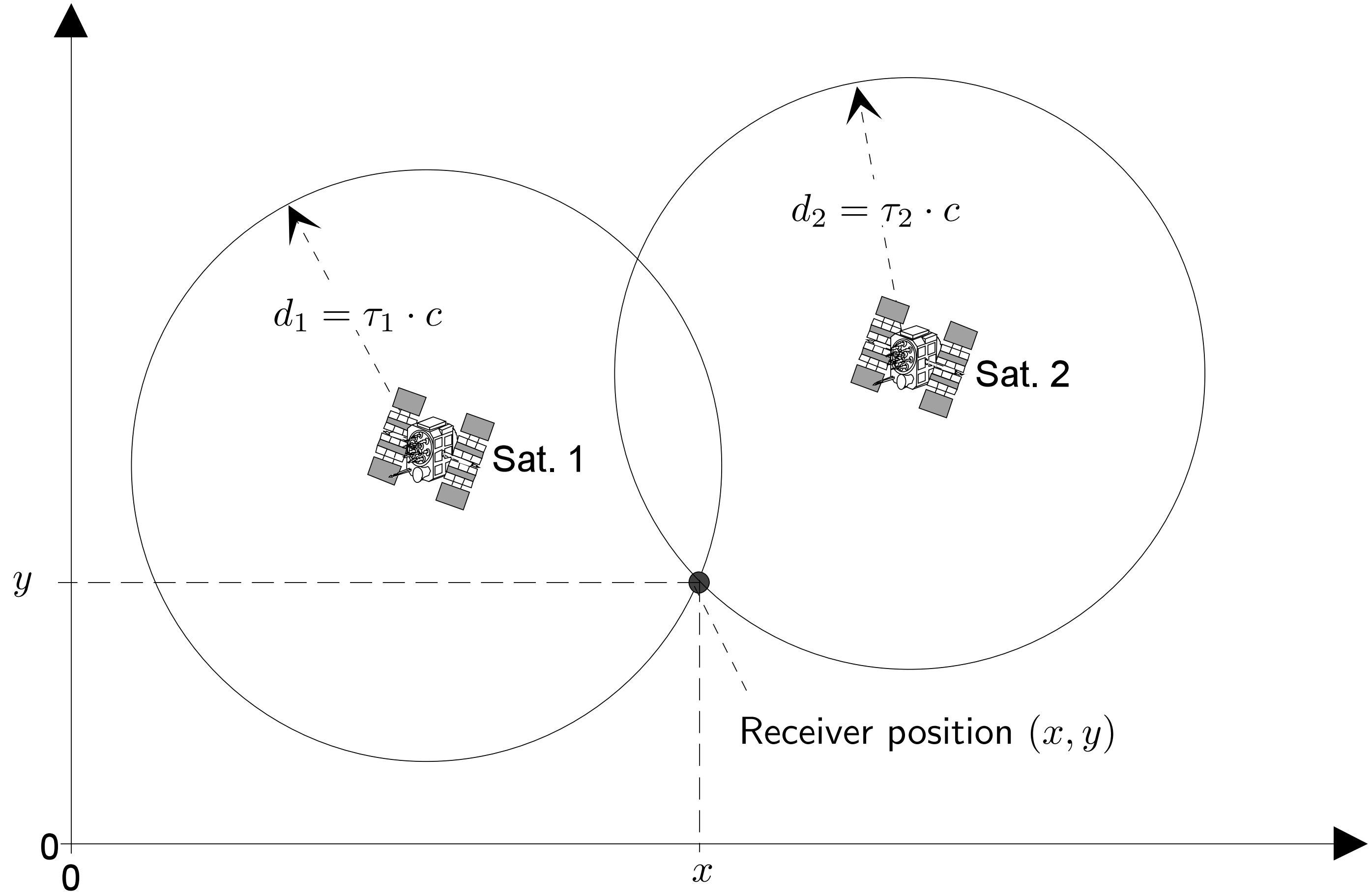


Signal transmition (start time)



Signal reception (stop time)

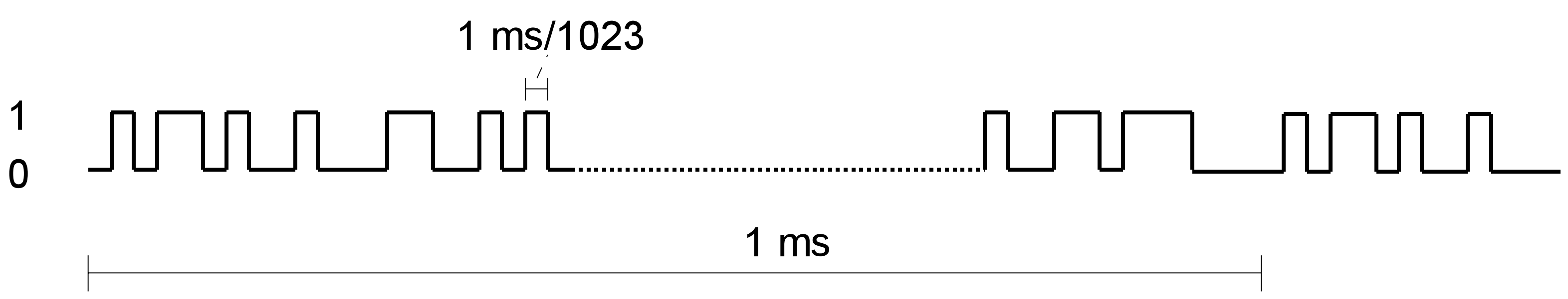
2D localization



Satellite signals

Each satellite transmits a unique 1 ms signature (pseudo random noise code (PRNC) of 1023 zeros and ones), repeated continuously

- Identification: the receiver knows from which satellite the signal originated
- Signal transit time measurement



Timing accuracy

Accurate measurement of signal transit time:

- 1μs error $\Rightarrow$ positional error 300 m
- Synchronised clocks are needed
- Atomic clocks in satellites
- Too costly ($50K to $100K) for receivers
- Secret to perfect timing is a fourth satellite

3D localization

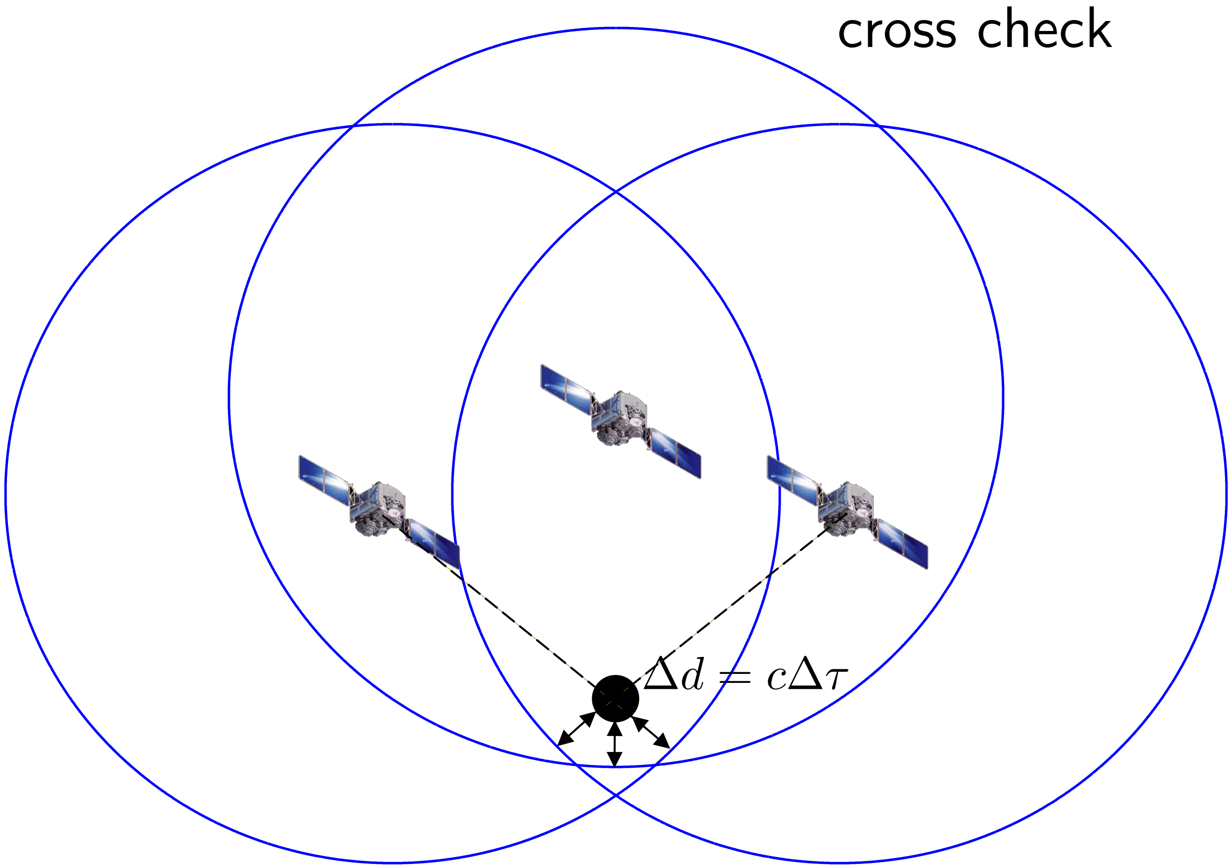
In order for the receiver to determine its position, it has to receive time signals $\tau_1, \ldots, \tau_4$ from four different satellites.

- Time at which the satellite transmits signals is known accurately due to the synchronised atomic clocks at the satellites
- The receiver is not synchronised to UTC and is slow or fast by $\Delta\tau$:

$$
\begin{aligned}
d_{i,\text{measured}} &= c(\tau_i + \Delta\tau) \\
&= d_i + \Delta d \\
&= \sqrt{(x_i - x)^2 + (y_i - y)^2 + (z_i - z)^2} + c\,\Delta\tau
\end{aligned}
$$

four unknowns

Time error



PulsarPlane project

PulsarPlane

Partners:

