Who am I?

Stavanger
Kristiansand
Göteborg
Gotland
Kalmar
Oland
TALLIN
ESTONIA
Saint Petersburg
Novgorod
Iaroslavl
Wick
Aberdeen
SCOTLAND
Perth
NORTH
SEA
DENMARK
COPENHAGEN
Malmö
BALTIC SEA
RIGA
LATVIA
Tver
MOSCOW
Riazan
British
Isles
Glasgow
Edinburgh
UNITED
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Newcastle
Belfast
IRELAND
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Cork
Liverpool
Manchester
WALES
ENGLAND
Birmingham
Cardiff
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Plymouth
Southampton
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VILNIUS
Kaluga
Tula
MINSK
BELARUS
Oriol
Gomel
Kursk
Belgorod
Hamburg
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POLAND
WARSAW
Elba
Oder
Vistula
Düsseldorf
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Stuttgart
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Carpathians Mts.
Dnieper
Dnipropetrovsk
Donetsk
Brest
PARIS
Danube
BRATISLAVA
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AUSTRIA
UKRAINE
Rennes
MOLDOVA
Munich
BUDAPEST
CHISINAU
Odessa
Nantes
FRANCE
BERNE
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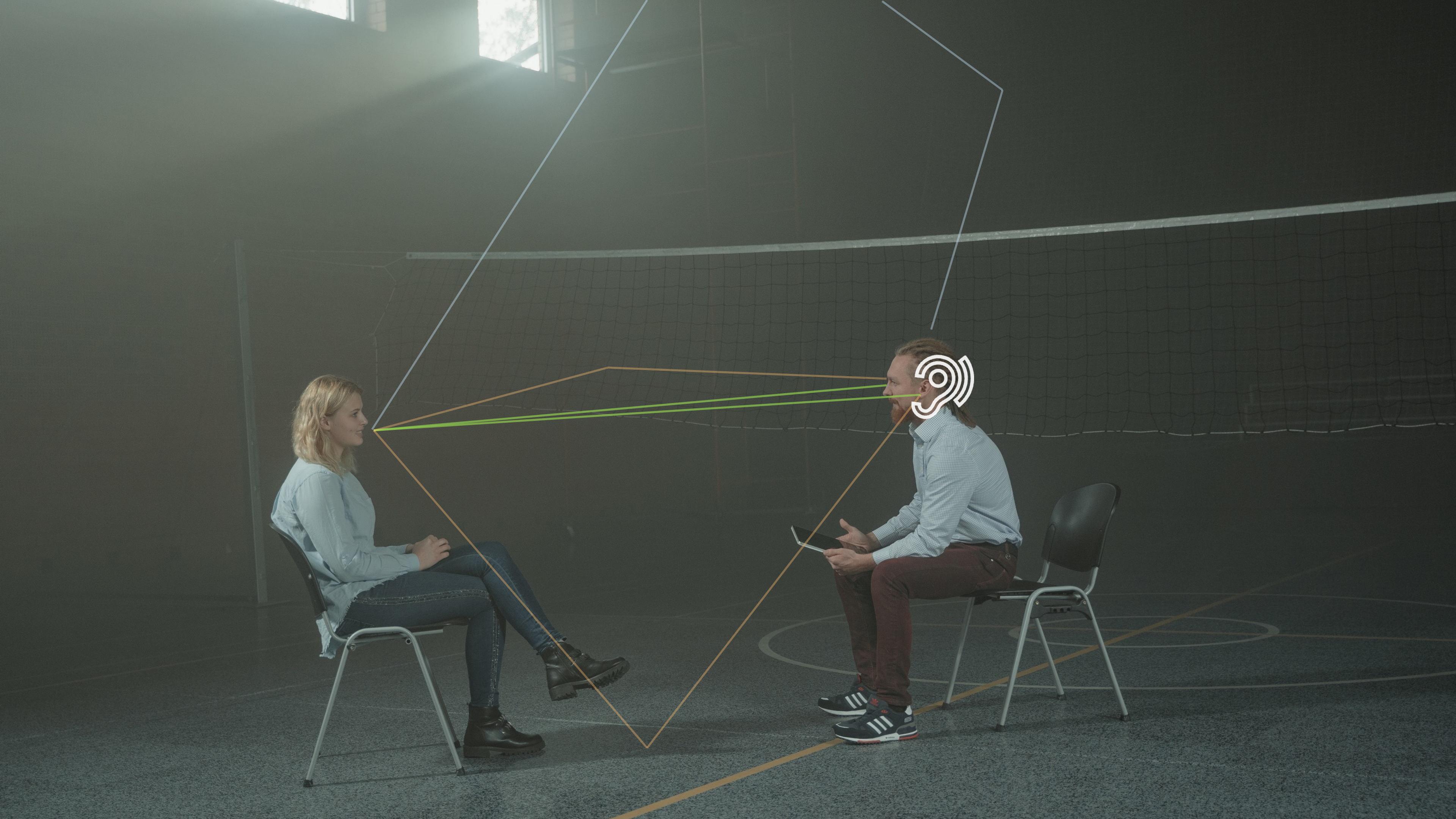
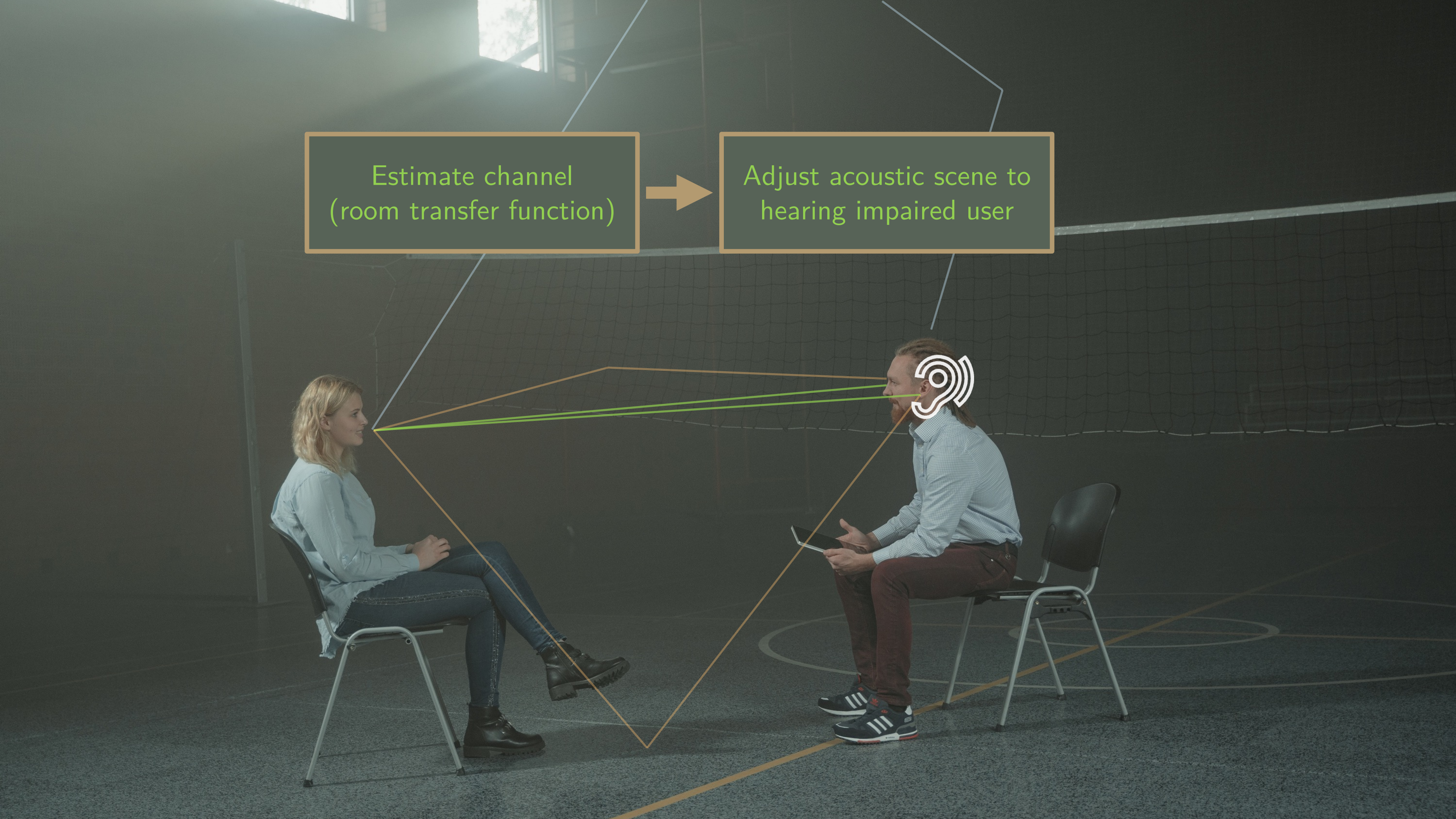
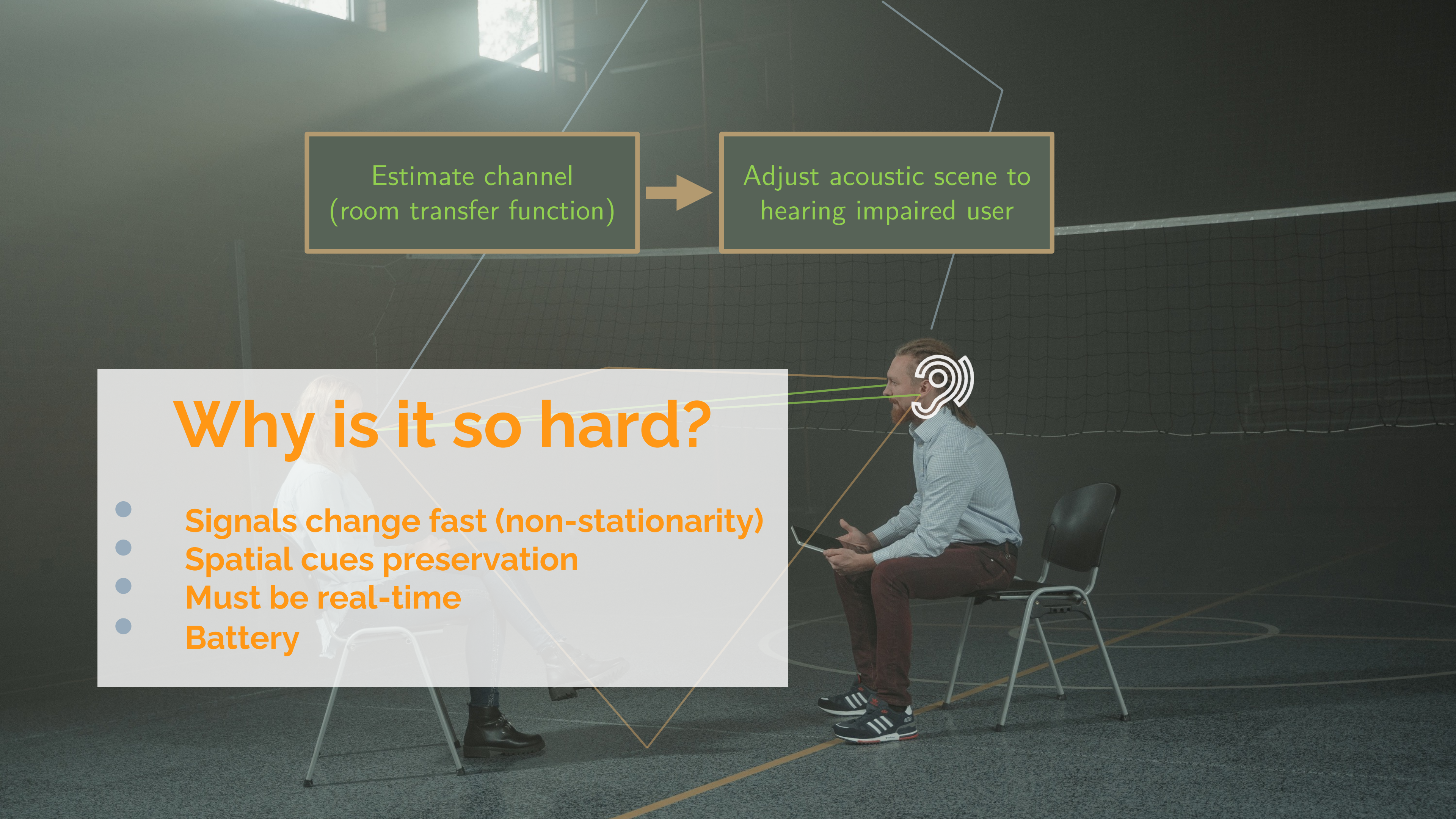
HUNGARY
ROMANIA
SWITZ.
Sea of Azov
SLOVENIA
LJUBLJANA
Lyon
Milano
ZAGREB
Venice
BELGRADE
Bordeaux
Rhone
Po
CROATIA
BUCHAREST
Sevastopol
Florence
BOSNIA & H.
SARAJEVO
SERBIA
Danube
Toulouse
Montpellier
ADRIATIC SEA
Pyrenees
Marseille
MONTENEGRO
SOFIA
BULGARIA
BLACK SEA
Bilbao
Monaco
ITALY
Vigo
AND.
Corsica
ROME
PODGORICA
SKOPJE
FYROM
Bosporus
Zaragoza
Porto
Barcelona
Naples
Bari
TIRANA
ALBANIA
MADRID
Caimbra
Tago
Valencia
Thessaloniki
ANKARA
PORTUGAL
Balearic Is.
TURKEY
SPAIN
Sardinia
LISBON
Izmir
Kayseri
Cordoba
Palermo
Messina
ATHENS
Konya
MEDITERRANEAN SEA
ALGIERS
Sicily
Peloponnesus
Adana
Seville
TUNIS
GREECE
Antalya
Gibraltar(UK)
Oran
Aleppo
Chlef
Constantine
VALETTA
AEGEAN SEA
Ceuta
NICOSIA
Tangier
TUNISIA
MALTA
CYPRUS
SYRIA
Melilla
Crete
RABAT
Fes
ALGERIA
MEDITERRANEAN SEA
LEBANON
BEIRUT
ATLANTIC
OCEAN




