Delft University of Technology
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ET 4386 Estimation and Detection

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Exercise 10.3

Given:

$$p(\mathbf{x}|\theta) = \exp\left(-\sum_{n=0}^{N-1}(x[n]-\theta)\right)U(\min(x[n])-\theta)$$

and

$$p(\theta) = \exp(-\theta)U(\theta).$$

Goal: Calculate $E[\theta|\mathbf{x}] = \int_\theta \theta p(\theta|\mathbf{x})d\theta$.

1. calculate $p(\mathbf{x})$

$$p(\mathbf{x}) = \int_0^{\min(x[n])} p(\mathbf{x},\theta)d\theta = \frac{\exp(-\sum_{n=0}^{N-1}x[n])}{N-1}\left(\exp\left((N-1)\min(x[n])\right)-1\right)$$

2. Determine $p(\theta|\mathbf{x}) = p(\theta,\mathbf{x})/p(\mathbf{x})$

$$p(\theta|\mathbf{x}) = \frac{\exp\left((N-1)\theta\right)U(\min(x[n])-\theta)U(\theta)}{\frac{1}{N-1}\left(\exp\left((N-1)\min(x[n])\right)-1\right)}$$

3. Calculate $E[\theta|\mathbf{x}] = \int_\theta \theta p(\theta|\mathbf{x})$

$$E[\theta|\mathbf{x}] = \int_\theta \theta p(\theta|\mathbf{x})d\theta = \frac{\min(x[n])}{1-\exp\left(-(N-1)\min(x[n])\right)} - \frac{1}{N-1}$$