SEVENTH FRAMEWORK PROGRAMME

TU Delft

NLR

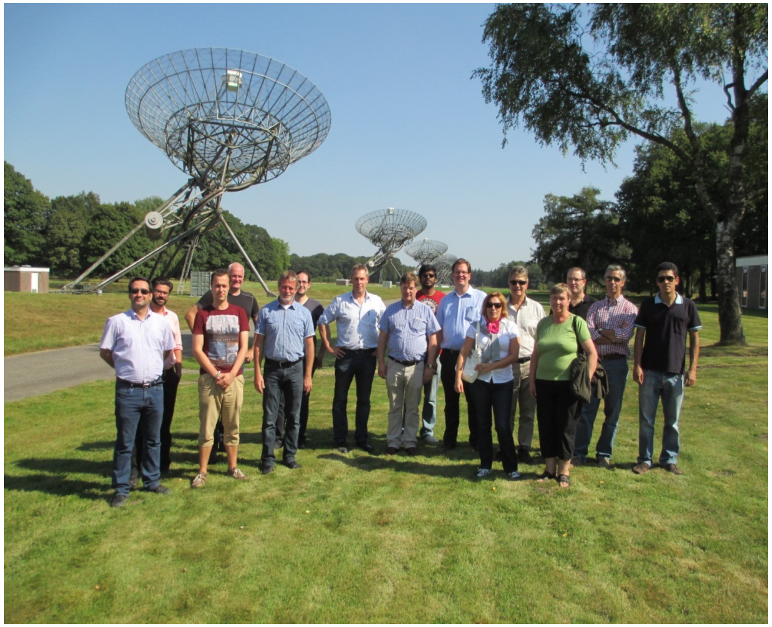
СВ. КЛИМЕНТ ОХРИДСКИ

University of Twente
The Netherlands

Aalto University

inesc id lisboa

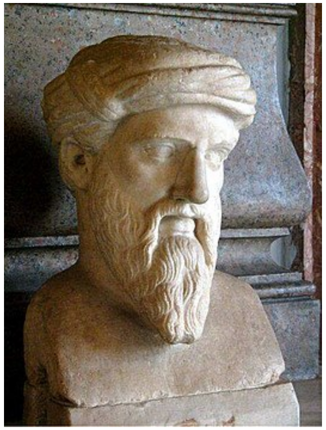
Kick-off: September 2013

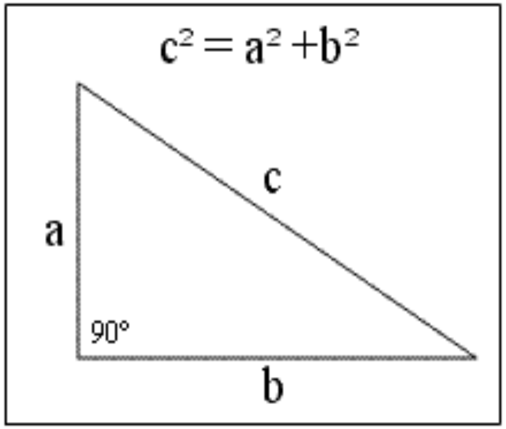


Key research question

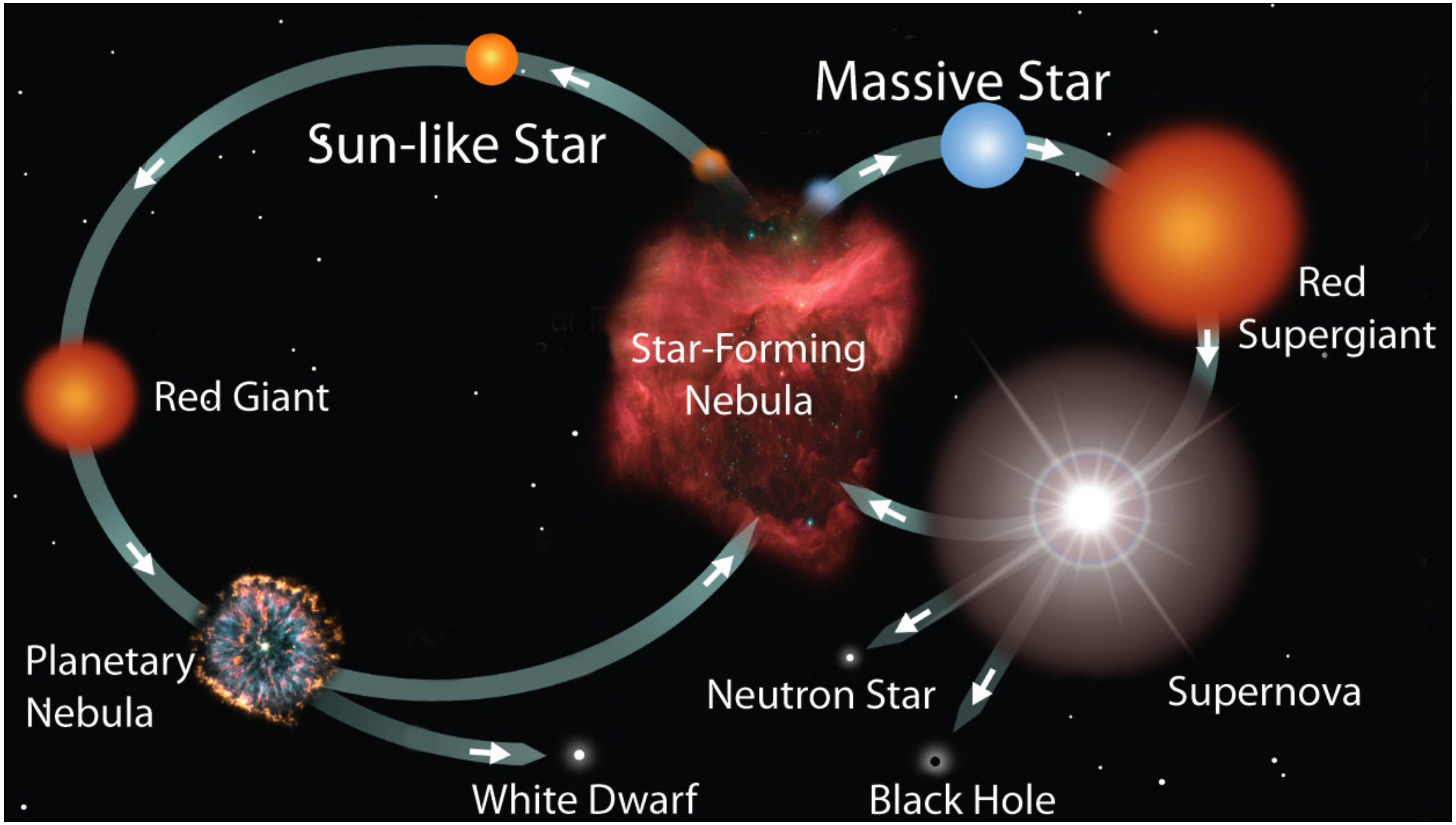
How and to what extent can nature (pulsars in radio domain) fulfill the required navigation performances for aviation?

E.g. PSR B0329+54 (2,643 ly). Aircraft would navigate using source signals that have been underway long before Christ was born...





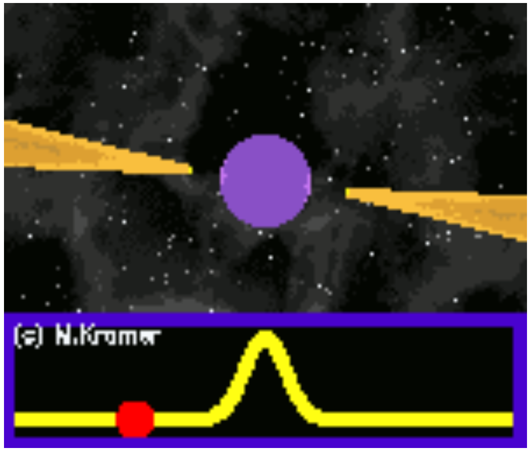
Life cycle of a star



Radio pulsars

Pulsars (pulsating star):

- Highly magnetized, rotating neutron star that emits a beam of electromagnetic radiation.
- Radiation can only be observed when the beam of emission is pointing toward the earth (lighthouse model)
- Wideband (100 MHz - 85 GHz)
- Extremely accurate pulse sources



Kramer (University of Manchester)

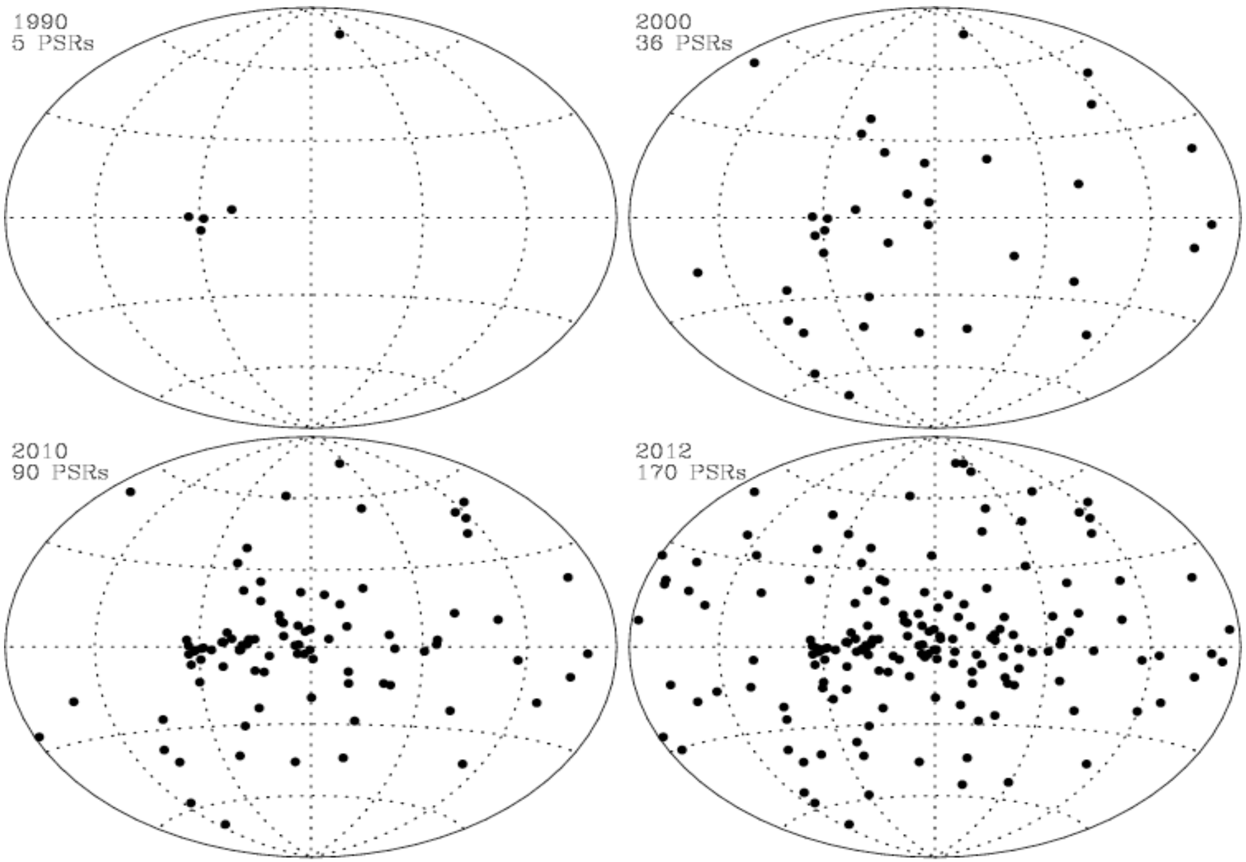
History of radio pulsars

- First discovered by Jocelyn Bell (November 28, 1967)
- For a short time scientists thought they might be coming from an extra-terrestrial civilisation
- Over 1500 pulsars detected
- Expected to discover thousands more during the next few years