Estimate channel
(room transfer function)
Adjust acoustic scene to
hearing impaired user


Estimate channel
(room transfer function)
Adjust acoustic scene to
hearing impaired user
Why is it so hard?
Signals change fast (non-stationarity)
Spatial cues preservation
Must be real-time
Battery

https://images.squarespace-cdn.com/content/v1/54d696e5e4b05ca7b54cff5c/1625158518155-U2HP9YODRMV1I3T83M4J/Higher+Order+Ambisonics+Microphones.jpg

Cramér-Rao bound for acoustic transfer function estimation

Giovanni Bologni (G.Bologni@tudelft.nl), Richard Hendriks, Richard Heusdens

18 December, 2023

TU Delft

On the menu today

~~**Intro**~~

Cramér-Rao bound for acoustic transfer function estimation

1. Parameter estimation & Cramér-Rao bound (CRB)
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Parameter estimation

A quantity θ needs to be estimated

Given a (random) model, what is best achievable accuracy on estimating unknown quantity?

Parameter estimation

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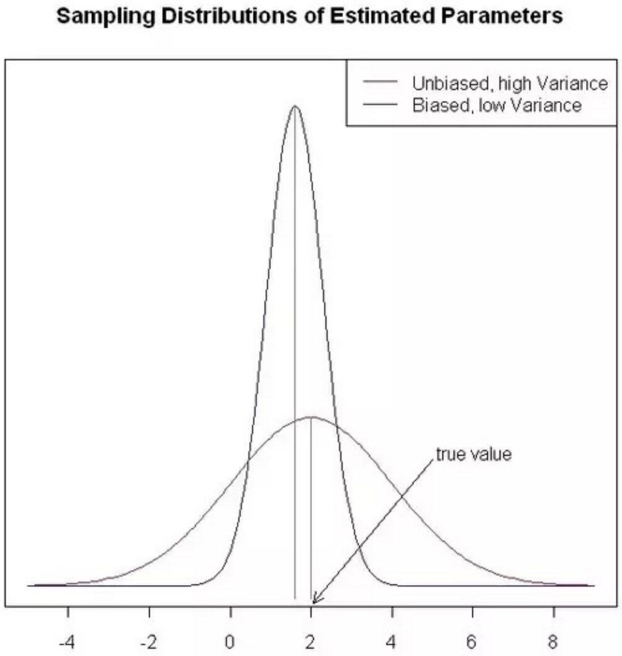
Given a (random) model, what is best achievable accuracy on estimating unknown quantity?

Best accuracy = minimum MSE = minimum variance (unbiased estimator)

$$\text{MSE}(\theta, \hat{\theta}) = \text{var}(\hat{\theta}) + \text{bias}^2(\hat{\theta}, \theta).$$



Pic: quora.com

Cramér-Rao bound

Under regularity assumptions on probability distribution $p(x;\theta)$,

$$\mathrm{var}(\hat{\theta}) \geq I^{-1}(\theta) = \frac{1}{-\mathbf{E}\left[\frac{\partial^2 \ln p(x;\theta)}{\partial \theta^2}\right]}.$$

Pic: gaussianwaves.com

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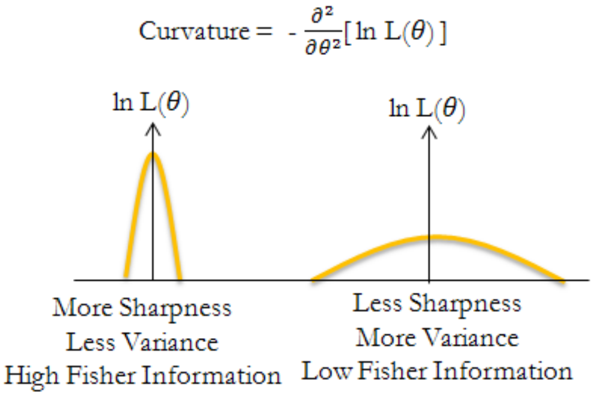
If PDF $\mathrm{p}(x;\theta)$ is influenced by parameter more, estimation will be more accurate



Pic: gaussianwaves.com

Deterministic function of parameter

Suppose we want to estimate a function $g(\theta)$ of the parameter

Example *measure parameter in noise, but instantaneous power desired:*

$$g(\theta) = \theta^2$$

In this case,

$$\text{var}(\hat{\theta}) \geq \frac{\left(\frac{\partial g}{\partial \theta}\right)^2}{-\mathbf{E}\left[\frac{\partial^2 \ln p(x;\theta)}{\partial \theta^2}\right]}.$$

