Mid-term Exam **Signal Processing** (EE2S31)

May 18, 2016
(13:30 - 15:30)

Important:

Make clear in your answer *how* you reach the final result; the road to the answer is very important (even more important that the answer itself).

Start every assignment on a new sheet. Even in the case you skip one of the exercises, you hand in an empty sheet with the number of the assignment you skipped.

Assignment 1: (9 p)

The joint probability density function of two variables X and Y is given by:

$$f_{X,Y}(x,y) = \begin{cases} ce^{-2x}e^{-3y} & \text{for} \quad 0 \leq x \leq y \leq \infty \\ 0 & \text{otherwise.} \end{cases}$$

(2 p) (a) Calculate the value of constant c.

(2 p) (b) Calculate the probability $P(Y > 10)$.

(2 p) (c) Show that $f_X(x)$ equals

$$f_X(x) = \frac{1}{3}ce^{-5x}.$$

(2 p) (d) Determine the conditional pdf $f_{Y|X}(y|x)$.

(1 p) (e) Argue whether or not X and Y are independent.

Assignment 2: (6 p)

Given is the stochastic process X(t)

$$X(t) = \begin{cases} At^2 & \text{for} \quad t \geq 0 \\ 0 & \text{for} \quad t < 0, \end{cases}$$

where A is a random variable with the following uniform distribution

$$f_A(a) = \begin{cases} c & \text{for} \quad 0 \leq a \leq 3 \\ 0 & \quad \text{otherwise.} \end{cases}$$

(1 p) (a) Give the value of constant c.

(1 p) (b) Plot three different possible realizations of this process.

(1 p) (c) Calculate the expected value of process $X(t)$.

(2 p) (d) Calculate the autocorrelation function $R_X(t, \tau)$.

(1 p) (e) Argue whether or not this process is stationary.

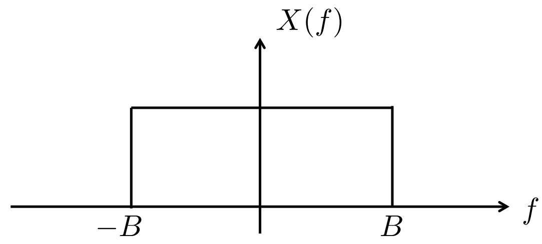

Figure 1: Spectrum $X(f)$.

Assignment 3: (15 p)

Consider a signal x of which its spectrum is given as depicted in Figure 1.

(2 p) (a) Is this signal x a discrete-time or continuous-time signal? Motivate your answer.

(2 p) (b) Is the signal x a periodic or a non-periodic signal? Motivate your answer.

Suppose we sample the signal $x(t)$ with sampling frequency $f_s = 3B$. Afterwards we can reconstruct the (analog) signal out of its samples, say $x_s(n)$, by a proper interpolation scheme.

(1 p) (c) What is the relation between $x(t)$ and $x_s(n)$?

(2 p) (d) Sketch the spectrum, say $X_s(f)$, of $x_s(n)$.

(2 p) (e) What is the minimum sampling frequency such that we can perfectly reconstruct $x(t)$ out of its samples $x_s(n)$ and give the corresponding reconstruction formula.

Assume we want to reconstruct the continuous-time signal using the interpolation function as depicted in Figure 2.

(2 p) (f) How does the continuous-time reconstructed signal look like if we reconstruct using the above mentioned interpolation function?

The reconstruction formula can be expressed in the Fourier domain and is given by

$$\tilde{X}(f) = X_s(f)G(f),$$

where $\tilde{X}(f)$ is the reconstructed signal, $X_s(f)$ is the spectrum of the discrete-time signal $x_s(n)$ and $G(f)$ is the Fourier transform of the interpolation function $g(t)$.

(2 p) (g) Compute $G(f)$.

(1 p) (h) Sketch the spectrum of $\tilde{X}(f)$.

(1 p) (i) Can we obtain a perfect reconstruction of $x(t)$ by using the interpolation function depicted in Figure 2? Motivate your answer.

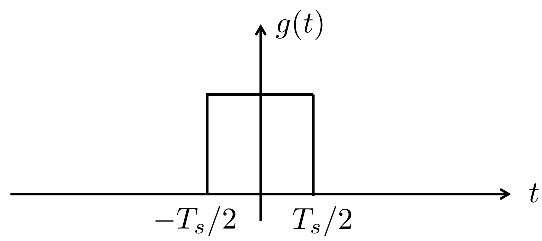

Figure 2: Interpolation function.