Estimation and Detection

Lecture 1: Introduction

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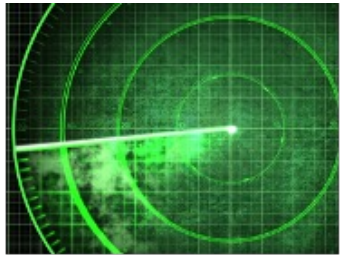
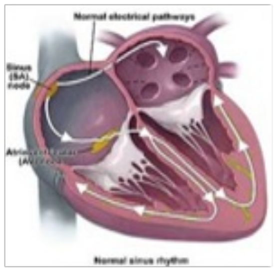
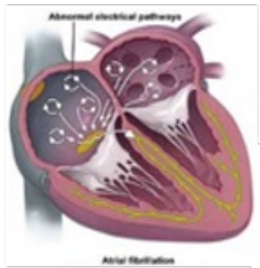
Introduction – Why Estimation & Detection

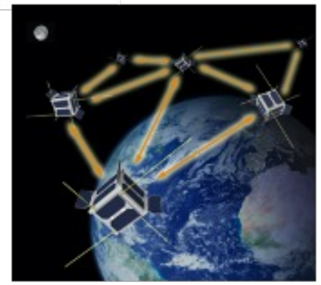
Many signal processing applications involve estimation and/or detection tasks, e.g.:

- Hearing aid: Estimation of target speech in a noisy environment.
- localisation of sensors or sources in a wireless sensor network.
- Estimation/detection of biomedical signals.
- Radar: Detection of a target.
- Car navigation.
- Etc.





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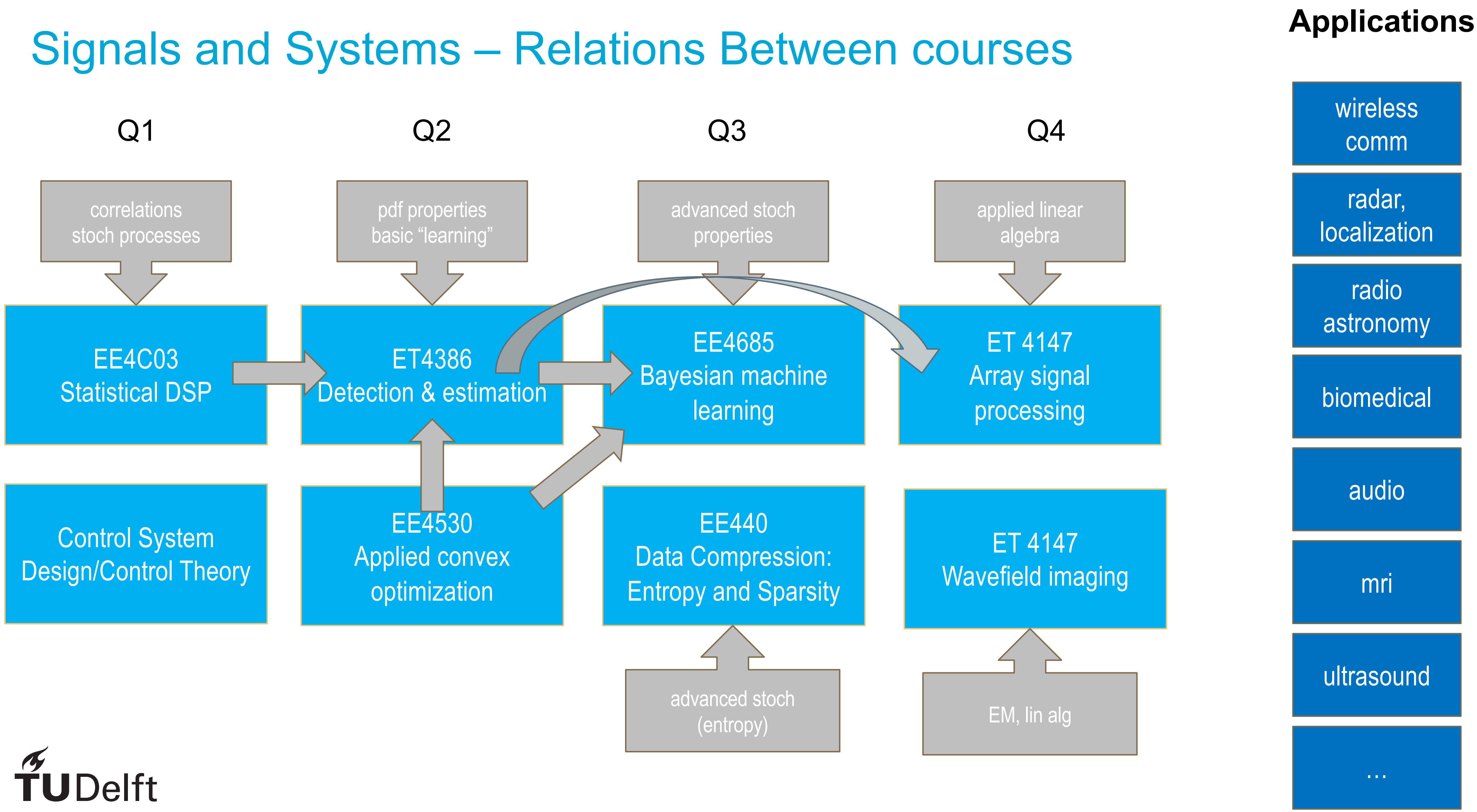
Signals and Systems – Relations Between courses