Cramér-Rao bound – multiple parameters

$\text{var}(\hat{\theta}) \geq I^{-1}(\theta) = \dfrac{1}{-\mathbf{E}\left[\dfrac{\partial^2 \ln p(x;\theta)}{\partial \theta^2}\right]}.$

Cramér-Rao bound – multiple parameters

The Fisher information matrix (FIM) is the negative expected Hessian of the log-likelihood function:

$$\mathbf{I}_\theta = -\mathbf{E}\big[\nabla_\theta \, \nabla_\theta^H \ln p(\mathbf{x})\big] = -\mathbf{E}\big[\nabla_\theta^2 \ln p(\mathbf{x})\big], \tag{1}$$

where the expectation is taken with respect to $p(\mathbf{x})$ and

$$[\nabla_\theta f]_i = \partial f / \partial \theta_i, \qquad [\nabla_\theta^2 f]_{ij} = \partial f / \partial \theta_i \partial \theta_j^*.$$

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The covariance matrix $\mathbf{R}_{\hat{\theta}}$ of any unbiased estimator $\hat{\theta}$ of θ satisfies $\mathbf{R}_{\hat{\theta}} \succeq \mathbf{I}_\theta^{-1}$.

$\mathbf{A} \succeq \mathbf{B}$ means $\mathbf{A} - \mathbf{B}$ is positive semidefinite with $\mathbf{A}$ and $\mathbf{B}$ Hermitian: $\mathbf{A} = \mathbf{A}^H$, $\mathbf{B} = \mathbf{B}^H$.

CRB – complex parameters

CRB described until now holds for real parameters.
How to extend to complex parameters z?

Two equivalent approaches:

- Consider real $Re(z)$ and imaginary part $Im(z)$ separately (cumbersome)
- Consider complex number z and its conjugate z^* (also cumbersome ☺ but generally easier)

Bos, A. van den. "A Cramer-Rao Lower Bound for Complex Parameters.", 1994
Brandwood, D. H. "A Complex Gradient Operator and Its Application in Adaptive Array Theory.", 1983

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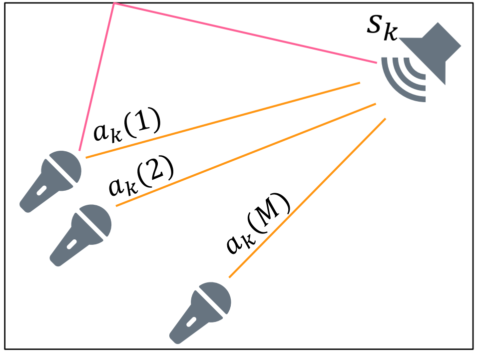
Case study – channel estimation

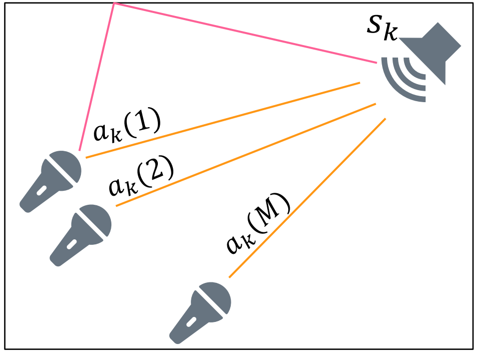


Signal model – frequency domain

Let a point source emit sound. The sound is measured by an array of M sensors. The received signal in the short-time Fourier transform (STFT) domain is

$$\mathbf{x}_k(l) = s_k(l)\,\mathbf{a}_k + \mathbf{v}_k(l) \in \mathbb{C}^M, \quad l = 1, \ldots, L, \; k = 1, \ldots, K$$

Our goal: recover transfer function $\mathbf{a}_k$ from noisy recording $\mathbf{x}_k$



Deriving CRB for channel estimation

$$\mathsf{var}(\hat{\theta}) \geq I^{-1}(\theta) = \frac{1}{-\mathbf{E}\left[\dfrac{\partial^2 \ln p(x;\theta)}{\partial \theta^2}\right]}.$$

$$\mathbf{R}_{\hat{\theta}} \succeq \mathbf{I}_{\theta}^{-1} = \left(-\mathbf{E}\left[\nabla_{\theta}^{2} \ln p(\mathbf{x})\right]\right)^{-1}.$$

Likelihood function

Collect IID measurements in data matrix $\boldsymbol{X}$

Assume noise $\boldsymbol{v}$ is complex circular Gaussian process

Unknown parameters $\theta = [\mathbf{a}^T \mathbf{a}^H]^T \in \mathbb{C}^{2M}$

Conditional likelihood is

$$p(\mathbf{X}; \theta, s(l)) = \frac{1}{|\pi \mathbf{R}|^L} \exp\left(-\sum_{l=1}^{L} (\mathbf{x}(l) - s(l)\mathbf{a})^H \mathbf{R}^{-1} (\mathbf{x}(l) - s(l)\mathbf{a})\right),$$

Stoica, P., and A. Nehorai. "Performance Study of Conditional and Unconditional Direction-of-Arrival Estimation." 1990

Log-likelihood and its derivatives

Define log-likelihood

$$L(\theta) = \ln p(\mathbf{X};\theta) = -L\ln|\pi\mathbf{R}| - \sum_{l=1}^{L}(\mathbf{x}(l) - s(l)\mathbf{a})^H\mathbf{R}^{-1}(\mathbf{x}(l) - s(l)\mathbf{a})$$

We then have

$$\nabla_{\mathbf{a}^*}L(\theta) = \mathbf{R}^{-1}\sum_{l=1}^{L}(s(l)^*\mathbf{x}(l) - |s(l)|^2\mathbf{a})$$

$$\nabla_{\mathbf{a}}L(\theta) = (\nabla_{\mathbf{a}^*}L(\theta))^*$$

$$-\mathbf{E}\big[\nabla_{\mathbf{a}^*}\nabla_{\mathbf{a}^*}^H L(\theta)\big] = -\mathbf{E}\big[\nabla_{\mathbf{a}^*}\nabla_{\mathbf{a}}^T L(\theta)\big] = \mathbf{E}\left[\mathbf{R}^{-1}\sum_{l=1}^{L}|s(l)|^2\right] = E_s L\mathbf{R}^{-1}$$

$$-\mathbf{E}\big[\nabla_{\mathbf{a}^*}\nabla_{\mathbf{a}}^H L(\theta)\big] = \mathbf{0}$$

$$E_s = \mathbf{E}\big[|s(l)|^2\big]$$

The Fisher information matrix (FIM) is the negative expected Hessian of the log-likelihood function:

