EE 4715 Array Processing

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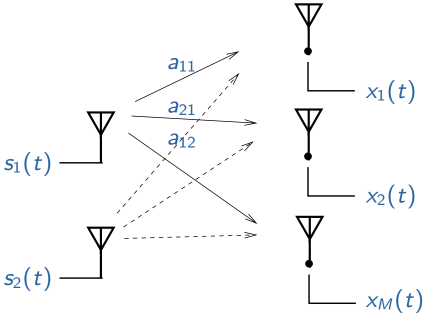
What is array processing?

In array processing, we consider **multiple antennas**: sampling in space.

We stack the output of antennas into a vector $\boldsymbol{x}(t)$. In simple cases, we have a linear model

$$\boldsymbol{x}(t) = \boldsymbol{A}\boldsymbol{s}(t) + \boldsymbol{n}(t)$$

where $\boldsymbol{s}(t)$: vector of source signals; $\boldsymbol{n}(t)$: vector of noise.



What is array processing?

$$\boldsymbol{x}(t) = \boldsymbol{A}\boldsymbol{s}(t) + \boldsymbol{n}(t)$$

- **Tools from linear algebra,** in particular matrix inversion, SVD and eigenvalue decompositions.

$$\boldsymbol{R_x} = \boldsymbol{A}\boldsymbol{R_s}\boldsymbol{A}^{\mathrm{H}} + \boldsymbol{R_n}$$

- **Tools from statistics:** usually covariance matrices and tools seen in Estimation & Detection.
- **Applications:** we will focus on wireless communication, radio astronomy, and acoustics (microphone arrays).

Diversity combining

With multiple antennas, we can **improve the SNR**:

- For a single signal $s(t)$ in noise, received over M antennas:

$$x_m(t) = s(t) + n_m(t)\,, \qquad m = 1, \cdots, M\,.$$

Assume signal power σ_s^2 and noise power σ_n^2, then the input SNR is

$$\text{SNR}_{in} = \frac{\sigma_s^2}{\sigma_n^2} \qquad \text{(per antenna)}$$

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- Let's average the M received signals:

  $$x(t) = \frac{1}{M}\sum_{m=1}^{M} x_m(t) = s(t) + \frac{1}{M}\sum_{m=1}^{M} n_m(t)$$

  The output SNR is

  $$\mathrm{SNR}_{out} = M\frac{\sigma_s^2}{\sigma_n^2} = M\,\mathrm{SNR}_{in} \qquad \text{(array gain)}$$

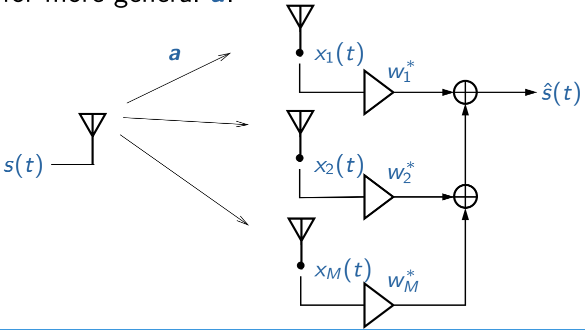
Diversity combining (cont'd)

Written differently:

$$\mathbf{x}(t) = \mathbf{a}s(t) + \mathbf{n}(t)\,, \qquad \text{where we had } \mathbf{a} = \mathbf{1} := \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix}$$

$$\hat{s}(t) = \mathbf{w}^{\mathrm{H}}\mathbf{x}(t)\,, \qquad \mathbf{w} = \frac{\mathbf{a}}{\|\mathbf{a}\|^2}$$

This also works for more general $\mathbf{a}$.



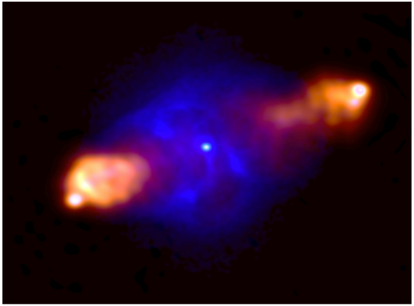
Diversity combining (cont'd)

- The vector $\boldsymbol{w}$ is known as a **beamformer**: a spatial filter.
  In this case, it is a matched filter ("maximum ratio combining").
- In wireless communication, this is used to combine multiple antennas, where some antennas may have poor reception due to **fading**.

Wavefield sampling

Using an array, we sample in space. We measure signals propagating in space: **wavefields**.

- Estimate directions, propagation delays, propagation velocities.
- Applications in **wavefield imaging**: radar, radio astronomy, ultrasound imaging, underwater acoustics, seismic exploration



Source separation

If $\boldsymbol{A}$ is square and invertible, then

$$\boldsymbol{x}(t) = \boldsymbol{A}\boldsymbol{s}(t) + \boldsymbol{n}(t) \quad \Rightarrow \quad \hat{\boldsymbol{s}}(t) = \boldsymbol{A}^{-1}\boldsymbol{x}(t)$$

Thus, we can separate a mixture of M incoming signals.

- Applications:
  - **wireless communication:** MIMO
  - **acoustic arrays:** speaker separation

Covariance models

For a stationary zero mean random process $\boldsymbol{x}(t)$, define the correlation matrix $\boldsymbol{R_x} = \mathrm{E}[\boldsymbol{x}(t)\boldsymbol{x}^{\mathrm{H}}(t)]$. With signals independent from the noise,

$$\boldsymbol{x}(t) = \boldsymbol{A}\boldsymbol{s}(t) + \boldsymbol{n}(t) \qquad \Rightarrow \qquad \boldsymbol{R_x} = \boldsymbol{A}\boldsymbol{R_s}\boldsymbol{A}^{\mathrm{H}} + \boldsymbol{R_n}$$

From an estimate of $\boldsymbol{R_x}$, we can try to identify

- $\boldsymbol{A}$: e.g. direction finding,
- $\boldsymbol{R_s}$: e.g. image formation,
- $\boldsymbol{R_n}$: noise power calibration

To enable estimation, we rely on structure present in $\boldsymbol{A}, \boldsymbol{R_s}, \boldsymbol{R_n}$, e.g. parametric models, diagonals (representing independence).

$\Rightarrow$ **Modeling** is important, the algorithms are based on it

Course outline

Models:

- Wave propagation
- Narrowband models
- Wideband models

Methods and algorithms:

- Beamforming and direction finding
- Factor analysis

Applications:

- Wireless communication
- Radio astronomy
- Microphone arrays

Course organization

- **Reader:** A.J. van der Veen, *"Array signal processing, an algebraic approach"*, TU Delft, 2022 (in progress)

- **Handouts:** related papers from literature

- **Exam:**
  - Take-home matlab assignments $\Rightarrow$ reports
  - Oral discussion about the reports