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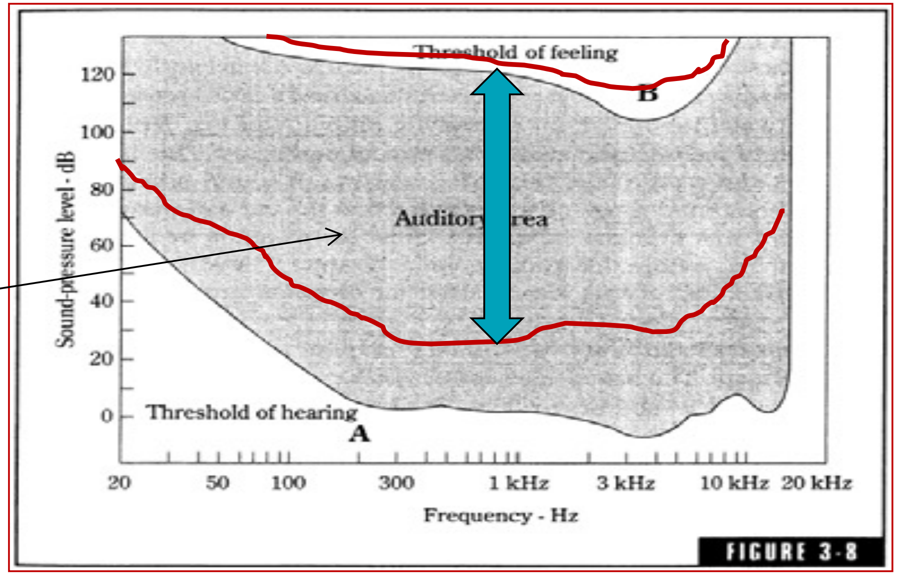
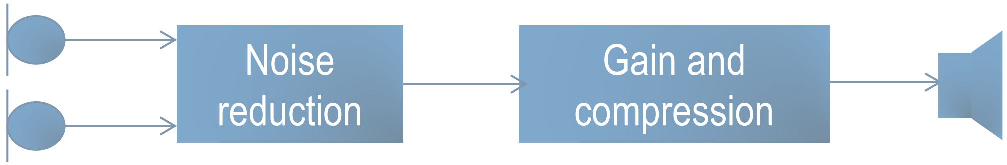
Example: Estimation for Hearing Aids