History of radio pulsars

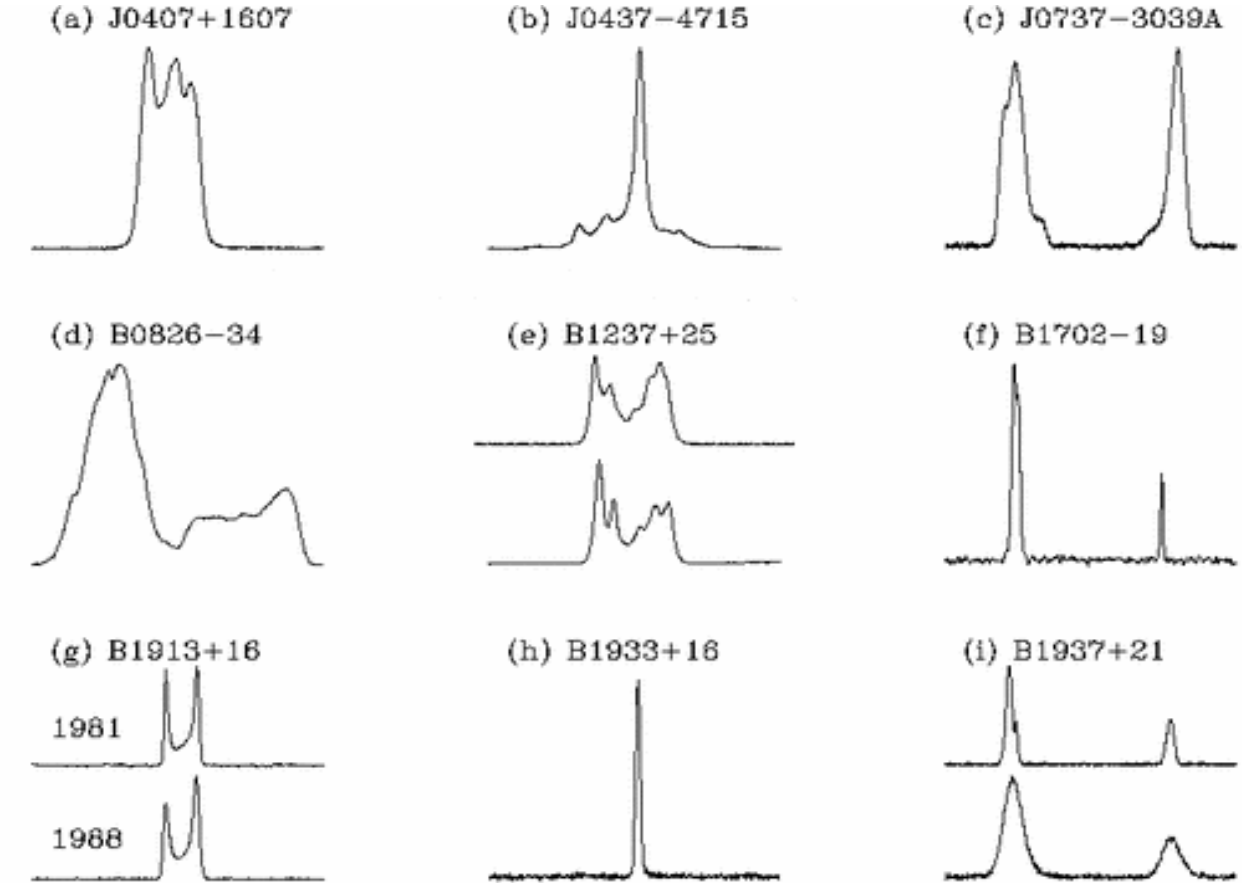
- They nicknamed the signal LGM-1, for "little green men"
- It was not until a second pulsating source was discovered in a different part of the sky that the "LGM hypothesis" was entirely abandoned
- Their pulsar was later dubbed CP 1919, and is now known by a number of designators including PSR 1919+21, PSR B1919+21 and PSR J1921+2153
- Although CP 1919 emits in radio wavelengths, pulsars have, subsequently, been found to emit in visible light, X-ray, and/or gamma ray wavelengths

Observing radio pulsars

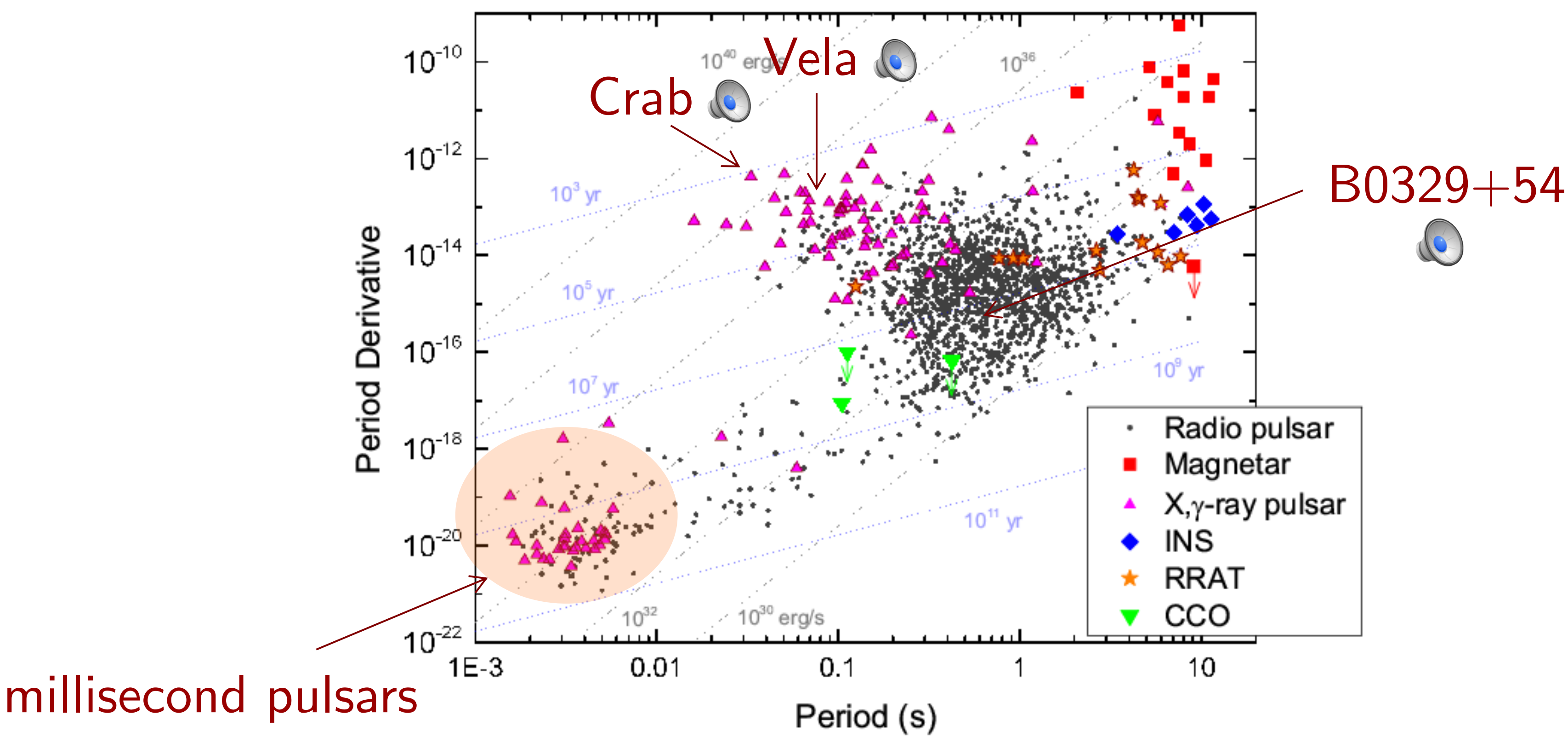


Handbook of Pulsar Astronomy, Lorimer & Kramer

Radio pulsars profiles



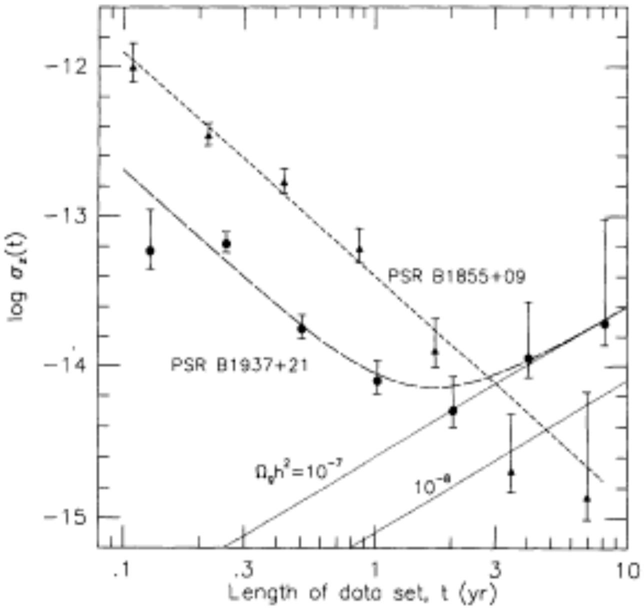
Radio pulsars



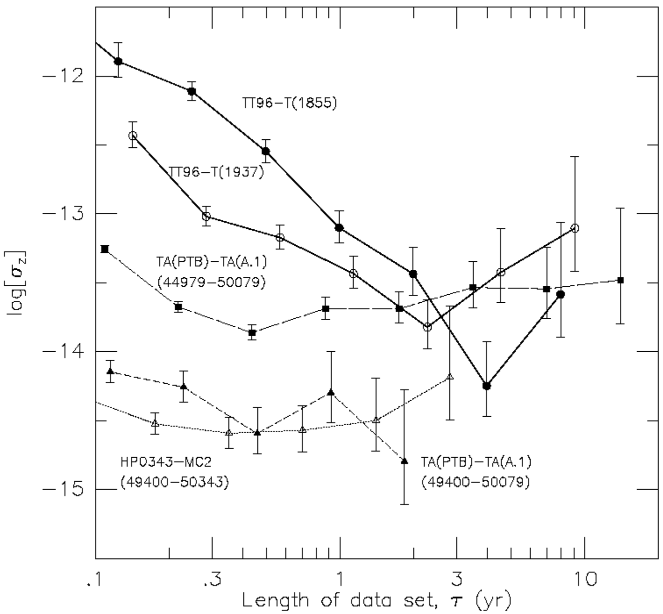
Radio pulsars

Variance of (millisecond) pulsars compared to atomic clocks:

pulsars



atomic clocks



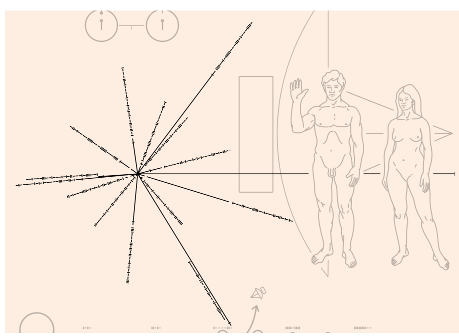
Why pulsar navigation?

Advantages of pulsar navigation:

- Completely independent from Earth
- No satellites involved
- Negligible infrastructure
- Always available
- Robust against solar flares and hostile attempt to disable them
- Wideband (jamming is difficult)

Why pulsar navigation?

- Pulsar maps have been included on the two Pioneer Plaques (1972/1973) as well as the Voyager Golden Record (1977).
- They show the position of the sun, relative to 14 pulsars, which are identified by the unique timing of their electromagnetic pulses, so that our position both in space and in time can be calculated by potential extraterrestrial intelligences.