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Fisher information matrix

With this, the Fisher information matrix is

$$\mathbf{I}_\theta = \begin{bmatrix} E_s L\mathbf{R}^{-*} & \mathbf{0} \\ \mathbf{0} & E_s L\mathbf{R}^{-1} \end{bmatrix}. \quad (1)$$

The block-diagonal matrix can be easily be inverted, leading to

$$\mathbf{I}_\theta^{-1} = \begin{bmatrix} \frac{1}{E_s L}\mathbf{R}^* & \mathbf{0} \\ \mathbf{0} & \frac{1}{E_s L}\mathbf{R} \end{bmatrix} \quad (2)$$

Variance is finally bounded as:

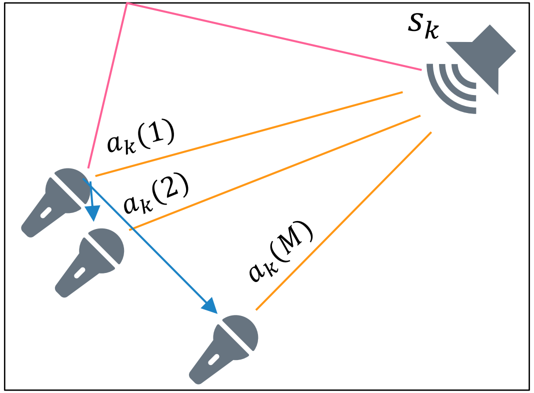
$$\text{var}(\hat{a}_i) \geq \frac{[R]_{ii}}{E_s L}, \quad i = 1, \ldots, M. \quad (3)$$

Deterministic function of parameter

ATFs are often estimated in relation to a reference microphone r, as in $g(\theta) = g(\mathbf{a}, \mathbf{a}^*) = \mathbf{a}/a_r$. In this case,

$$\mathbf{R}_{g(\theta)} - (\nabla_\theta \mathbf{g})\mathbf{I}_\theta^{-1}(\nabla_\theta^H \mathbf{g}) \geq 0, \tag{1}$$

where $\mathbf{R}_{g(\theta)}$ is the covariance matrix of $g(\theta)$.



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where $\mathbf{R}_{g(\theta)}$ is the covariance matrix of $g(\theta)$. Choosing $r = 1$, Jacobian is

$$\nabla_\theta \mathbf{g} = \begin{bmatrix} \nabla_{\mathbf{a}} \mathbf{g} & \nabla_{\mathbf{a}^*} \mathbf{g} \end{bmatrix} \tag{2}$$

$$= \left[\begin{array}{ccccc|c} 0 & 0 & 0 & & 0 & \\ -\theta_2/|\theta_1|^2 & 1/\theta_1 & 0 & \dots & 0 & \\ -\theta_3/|\theta_1|^2 & 0 & 1/\theta_1 & \dots & 0 & \mathbf{0}_{M\times M} \\ \vdots & & & \ddots & \vdots & \\ -\theta_M/|\theta_1|^2 & 0 & \dots & 0 & 1/\theta_1 & \end{array}\right], \tag{3}$$

where $[\nabla_\theta \mathbf{f}]_{ij} = \partial f_i/\partial \theta_j$.

Experiments

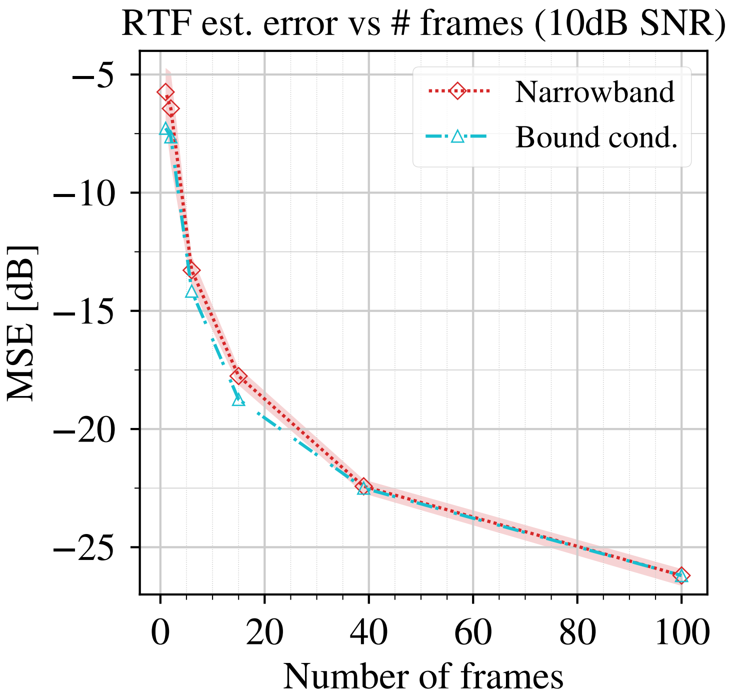
Settings

- 3 microphones,
  random ATF,
  covariance matrices estimated from N snapshots,
  results averaged over 1000 experiments