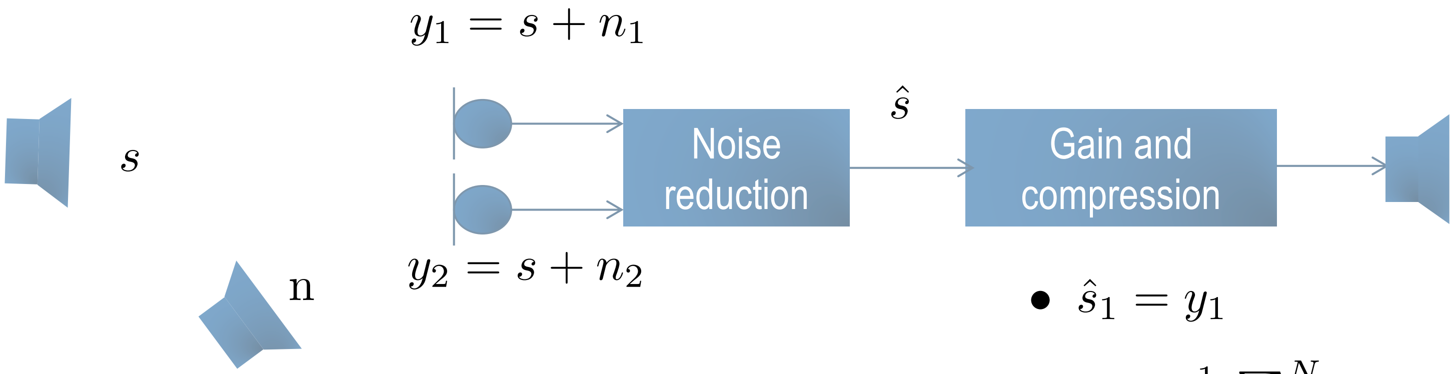
- Sensitivity
- Dynamic range



- Due to the gain, inaudible noise/disturbances can suddenly become audible.
- Hearing impaired people have a worse spectral and temporal resolution, and therefore more difficulties to understand a target under noisy conditions.

Example: Estimation

A simplified model:



- $\hat{s}_1 = y_1$
- $\hat{s}_2 = \frac{1}{N}\sum_{i=1}^{N} y_i$
- How good are these estimators?
- Are there better estimators?

How to determine an estimate $\hat{s}$?

What about $\hat{s}_3 = \frac{\sum_{i=1}^{N} \frac{y_i}{\sigma_{n_i}^2}}{\sum_{i=1}^{N} \frac{1}{\sigma_{n_i}^2}}$?

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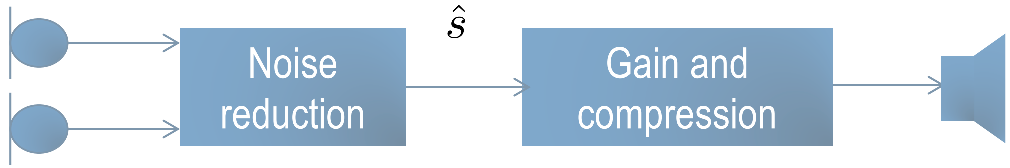
Example: Estimation

$y_1 = s + n_1$



$y_2 = s + n_2$

let's assume

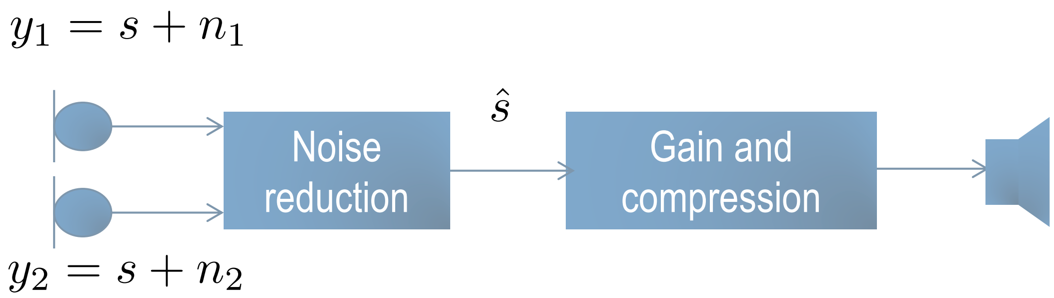
- s is deterministic
- Noise n_i at every microphone is a zero-mean iid random process.

- $\hat{s}_1 = y_1$
- $\hat{s}_2 = \frac{1}{N}\sum_{i=1}^{N} y_i$
- $\hat{s}_3 = \frac{\sum_{i=1}^{N} \frac{y_i}{\sigma_{n_i}^2}}{\sum_{i=1}^{N} \frac{1}{\sigma_{n_i}^2}}$

$Var[\hat{s}]$?

- $Var[\hat{s}_1] = \sigma_{n_i}^2$ If indeed iid $\Rightarrow Var[\hat{s}_1] = \sigma_n^2$
- $Var[\hat{s}_2] = Var[\frac{1}{N}\sum_{i=1}^{N} y_i] = \frac{1}{N^2}\sum \sigma_{n_i}^2$. If indeed iid $\Rightarrow Var[\hat{s}_2] = \frac{\sigma_n^2}{N}$
- $Var[\hat{s}_3] = Var\left[\frac{\sum_{i=1}^{N} \frac{y_i}{\sigma_{n_i}^2}}{\sum_{i=1}^{N} \frac{1}{\sigma_{n_i}^2}}\right] = \frac{1}{\sum \frac{1}{\sigma_{n_i}^2}}$. If indeed iid $\Rightarrow Var[\hat{s}_3] = \frac{\sigma_n^2}{N}$

Example: Estimation



What if the noise is not iid, but only independent?