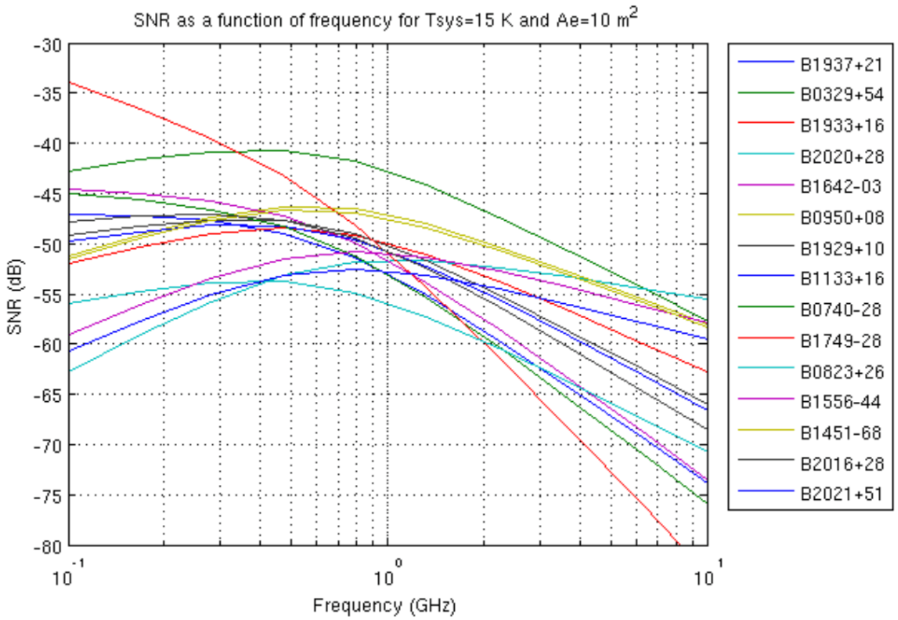


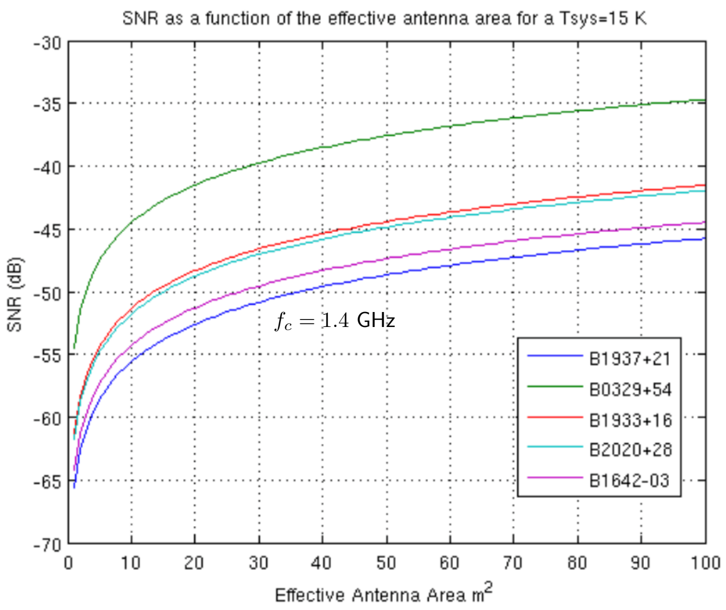
Detection of radio pulsars

Data acquisition

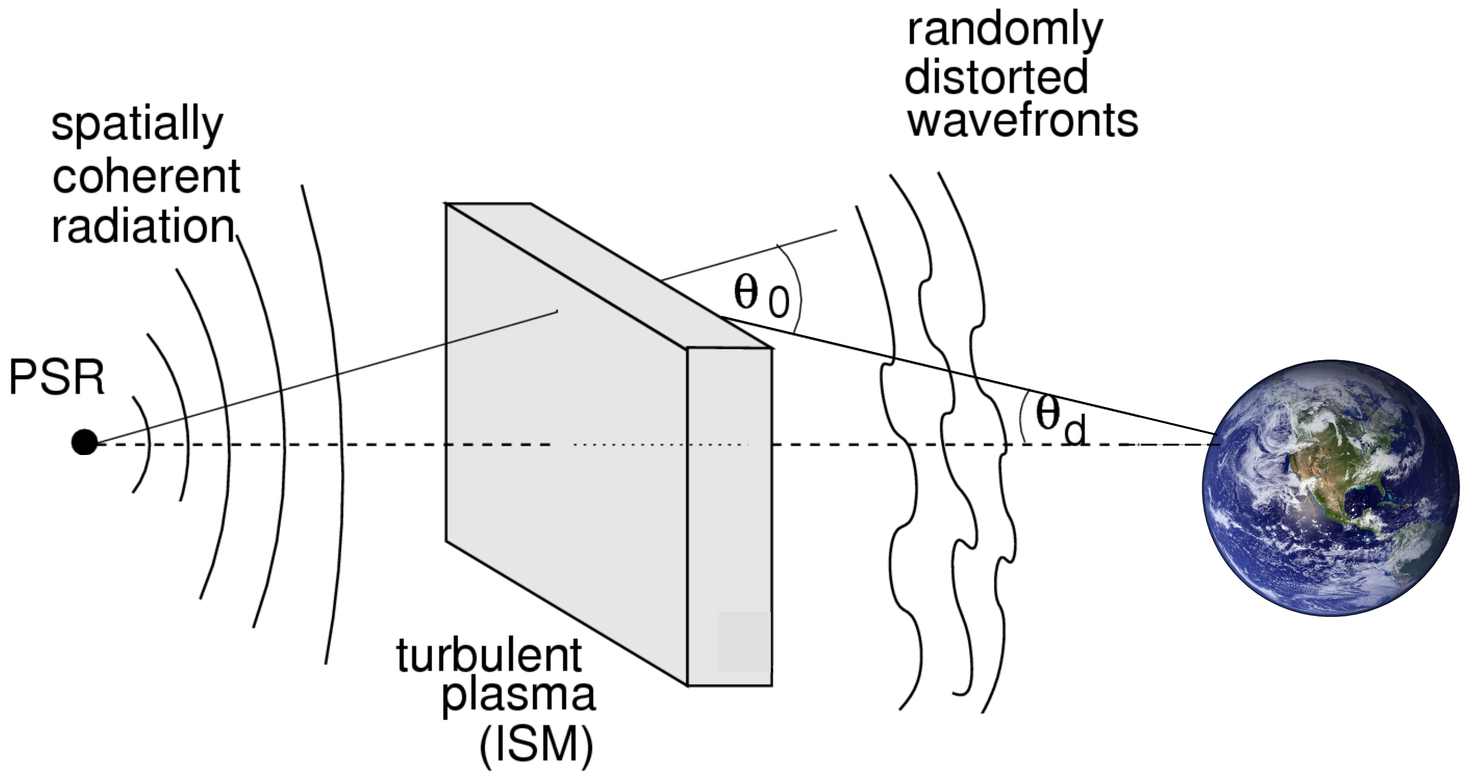
- (Average) signal strength of pulsars is expressed in terms of milli-Jansky (mJy)
- 1 Jy $= 10^{-26}\ W/\mathrm{m}^2/\mathrm{Hz}$
- Strongest pulsar (Vela): 5000 mJy
- Usually in the order 1-10 mJy
- Mobile phone placed on the moon is by far the strongest signal!