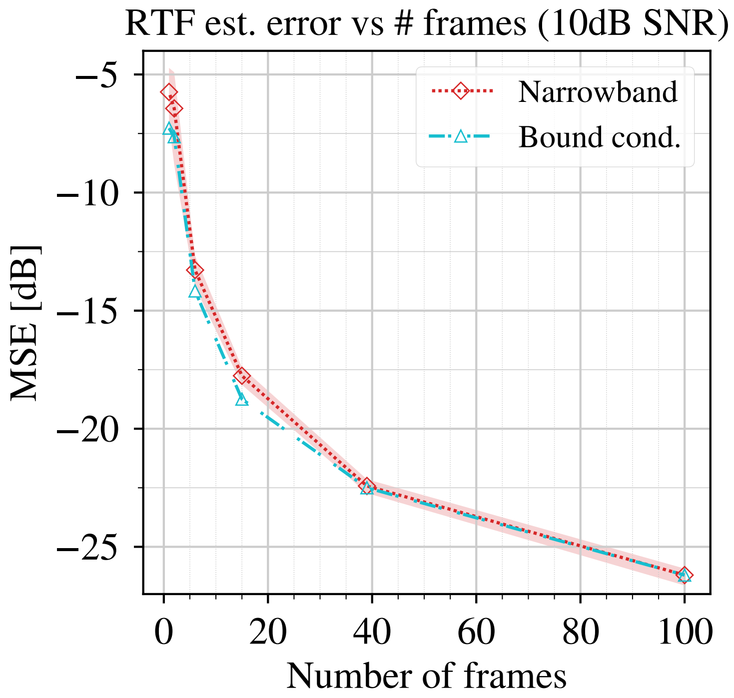
Performance metric

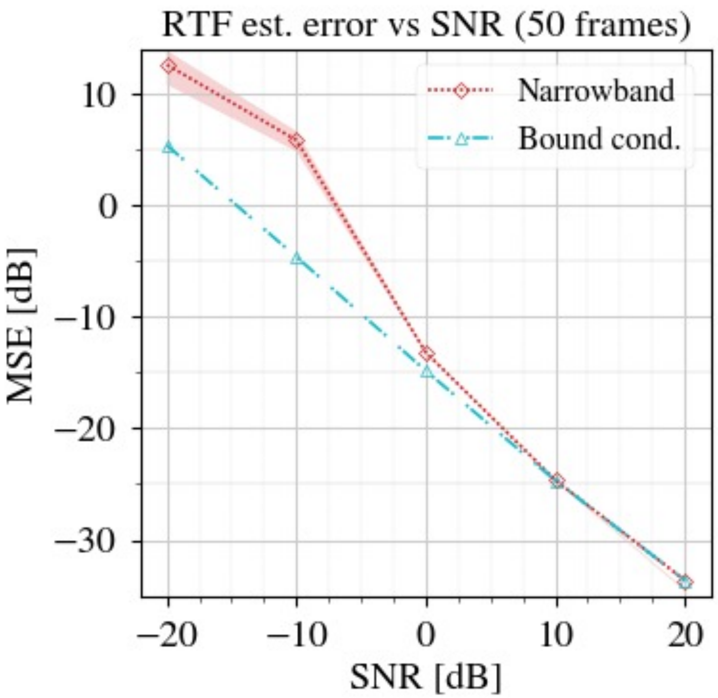
- Mean-squared error (in dB) between actual and estimated RTFs