- $\sigma^2_{n_1} = \sigma^2_n$ and $\sigma^2_{n_2} = 2\sigma^2_n$

$Var[\hat{s}]$?

- $Var[\hat{s}_1] = \sigma^2_n$
- $Var[\hat{s}_2] = Var[\frac{1}{N}\sum_{i=1}^N y_i] = \frac{3\sigma^2_n}{4}$.
- $Var[\hat{s}_3] = Var\left[\frac{\sum_{i=1}^N \frac{y_i}{\sigma^2_{n_i}}}{\sum_{i=1}^N \frac{1}{\sigma^2_{n_i}}}\right] = \frac{2\sigma^2_n}{3}$.

Questions:

- How to derive good (optimal) estimators?
- How to quantify their performance?
- What is the best estimator for a certain estimation task?

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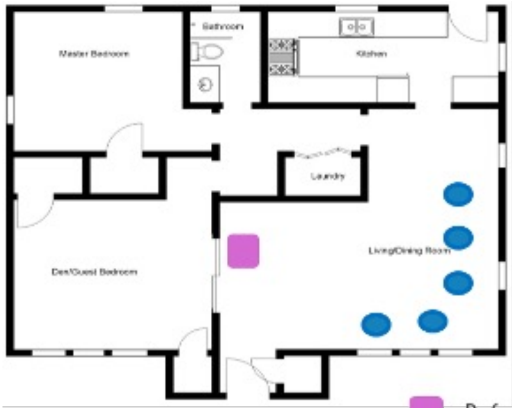
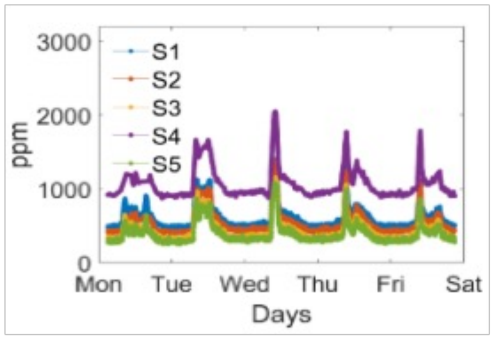
Example: Sensor Calibration

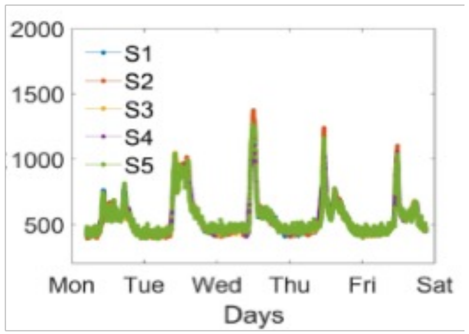


Reference Sensor

CO2 sensor



Raw Sensor data

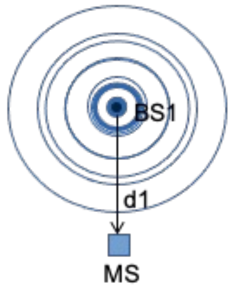


Ref. based calibration

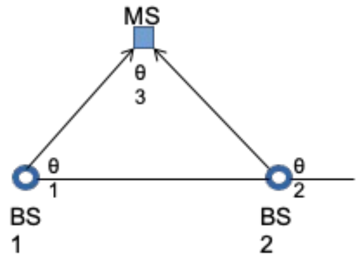
How can we *accurately* estimate the gain and offset of air-quality sensors ?