Data acquisition

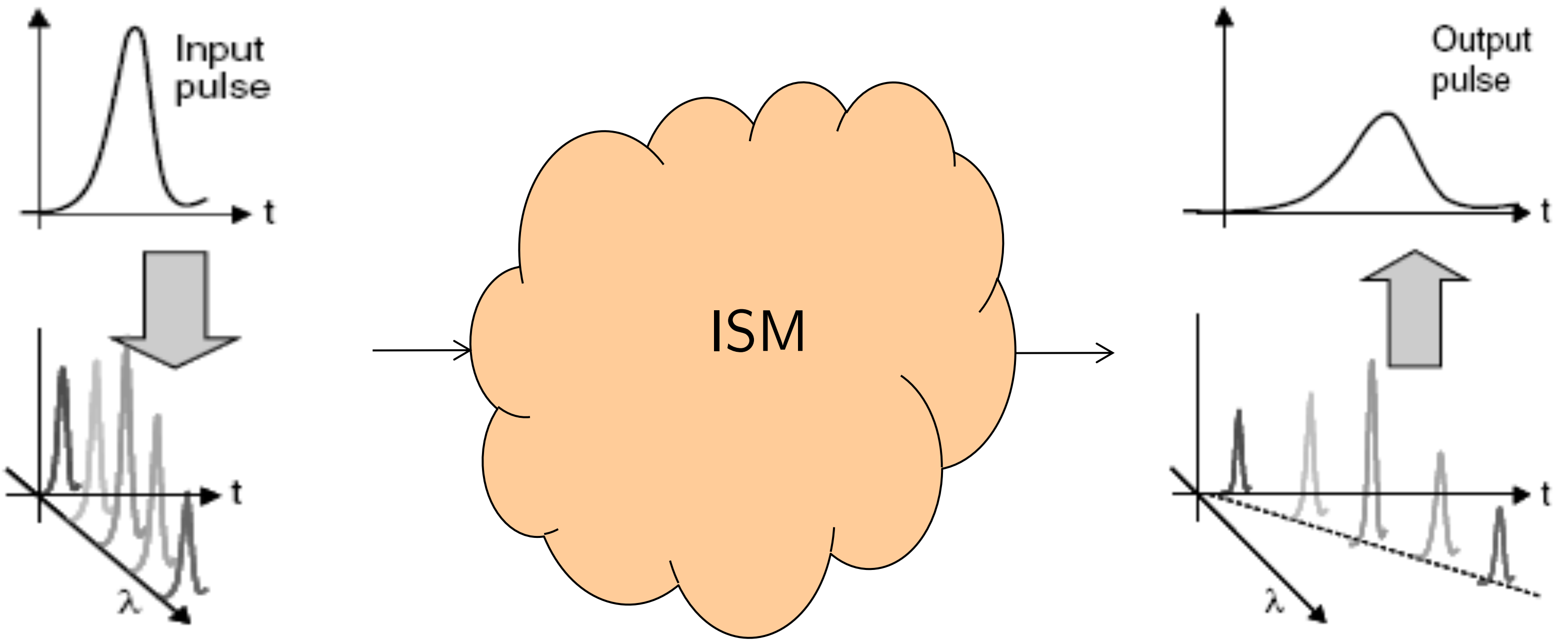




Propagation effects



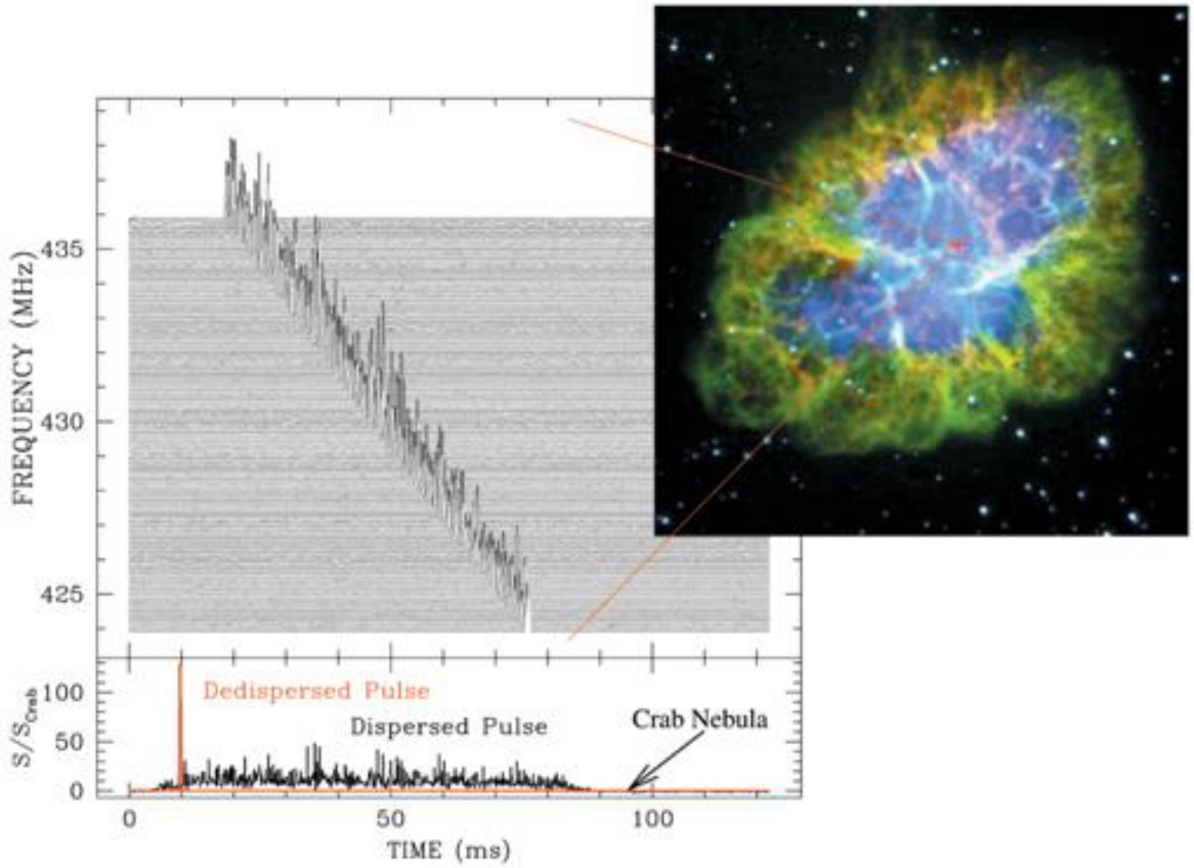
Dispersion



Phase velocity is frequency dependent

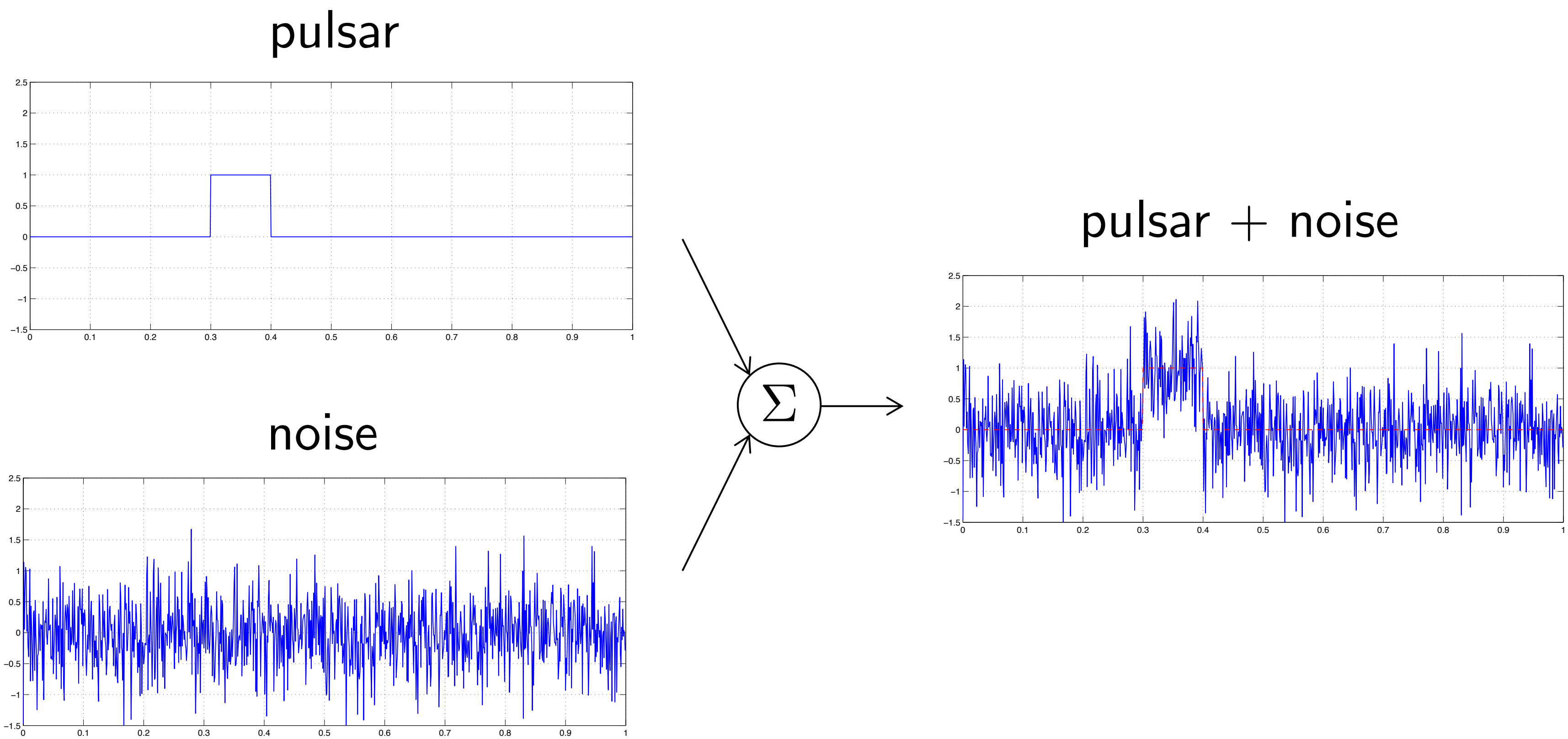
The ISM acts as an all-pass filter!

Dispersion



FREQUENCY (MHz)
435
430
425
S/S_{Crab}
100
50
0
Dedispersed Pulse
Dispersed Pulse
Crab Nebula
0
50
100
TIME (ms)

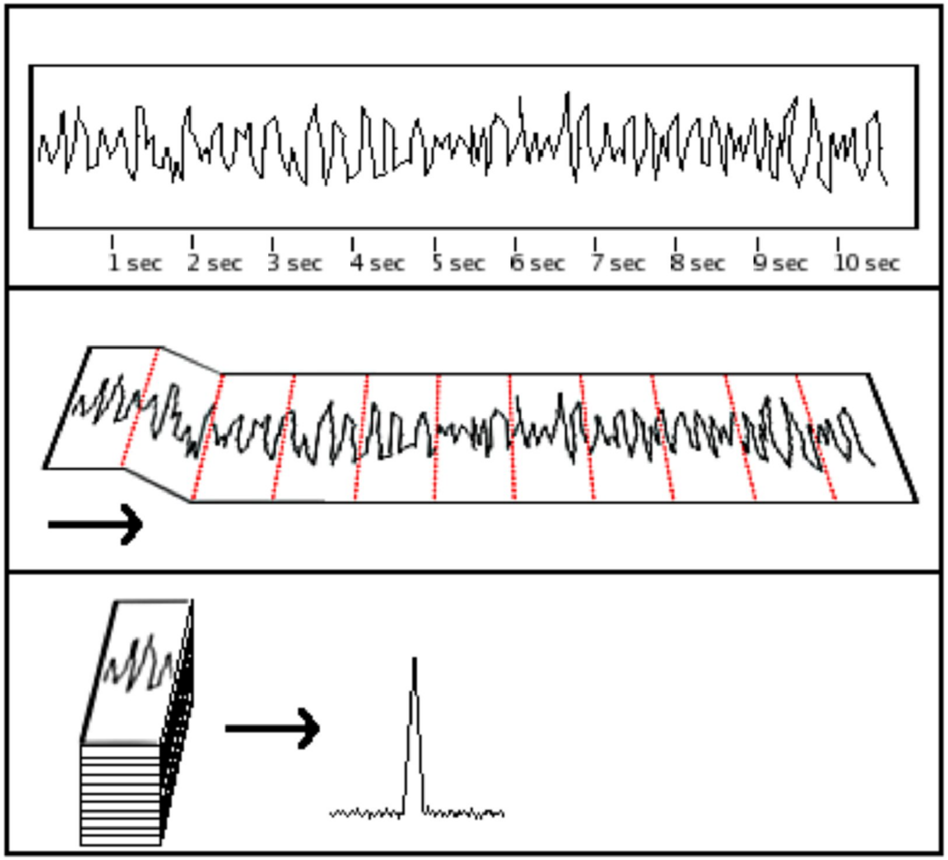
Data acquisition



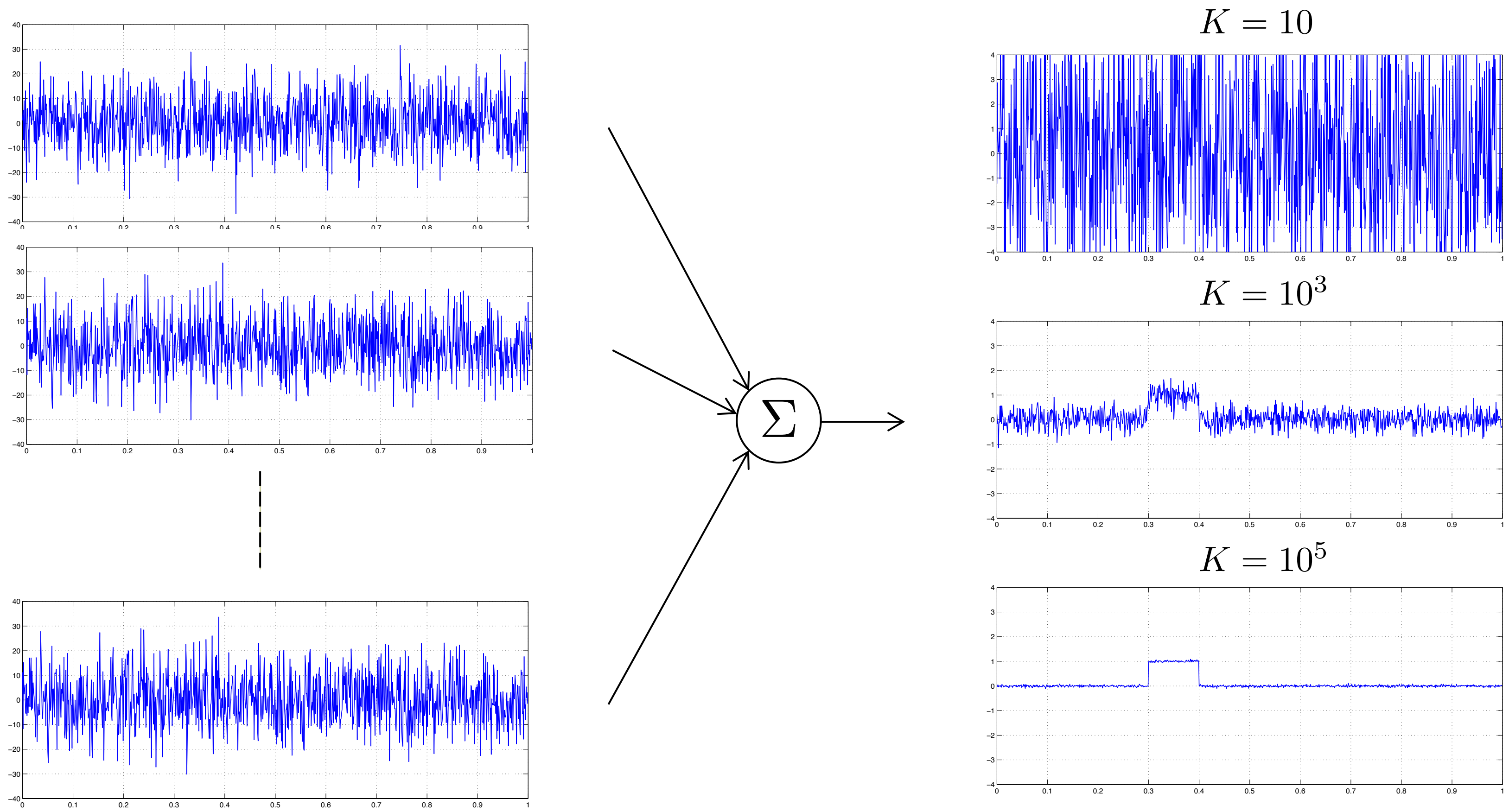
Data acquisition

- The epoch folding technique assumes that we know the periodicity of the underlying signal, say T_0
- Detection of pulsar signal by averaging signal segments of length T_0
  - Noise signal will be cancelled out
  - T_0-periodic pulsar signal will be added constructively
- Performance increases with increasing number of frames K
- K large implies a large integration time