Experiments



Experiments





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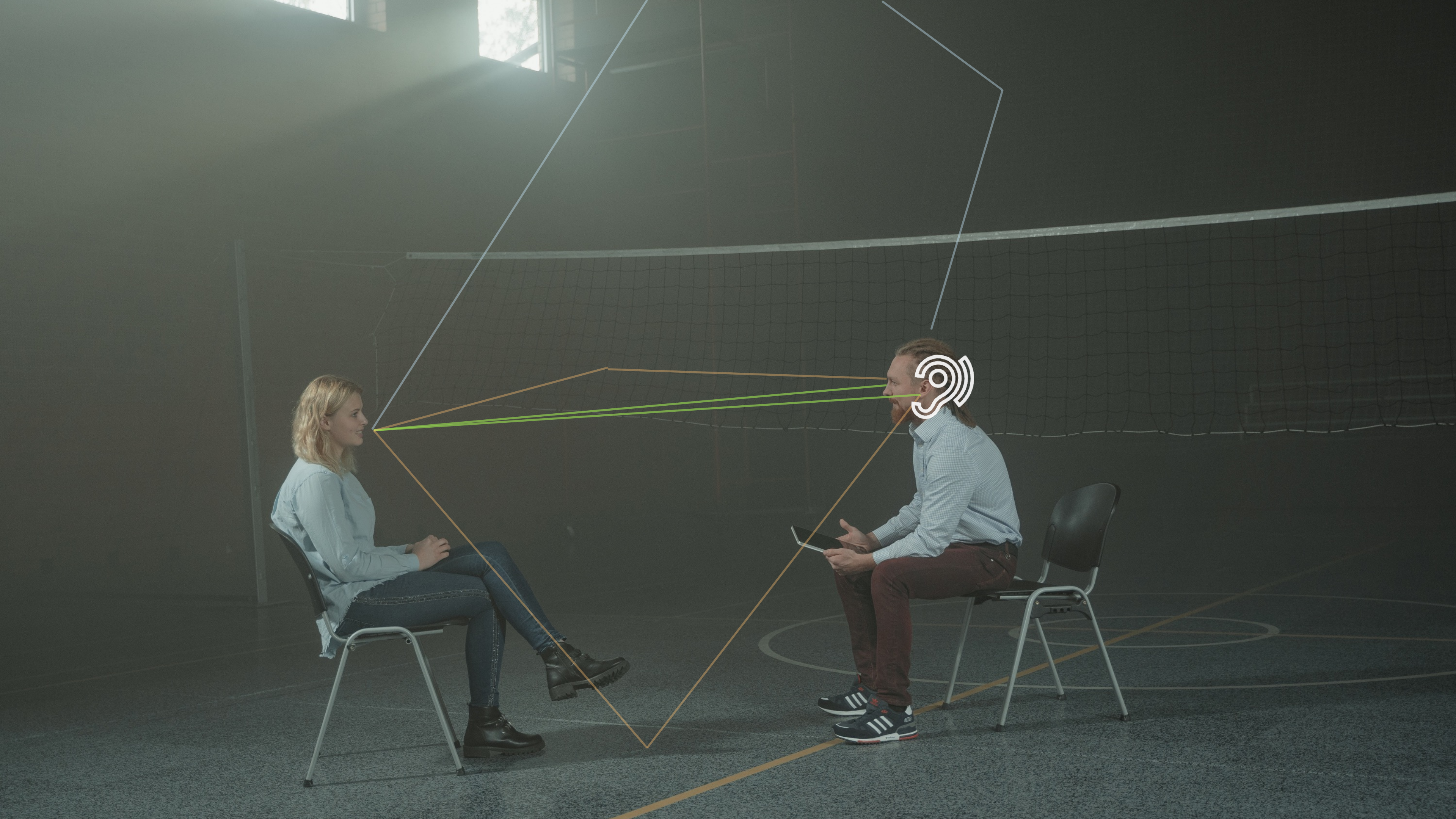
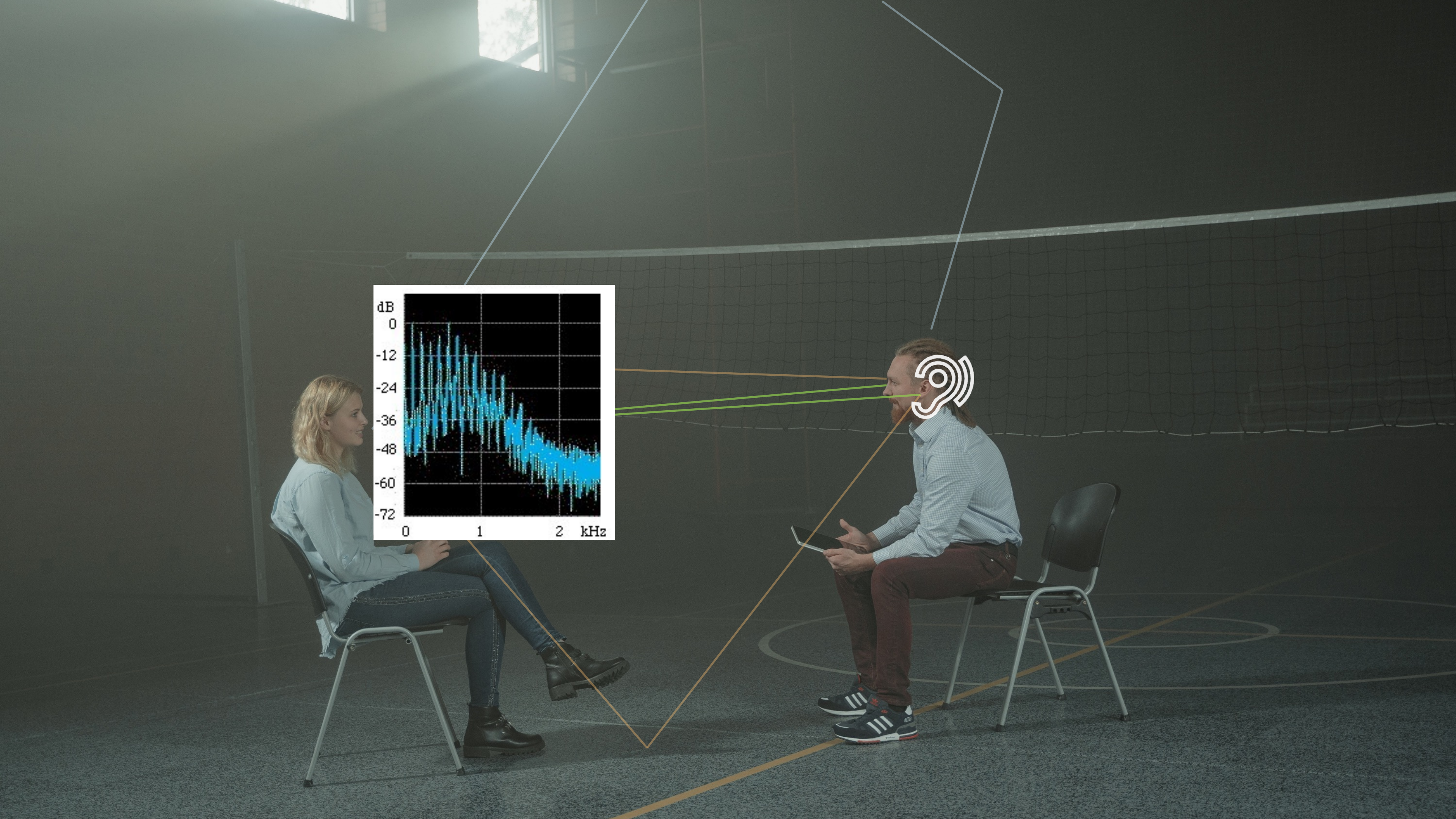
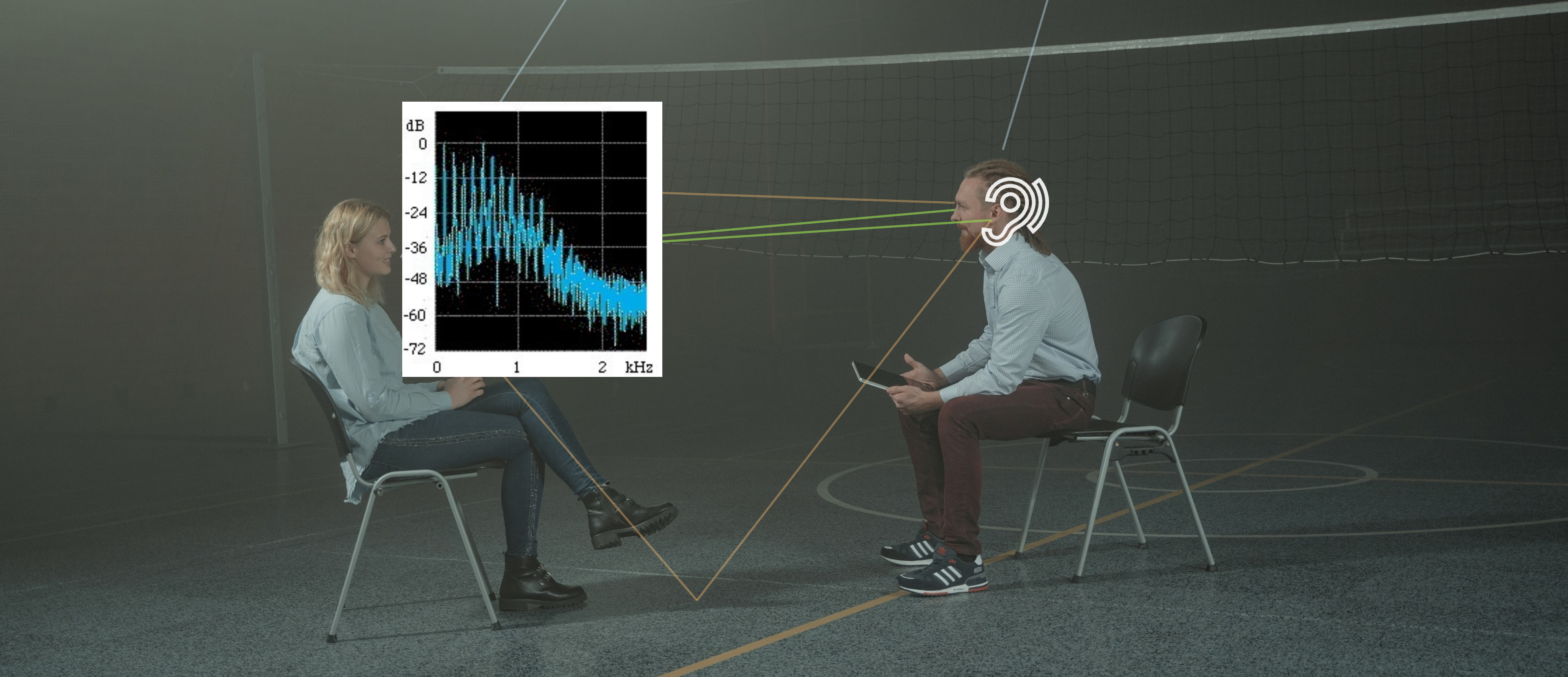