Example: Navigation

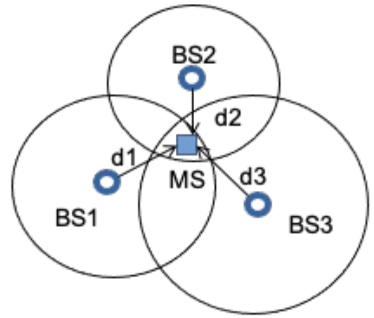
- How can we estimate the location of mobile stations (MS) ?
- Measurements: RSS, distances, angles, . . .
- Applications: GPS-based, Indoor, Underwater, Satellite swarms, Drones



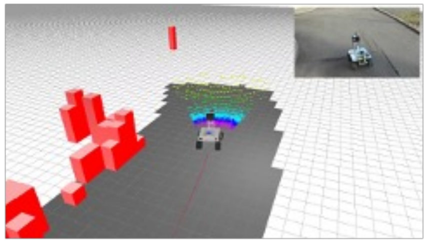
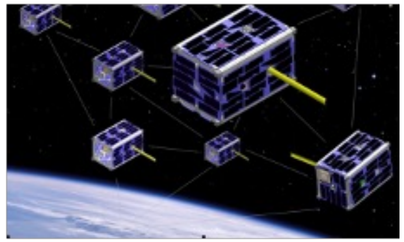
Received Signal Strength
(RSS)

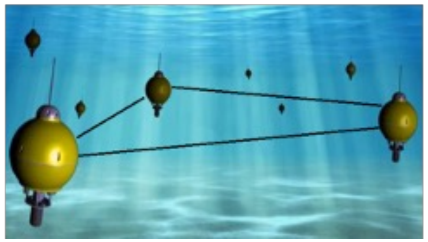


Angle of Arrival
(AoA)



Time of Arrival
(ToA)

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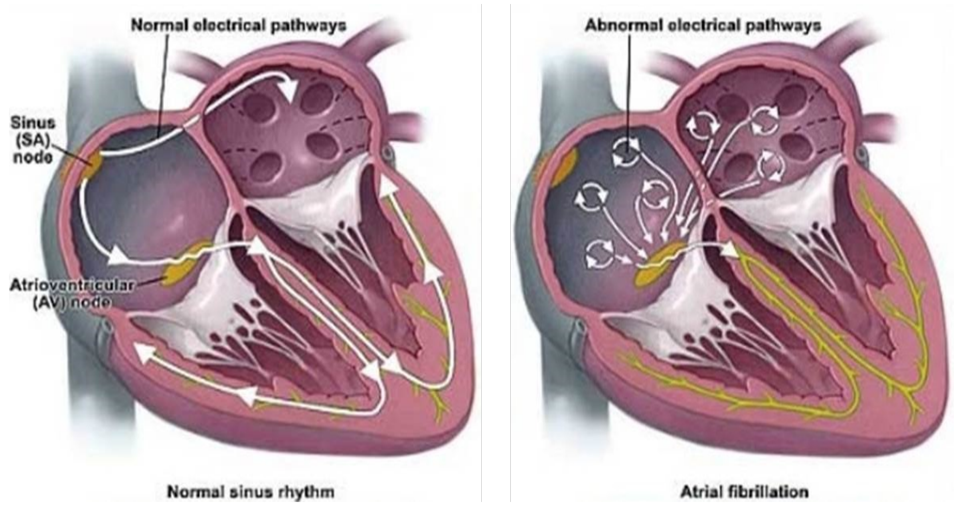
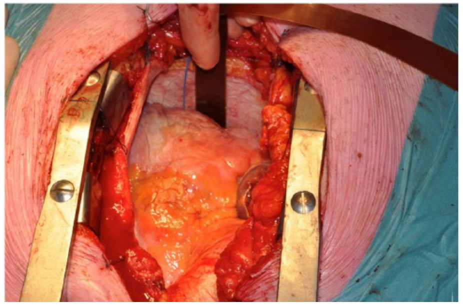
Example: Biomedical Signal Processing – Atrial Fibriallation

- Atrial fibrillation (AF) is one of the most common age related cardiac arrhythmia
- AF is characterized by rapid and irregular electrical activity of the atria.
- AF is rooted in impaired electrical conduction known as electropathology.