Epoch folding



Epoch folding



Detection of pulsars

Some observations:

- Pulsar signals are weak (SNR = -35 $\cdots$ -90 dB)
- Detecting them requires antennas with large aperture, large integration times, advanced algorithms or a combination of both
- "Peaked" signals are easier to detect $\Rightarrow$ not true!
- Traditionally, astronomers use a de-dispersion algorithm before epoch folding, which is a time- and processing power-consuming approach ($\approx 80\%$ of computational complexity)

Detection of deterministic signal

Consider the more general signal detection problem of detecting a *known deterministic signal* s *with unknown time-of-arival (TOA)* τ_0 in white, zero mean, Gaussain noise (z). The detection problem is to distinguish between the hypotheses

$$\mathcal{H}_0 : x = z$$
$$\mathcal{H}_1 : x = z + D^{\tau_0} s$$

where $x, z, s \in \mathbb{R}^N$ and D is the shift operator. τ_0 is the unknown time-of-arrival (delay)

Detection of deterministic signal

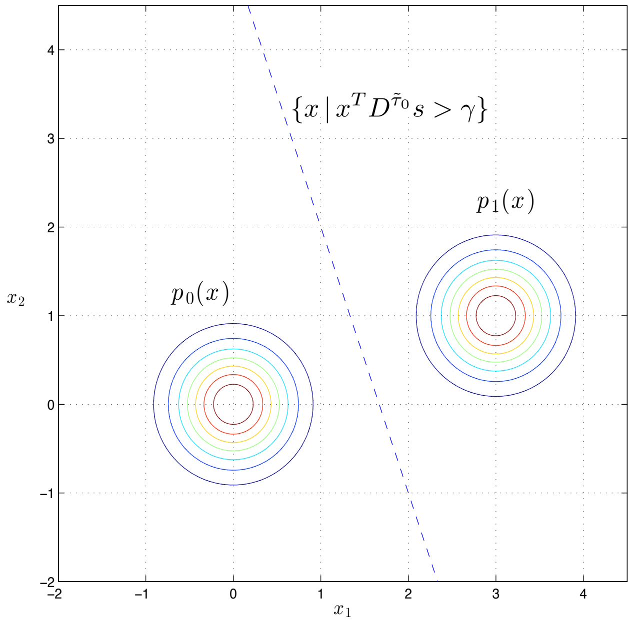
If the probability density function of the signals under the hypotheses $\mathcal{H}_0$ and $\mathcal{H}_1$ are denoted by $p_0(x)$ and $p_1(x)$, respectively, the optimal (Neyman-Pearson) detector decides $\mathcal{H}_1$ if

$$L_G(x) = \frac{p_1(x;\tilde{\tau}_0)}{p_0(x)} > \xi$$

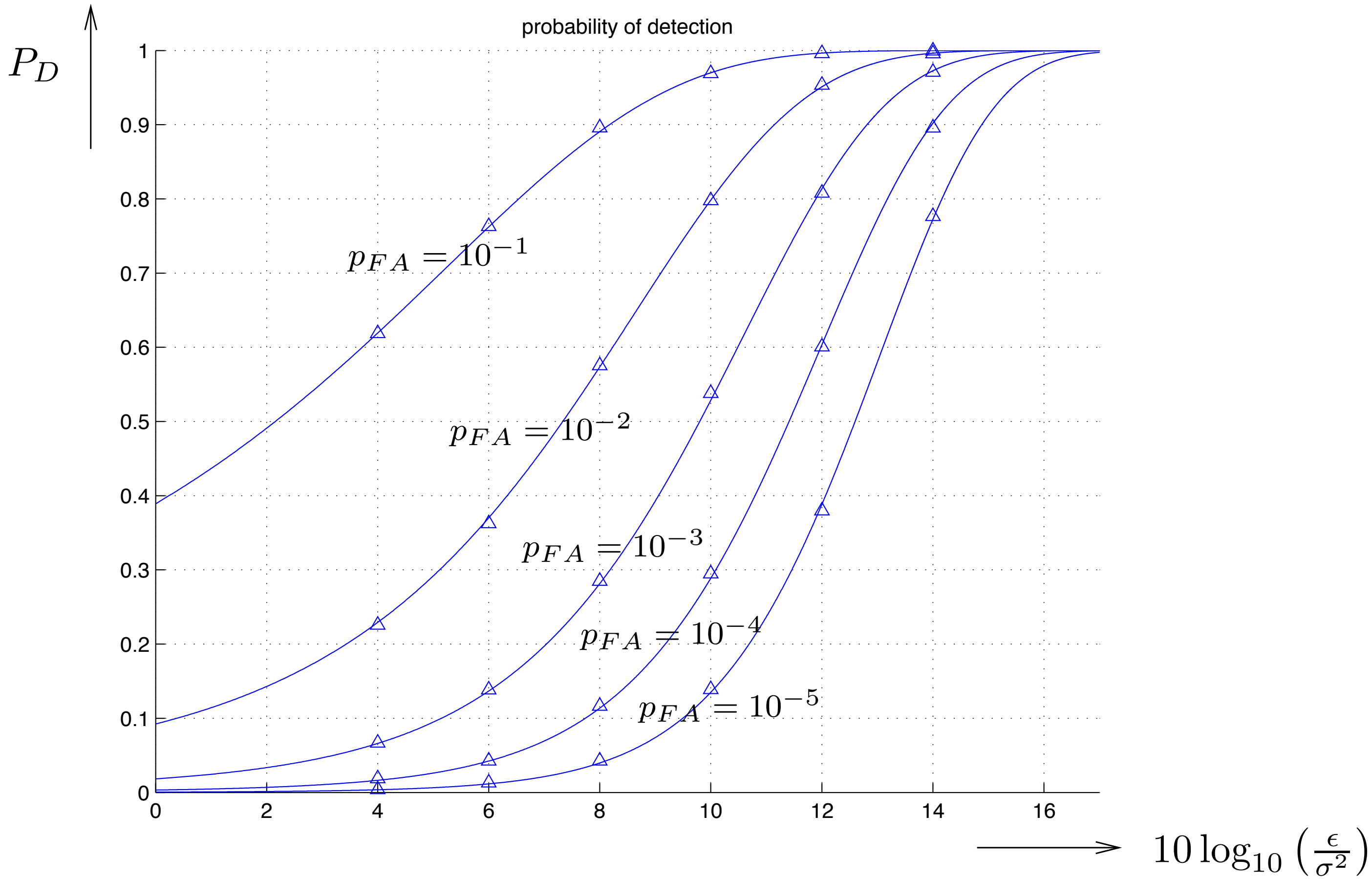
where $\tilde{\tau}_0$ denotes the ML estimate of τ_0

Detector depends on the ENR $\eta = \epsilon/\sigma^2$

$$(\epsilon = \|s\|^2)$$



Detection performance



Unknown TOA

We have

$$p_1(x) = \frac{1}{(2\pi\sigma^2)^{\frac{N}{2}}} \exp\left(-\frac{(x - D^\tau s)^T (x - D^\tau s)}{2\sigma^2}\right)$$

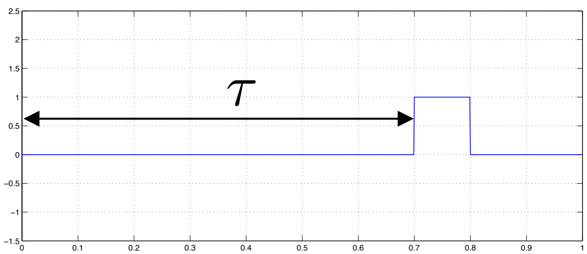
Hence,

$$\begin{aligned}\tilde{\tau}_0 &= \arg\min_{\tau} \|x - D^\tau s\|_2^2 \\ &= \arg\max_{\tau} x^T D^\tau s\end{aligned}$$

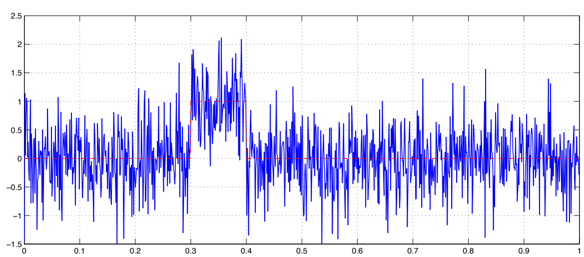
In other words, the MLE of τ_0 is the shift-value that gives highest correlation between x and $D^\tau s$ (matched filter)

Unknown TOA

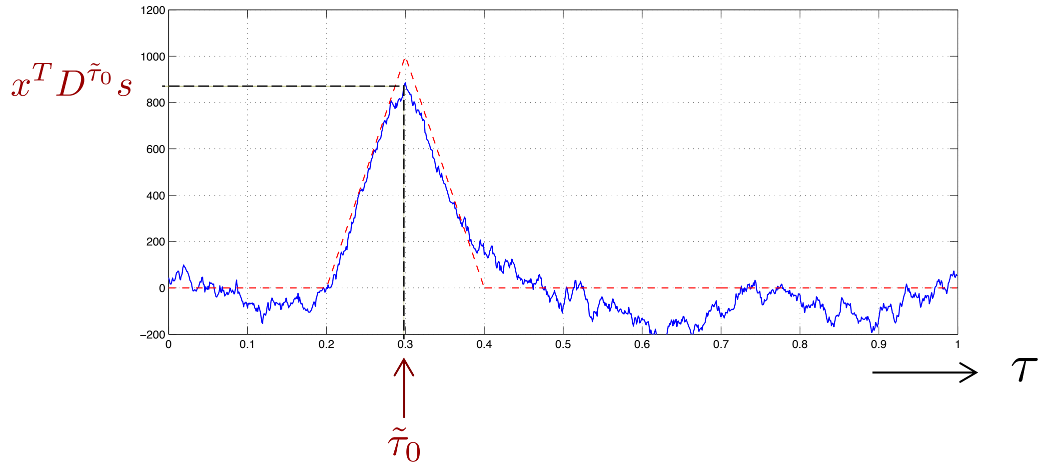
$$D^{\tau} s$$



$$x = z + D^{\tau_0} s$$

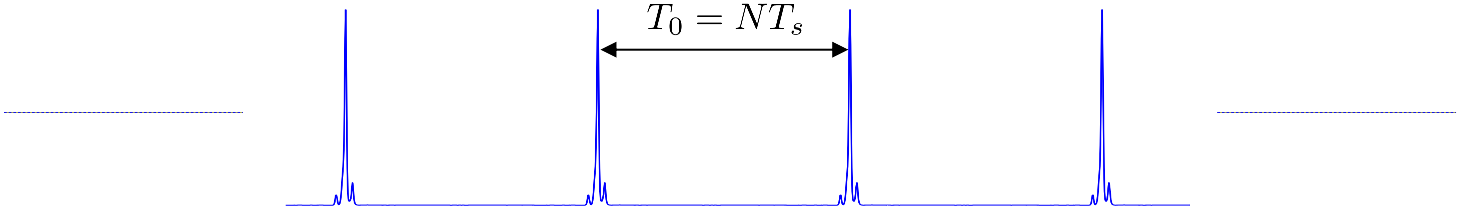


$$x^T D^{\tau} s = \sum_{n=0}^{N-1} x(n) s(n-\tau)$$



Relation to epoch folding

$$s(n) = s(n + kN)$$



K epochs, $M = KN$ samples in total

Matched filter: $x^T D^\tau s = \frac{1}{M} \sum_{m=0}^{M-1} x(m) s(m - \tau)$

Relation to epoch folding

Epoch folding:

$$
\begin{aligned}
\bar{x}_K(n) &= \frac{1}{K}\sum_{k=0}^{K-1} x(n+kN) \\
&= \frac{1}{K}\sum_{k=0}^{K-1} D^{\tau_0} s(n+kN) + \frac{1}{K}\sum_{k=0}^{K-1} z(n+kN) \\
&= D^{\tau_0} s(n) + \bar{z}(n)
\end{aligned}
$$