dB
0
-12
-24
-36
-48
-60
-72
0
1
2
kHz

Can acoustic parameter estimation be improved by exploiting “hidden” correlations across frequencies?



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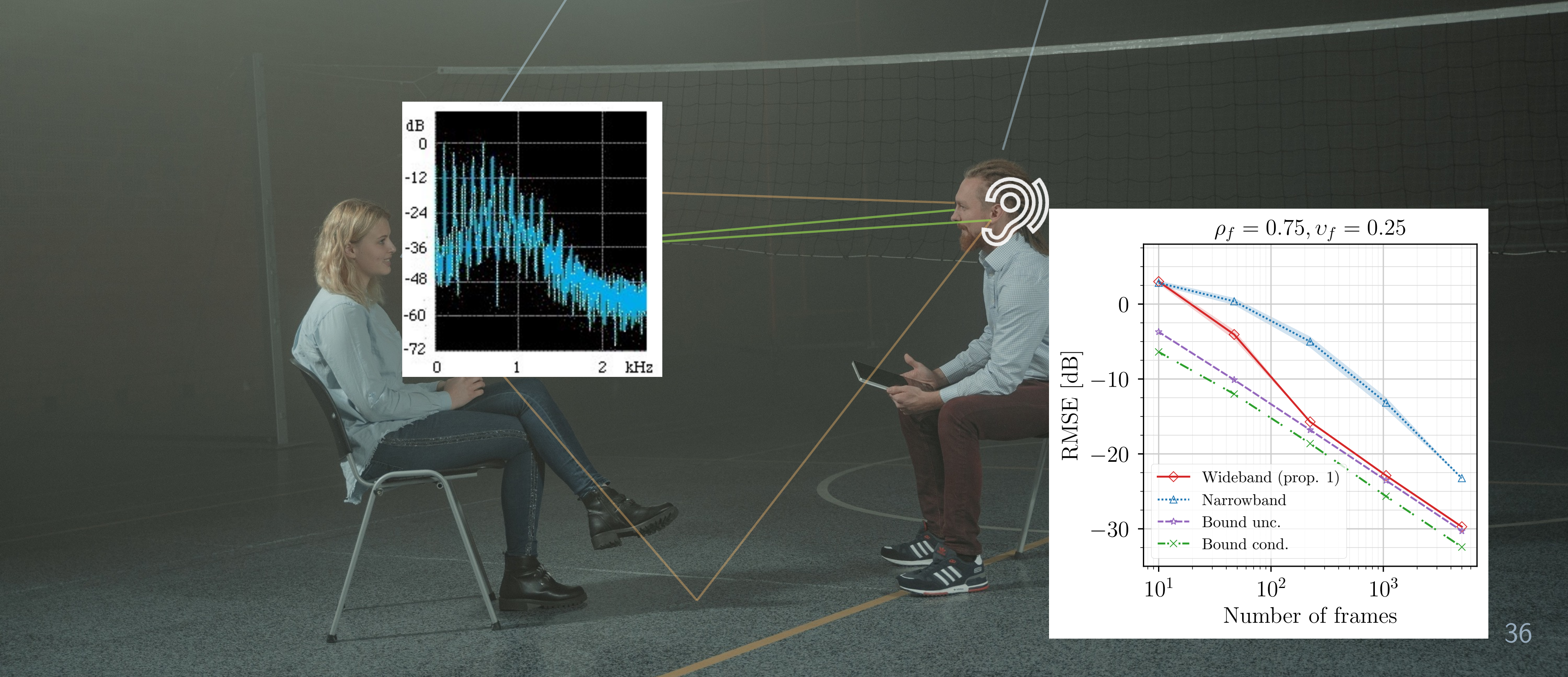
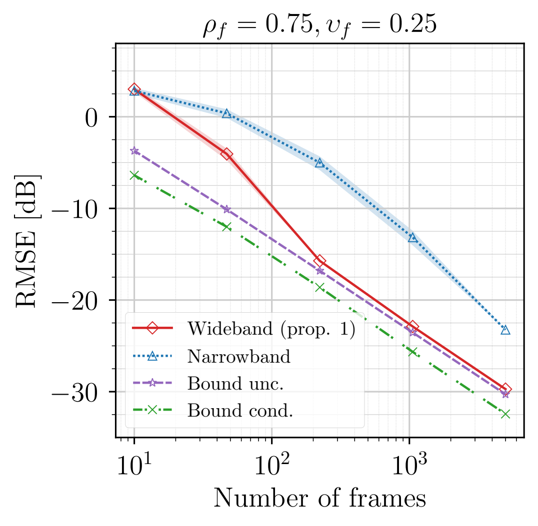


Open questions - theses projects

- How to find optimal RTF estimation algorithm?
  - Maximum likelihood [1], tensor methods
- *Wideband* beamforming?
  - *Wideband* MVDR with cue preservation
  - ATF informed DNN [2]
- How to estimate spectral correlation covariance?
  - Cyclostationary model [3]



[1] Y. Laufer and S. Gannot, "Scoring-Based ML Estimation and CRBs for Reverberation, Speech, and Noise PSDs in a Spatially Homogeneous Noise Field," TALSP 2020,

[2] A. Briegleb, M. M. Halimeh, and W. Kellermann, "Exploiting Spatial Information with the Informed Complex-Valued Spatial Autoencoder for Target Speaker Extraction," ICASSP 2023

[3] J. Antoni, "Cyclostationarity by examples," Mechanical Systems and Signal Processing 2009

Recap

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