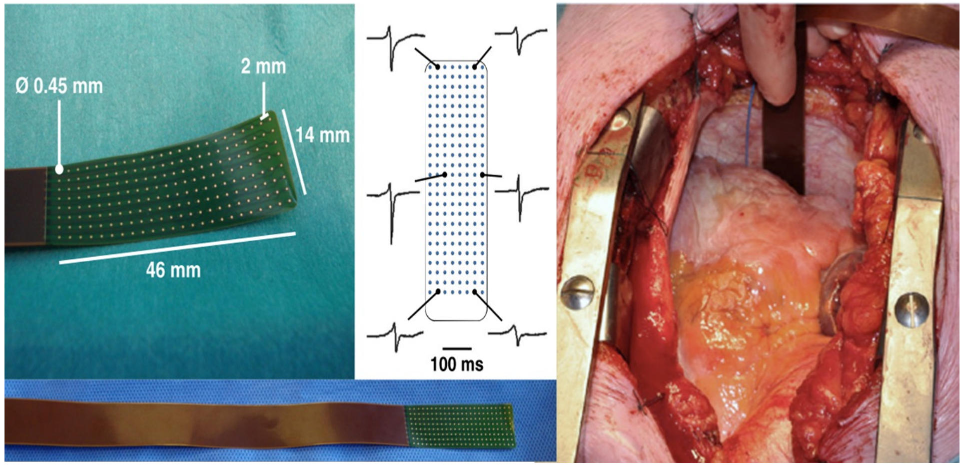
Example: Biomedical Signal Processing – Atrial Fibriallation



- Atrial fibrillation
- Common treatment: Ablation. Only successful in 50 % of cases. . .
- Research project on atrial fibrillation:
  - Measure signals directly on the heart using a high dimensional sensor (188).
  - Develop model that describes how action potentials generated by cells is measured by electrodes.
  - Estimate underlying cell characteristics.

Atrial Electrograms (EGMs)

An electrogram is a record of changes in the electrical potential of the (many) cells in the neighborhood of an electrode that is positioned on the heart surface.



To understand which (group of) cells are problematic, we would like to *detect* when each cell is being activated and *estimate* the underlying conductivity of the cell.

Modeling the Electrogram

$$\mathbf{\Phi} = k\,\mathbf{R}\,\mathbf{D}_{\boldsymbol{\sigma}}\,\mathbf{V},$$

- $\mathbf{\Phi}_{M\times T}$: electrogram array
- $\mathbf{R}_{M\times N}$: matrix containing the inverse cell distances from the electrodes
- $\mathbf{D}_{\boldsymbol{\sigma}} = \mathbf{D}_x \mathrm{diag}(\boldsymbol{\sigma}_{xx})\mathbf{D}_x + \mathbf{D}_x \mathrm{diag}(\boldsymbol{\sigma}_{xy})\mathbf{D}_y + \mathbf{D}_y \mathrm{diag}(\boldsymbol{\sigma}_{yx})\mathbf{D}_x + \mathbf{D}_y \mathrm{diag}(\boldsymbol{\sigma}_{yy})\mathbf{D}_y$
- $\mathbf{V}_{N\times T}$: matrix containing all per cell potentials

Physical modelling from signal processing perspective to be able to formulate an estimation problem.

Compact Matrix Model:

$$\mathrm{vect}(\mathbf{\Phi}) = \left(\mathbf{V^T}_{\boldsymbol{\tau}} \otimes (k\,\mathbf{R})\right)\mathrm{vect}(\mathbf{D}) = \mathbf{M}_{\boldsymbol{\tau}}\boldsymbol{\sigma} \qquad (1)$$

$$\hat{\boldsymbol{\sigma}} = \arg\min_{\boldsymbol{\sigma}} \|\phi - \mathbf{M}_{\boldsymbol{\tau}}\boldsymbol{\sigma}\|_2^2$$