Hence

$$\lim_{K\to\infty} \bar{x}_K = D^{\tau_0} s.$$

Relation to epoch folding

$$x^T D^\tau s = \frac{1}{M}\sum_{m=0}^{M-1} x(m)s(m-\tau)$$

$$= \frac{1}{KN}\sum_{k=0}^{K-1}\sum_{n=0}^{N-1} x(n+kN)s(n-\tau+kN)$$

$$= \frac{1}{N}\sum_{n=0}^{N-1} \underbrace{\left(\frac{1}{K}\sum_{k=0}^{K-1} x(n+kN)\right)}_{\bar{x}_K(n)\text{ : epoch-folded observation}} s_0(n-\tau)$$

$$= \frac{1}{N}\sum_{n=0}^{N-1} \bar{x}_K(n)s_0(n-\tau) \quad \text{(one-period matched filter)}$$

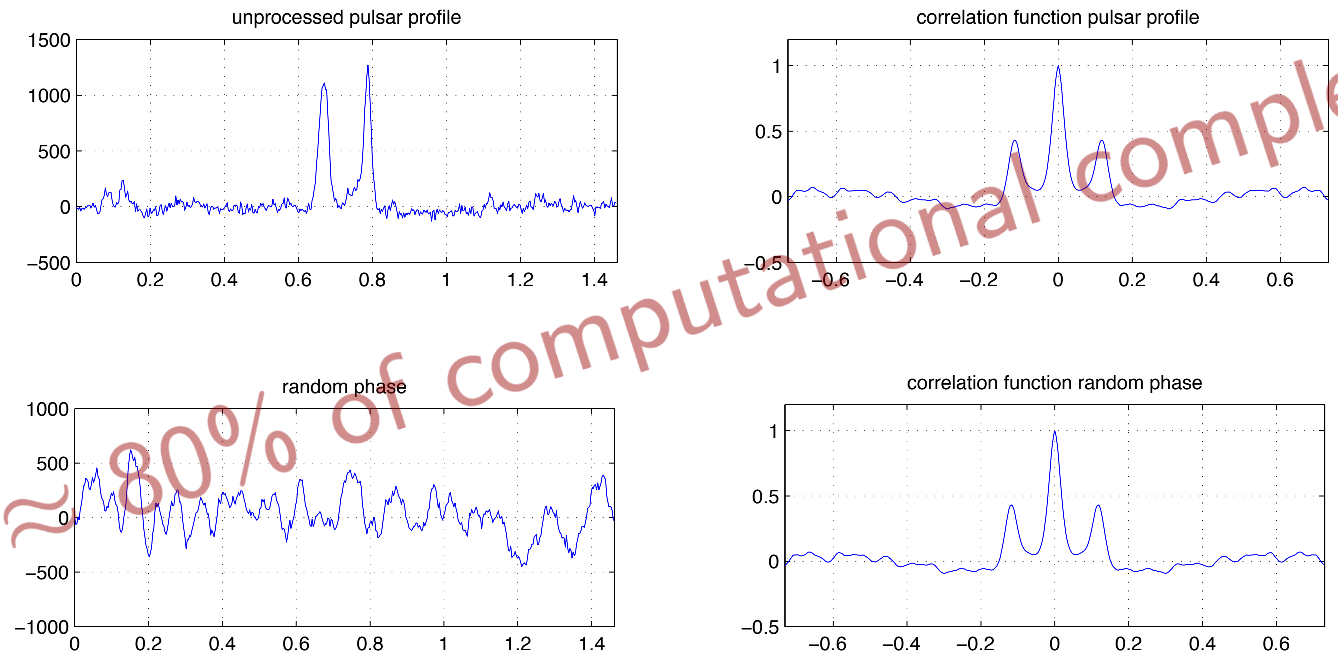
Detection of deterministic signal

Conclusions:

- Detector depends on the ENR $\eta = K\epsilon_0/\sigma^2$
- η can be increased by either increasing K (increasing the integration time) or increasing ϵ_0 which is done by increasing the sampling frequency f_s (more data samples per time unit)
- Trade-off between K and f_s
- Performance independent of phase

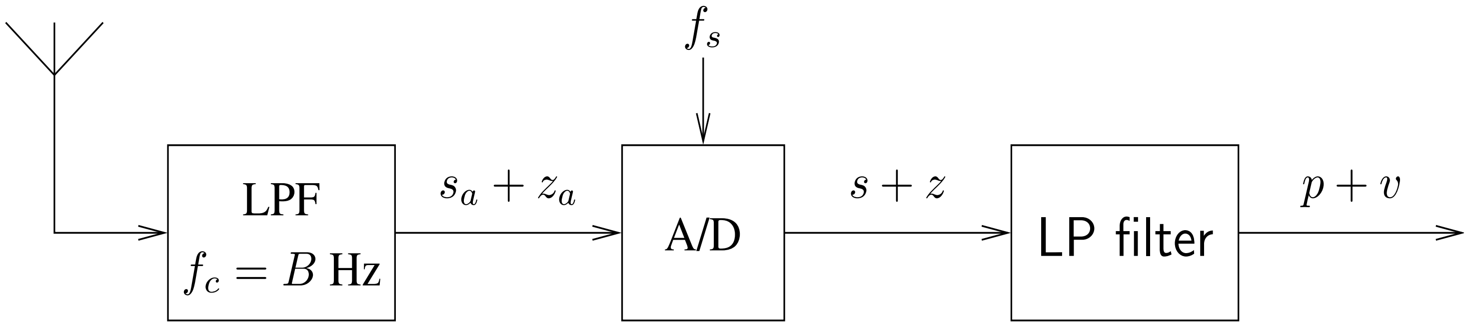
Dispersion

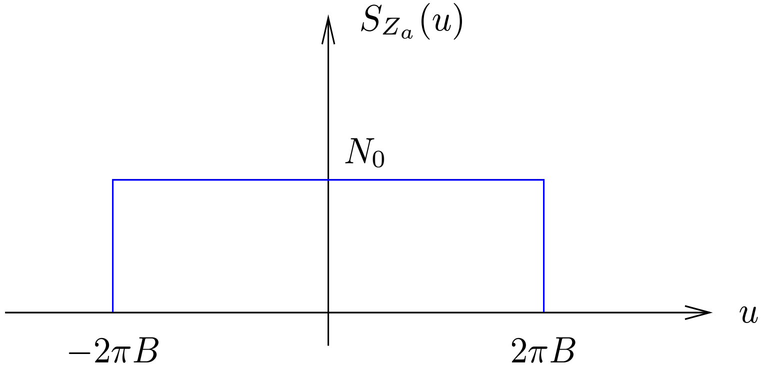
Correlation independent of phase ⇒ dispersion has NO effect!

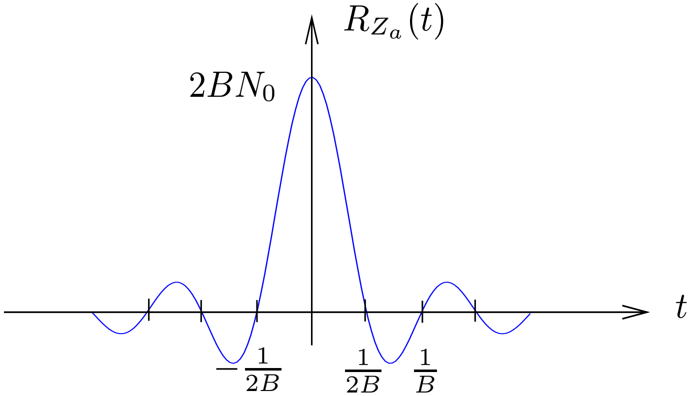


safes ≃ 80% of computational complexity

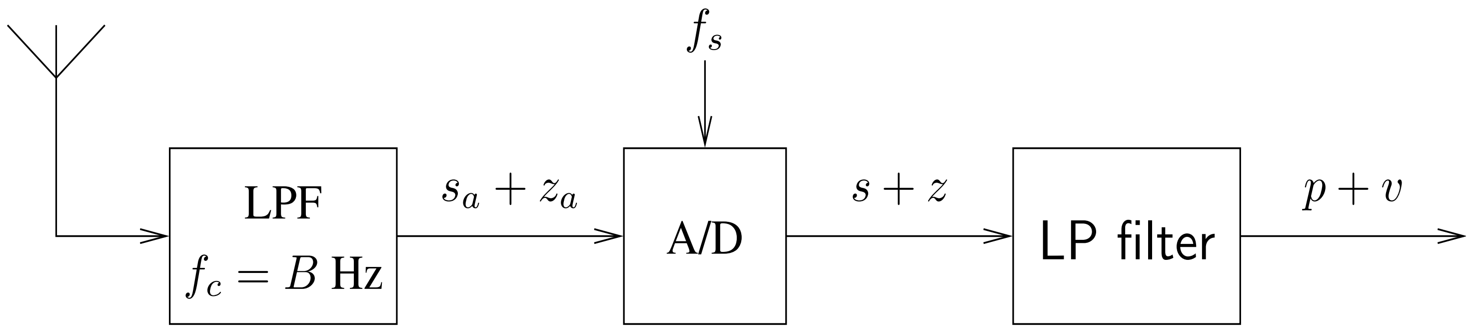
Effect of sampling rate







Effect of sampling rate



$$R_Z(k) = \mathrm{E} Z_n Z_{n+k} = \mathrm{E} Z_a(nT_s) Z_a((n+k)T_s) = R_{Z_a}(kT_s)$$

- sampling at Nyquist rate ($f_s = 2B$):

$$\eta = \frac{\epsilon_s}{\sigma_z^2} = \frac{f_s\,\epsilon_a}{2BN_0} = \frac{\epsilon_a}{N_0}$$

large bandwidth increases η

Estimation of TOA

Suppose we wish to estimate an unknown deterministic parameter θ from noisy measurements x.

The variance of *any unbiased estimator* $\tilde{\theta}$ is bounded from below by the reciprocal of the Fisher information $I(\theta)$. That is

$$\mathrm{var}(\tilde{\theta}) \geq \frac{1}{I(\theta)}, \qquad \text{Cramer-Rao lower bound}$$

where

$$I(\theta) = E\left(\left(\frac{\partial \log p(x;\theta)}{\partial \theta}\right)^2\right) = -E\left(\frac{\partial^2 \log p(x;\theta)}{\partial \theta^2}\right)$$

Estimation of TOA

Maximum likelihood estimator:

$$\tilde{\tau}_0 = \arg\max_{\tau} x^T D^\tau s$$

Cramer-Rao bound:

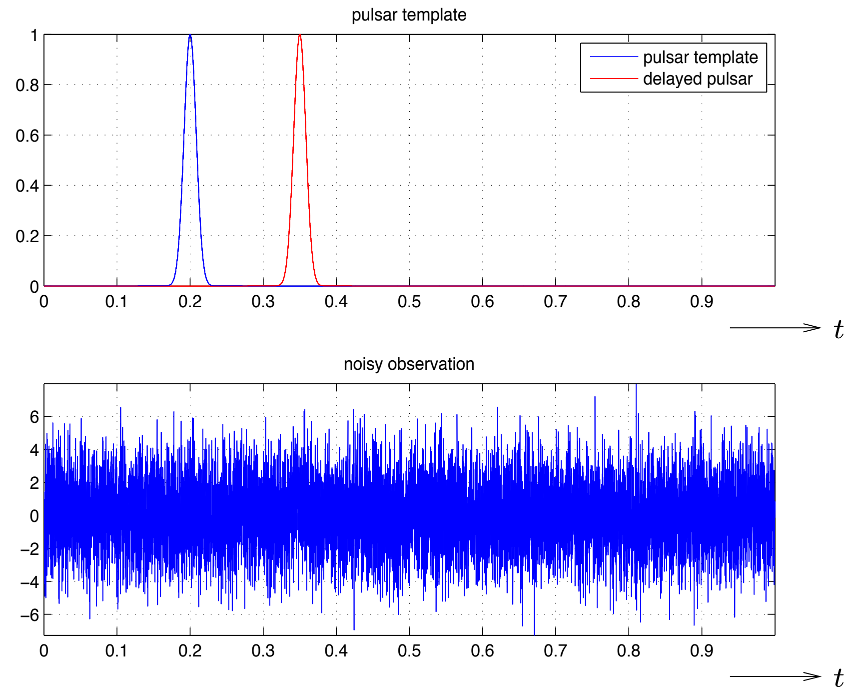
independent of phase

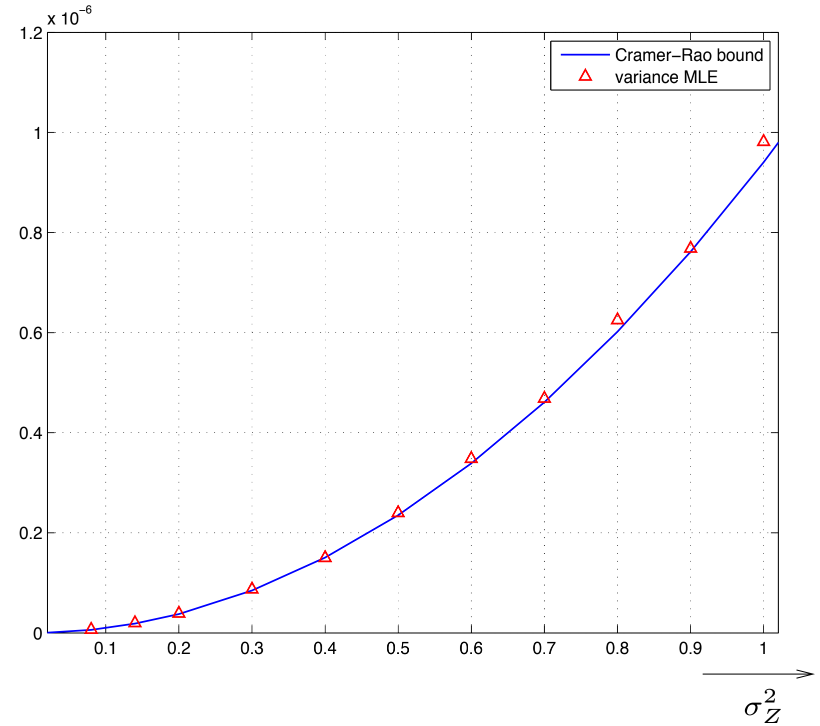
$$\mathrm{var}(\tilde{\tau}_0) \geq \frac{N_0}{\epsilon_a \, \mathrm{var}(|\hat{s}(f)|^2)} = \frac{1}{\eta \, \mathrm{var}(|\hat{s}(f)|^2)}$$

Maximum likelihood estimator is asymptotically optimal:

- unbiased estimator
- Cramer-Rao bound is tight

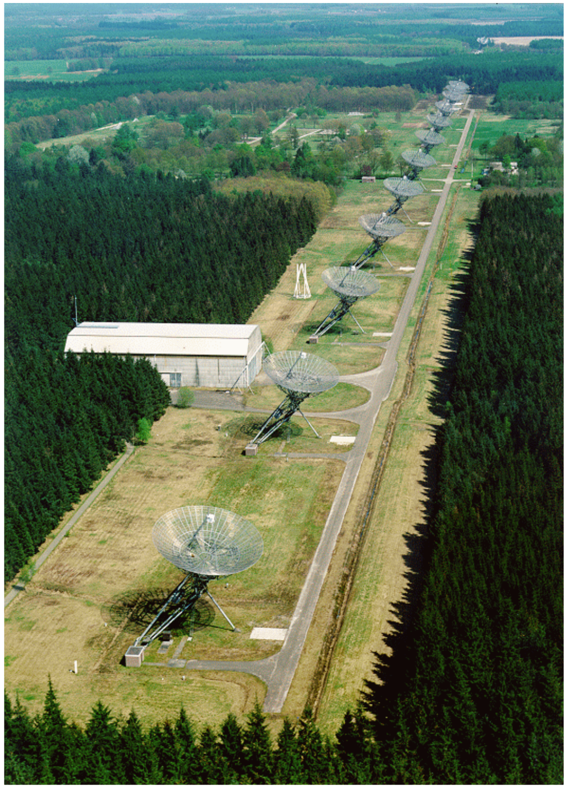
Estimation of TOA





Data Aquisition

- Real data obtained from the Westerbork Synthesis Radio Telescopes in Westerbork (Drenthe, The Netherlands)
- Linear array of 14 antennas with a diameter of 25 metres arranged on a 2.7 km East-West line.



Observed Pulsar

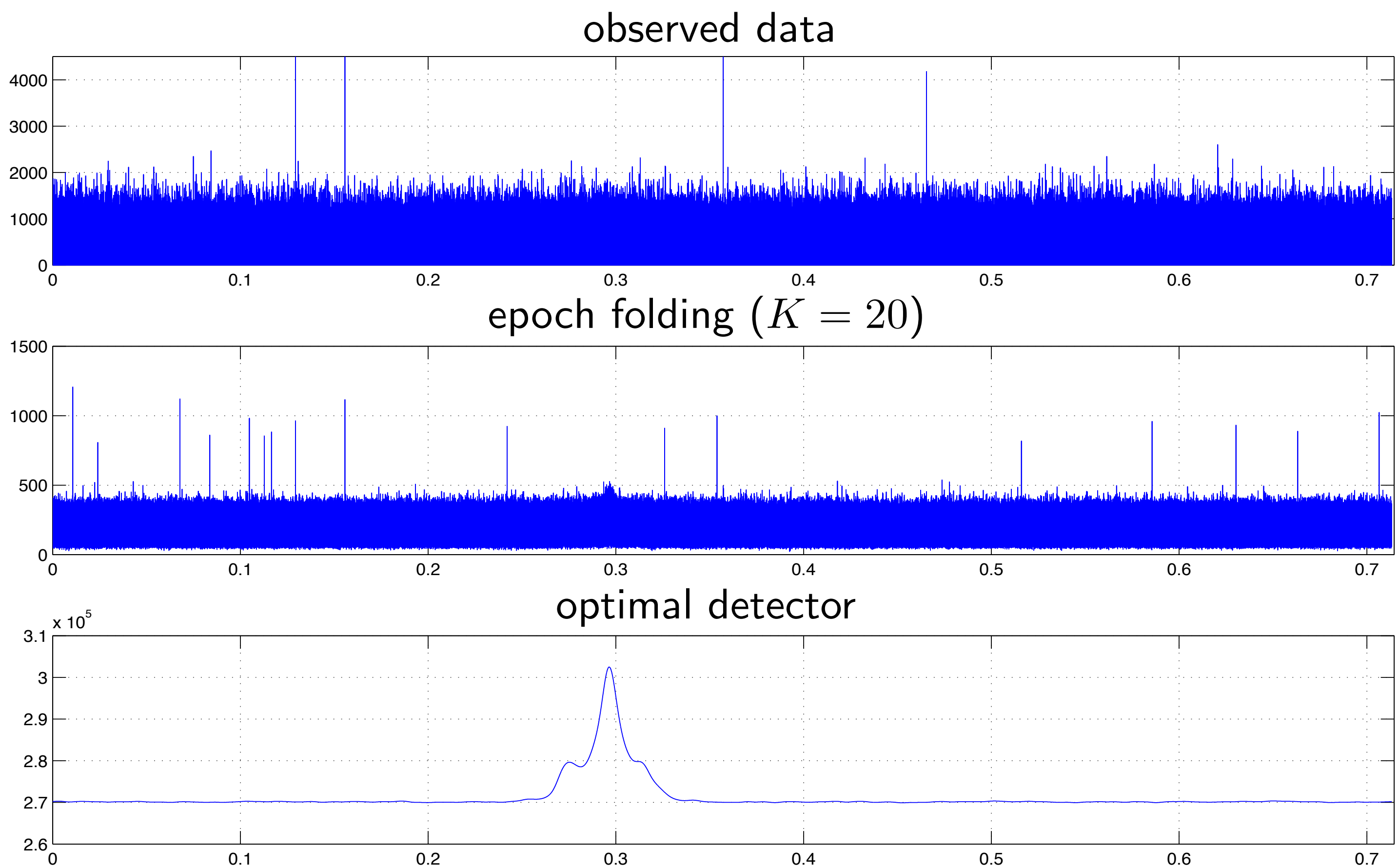
- Observed pulsar B0329+54
- Pulse period $T = 0.7145$ s
- Dispersion measure $\mathrm{DM} = 26.77\ \mathrm{pc/cm^3}$
- Flux density 203 Jansky@1400 MHz
- SNR original data -35 dB
- Pulsar template:

http://www.jb.man.ac.uk/research/pulsar/Resources/epn/epndb/B0329+54

Measured Data

- Recordings made on February 2$^{\mathrm{nd}}$, 2012
- Real voltage signals in X and Y polarization (V_x, V_y)
- Total observation of 14 s (20 periods)
- Central frequency of the acquisition 1.33 GHz
- Bandwidth 20 MHz
- Sampling frequency $f_s = 40$ MHz.
- Both polarizations are coherently de-dispersed
- Signal power obtained by $V_x^2 + V_y^2$

Measured Data



Conclusions

- Radio pulsars generate extremely accurate periodic signals, making them potential candidates for navigation
- Difficult to detect due to the propagation effects of the interstellar medium
- Optimal detection involves correlation with known template
- Optimal detection does not depend on waveform, but on the autocorrelation of it (no need for de-dispersion)
- Detection performance can be improved by increasing the sampling frequency

Graduation project

In cooperation with the Netherlands Defence Academy (NLDA) and the Netherlands Aerospace Centre (NLR):

Feasibility study of pulsar navigation:

- Effect of receiver bandwidth, antenna size, etc. on
  - Detection performance
  - Location accuracy
  - Timing accuracy
- Implementation of (very) wideband DA convertor