Formulated estimation problem

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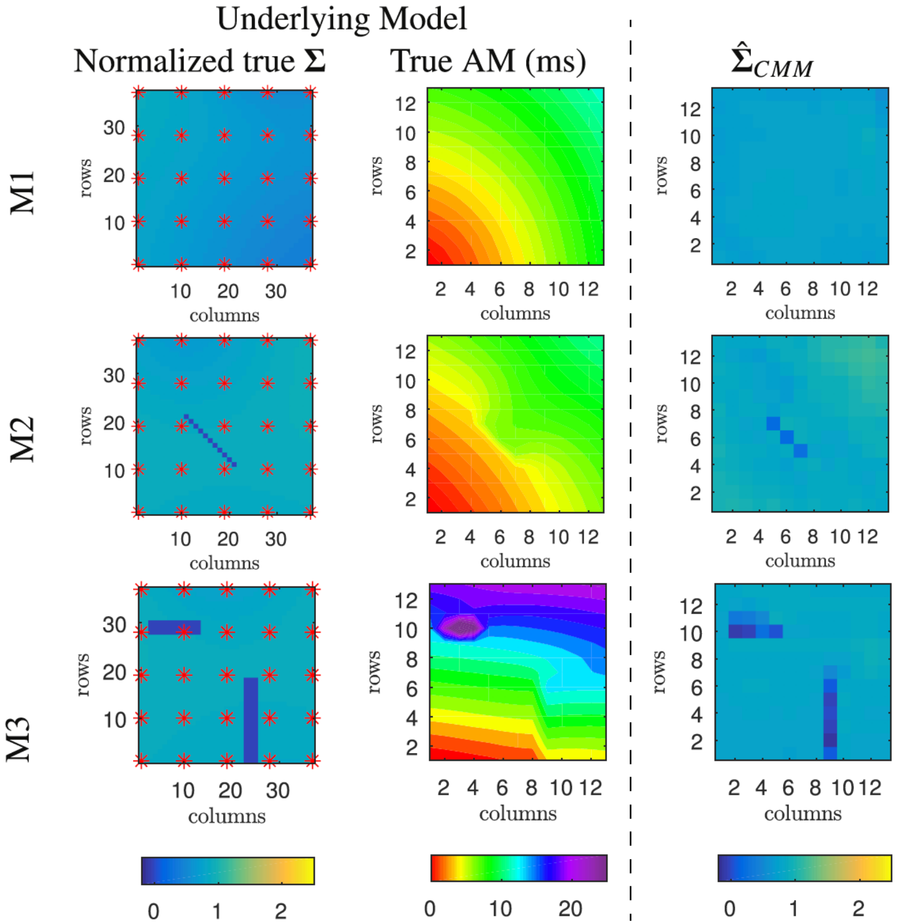
Simulation on Conductivity Estimation



This Course

Theory on estimation and detection:

- Generally, the (optimal) solution to estimation and detection problems depends on the underlying signal model.
- Data is often random: Random noise and sometimes random target as well. Models are therefore often based on statistical descriptions.

Key Questions:

- How to determine optimal estimators.
- What is the estimation bound?
- How to determine the optimal detector?
- How to express the detection performance?

This Course – Estimation Theory

An example problem formulation:
Consider the estimation of θ from observations

$$y[n] = \theta + w[n]$$

where the noise $w[n]$ is typically (assumed to be) stochastic. However, the unknown parameter θ, can be

- a deterministic quantity $\Rightarrow$ Classical estimation (lectures 1 - 5), or
- a random quantity $\Rightarrow$ Bayesian estimation (lectures 6 - 8).

This Course – Detection Theory

An example problem formulation:
Determine whether a certain signal that is embedded in noise is present or not.

$$\begin{array}{ll} \mathcal{H}_0 & x[n] = w[n] \\ \mathcal{H}_1 & x[n] = s[n] + w[n] \end{array}$$

Typical detector:

$$T(x[n]) > \gamma$$

- How to make an optimal decision?i.e., how to chose γ ?
- Detection for deterministic signals (lecture 9)
- Detection for random signals (lecture 10)
- Detection with unknown parameters (lecture 11)

Course Information - 1

- Course website: https://cas.tudelft.nl/Education/courses/et4386/index.php
- Brightspace
- Exam:
  - Written
  - closed book, with one 2-sided HANDWRITTEN A4 formula sheet.
  - Mini-project counts for 20 % of the final grade.

Course Information - 2

- Books:
  - Fundamentals of Statistical Signal Processing, Volume I: Estimation Theory; S.M. Kay, Prentice Hall 1993; ISBN-13: 978-0133457117.
  - Fundamentals of Statistical Signal Processing, Volume II: Detection Theory; S.M. Kay, Prentice 1993; ISBN-13: 978-0135041352.
- Instructors:
  - Richard C. Hendriks
  - Raj Rajan
- Contact: et4386-EWI@TuDelft.nl

Course Information – Mini Projects

- Grade for the project counts for 20 % of the final grade.
- Can be carried out in pairs of students.
- There will be several projects to choose from. Sign up before December 1st (via brightspace).
- Final report deadline: before January 9th! (via brightspace)
- Access to final exam will only be granted if report is handed in!

Course Information – Lectures

- Lecture schedule on course website.
- We assume that when you come to the lecture, you have seen the corresponding video lectures.
- During the "live" lectures we will give a summary, emphasize on important topics, answer questions, and give exercises.

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