

| Combinational Logic - Outline |  |  |
| :---: | :---: | :---: |
| ■ Conventional Static CMOS basic principles |  |  |
| ■ Complementary static CMOS |  |  |
| -Complex Logic Gates |  |  |
| -VTC, Delay and Sizing |  |  |
| ■ Ratioed logic |  |  |
| $\square$ Pass transistor logic |  |  |
| $\square$ Dynamic CMOS gates |  |  |
|  | ${ }_{123024}$ |  |

Combinational vs. Sequential Logic


Complementary static CMOS
■ Complex Logic Gates
■ VTC, Delay and Sizing



PMOS is better pull-up
■ NMOS is better pull-down
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## Complementary Static CMOS (2)

■ Conduction of PUN and PDN must be mutually exclusive

- PUN is dual (complement) network of PDN series $\Leftrightarrow$ parallel
nmos $\Leftrightarrow$ pmos
- Complementary gate is inverting
- No static power dissipation
- Very robust
- Wide noise margin

■ Need 2N transistors for N -input gate

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Improved Gate Level Implementation
$\square$ Using DeMorgan $A+B C=\overline{\bar{A} \cdot \overline{B C}}$

\{TPS\}: Can this be further improved?

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## CMOS Complex Gate Sizing

Assume all transistors will have mininum length $L$

- Determine $W_{n}$ for PDN transistor of inverter that would give the desired 'drive strength'
■ For each transistor in PDN of complex gate do the following:
■ Determine the length $I$ of the longest PDN chain in which it participates
- Set $W=I W_{n}$

Repeat this procedure for PUN, using $W_{p}$ for PUN transistor of inverter.


W/L ratios
■ what are the W/L of 2-input NAND for the same drive strength?

0 -th order calculation





## Combinational Path Sizing for Timing



Given: $\mathrm{C}_{\mathrm{L}}, \mathrm{S}_{1}$
Determine $\mathbf{S}_{2}, \mathbf{S}_{3}, \mathbf{S}_{4}$ to minimize delay
We know how to optimally size string of inverters: make equal stage delays
\{TPS\}: What is different in comparison to string of inverters?


## Logical Effort Methodology

| Inverter delay: | $d_{i n v}=1+f / \gamma$ |  |
| :--- | :--- | :--- |
| Gate delay: | $d_{\text {gate }}=p+\boldsymbol{g} / \gamma=p+\boldsymbol{h} / \gamma$ |  |

```
Logical Effort Methodology Definitions:
p parasitic delay
    - ratio of intrinsic delay compared to inverter
    - ratio of output cap for same drive strength
g logical effort
    - how much more load the gate creates
    - ratio of input cap for same drive strength
h gate effort, h=gf
```

[Logical Effort - Designing Fast CMOS Circuits, Sutherland, Sproul, Harris]
Beware: compared to most texts, incl. Sutherland, Rabaey swaps definition of $f$ and $h$
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## Multistage Networks

$$
\text { Delay }=\sum_{i=1}^{N}\left(p_{i}+g_{i} \cdot f_{i}\right)
$$

Normalized w.r.t. unit delay, assume $\gamma=1$
Stage effort: $\quad h_{i}=g_{i} f_{i}$
Path electrical effort: $F=f_{1} f_{2} \ldots f_{N}=C_{\text {out }} / C_{\text {in }}$
Path logical effort: $\quad G=g_{1} g_{2} \ldots g_{N}$
Path effort: $\quad H=G F$
Path delay $\quad D=\Sigma d_{i}=\Sigma p_{i}+\Sigma h_{i}$

## Optimum Effort per Stage

When each stage bears the same effort, optimal effort $h_{*}$ :

$$
\begin{gathered}
h_{*}^{N}=H \\
h_{*}=\sqrt[N]{H}
\end{gathered}
$$

Stage efforts: $g_{1} f_{1}=g_{2} f_{2}=\ldots=g_{N} f_{N}=h$
Effective fanout of each stage: $\boldsymbol{f}_{\boldsymbol{i}}=\boldsymbol{h}_{\boldsymbol{*}} / \boldsymbol{g}_{\boldsymbol{i}}$
Minimum path delay
larger fanout for simpler stages

$$
\hat{D}=\sum\left(g_{i} f_{i}+p_{i}\right)=N H^{1 / N}+P
$$

## Combinational Path Sizing for Timing



| $\mathrm{g}_{1}=1$ | $\mathrm{~g}_{2}=4 / 3$ | $\mathrm{~g}_{3}=5 / 3$ | $\mathrm{~g}_{4}=1$ | $\mathrm{G}=\Pi \mathrm{g}_{\mathrm{i}}=20 / 9$ |
| :---: | :---: | :---: | :---: | :---: |
| $\mathrm{f}_{1}=\mathrm{S}_{2}$ | $\mathrm{f}_{2}=\mathrm{S}_{3} / \mathrm{S}_{2}$ | $\mathrm{f}_{3}=\mathrm{S}_{4} / \mathrm{S}_{3}$ | $\mathrm{f}_{4}=36.45 / \mathrm{S}_{4}$ | $\mathrm{~F}=\Pi \mathrm{f}_{\mathrm{i}}=36.45$ |

$\mathrm{H}=\mathrm{FG}=81 \quad h_{*}=\sqrt[N]{H}=\sqrt[4]{81}=3$
$\mathrm{f}_{1} \mathrm{~g}_{1}=3 \Rightarrow \mathrm{~S}_{2}=3$
$\mathrm{f}_{2} \mathrm{~g}_{2}=3 \Rightarrow \mathrm{~S}_{3}=27 / 4=6.75$
$\mathrm{f}_{3} \mathrm{~g}_{3}=3 \Rightarrow \mathrm{~S}_{4}=12.15$

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## Logical Effort - Summary

■ Numerical logical effort characterizes gates
■ Inverters and NAND2 best for driving large caps

- NANDs are faster than NORs in CMOS
- Extension needed (see book) for branching
- Simplistic delay model
$■$ Neglects input rise time effects
- Interconnect

■ Iteration required in designs with wire

- Maximum speed only
$■$ Not minimum area/power for constrained delay



## DYNAMIC POWER DISSIPATION

Dynamic Power Dissipation
Power $=$ Energy/transition $\cdot$ Transition rate

$$
\begin{array}{ll}
=C_{L} V_{D D}^{2} \cdot f_{0 \rightarrow 1} & \text {-Transistor Sizing } \\
=C_{L} V_{D D}^{2} \cdot f \cdot p_{0 \rightarrow 1} & \text { - Physical capacitance } \\
=C_{\text {switched }} V_{D D}^{2} \cdot f & \text { - Short-circuit power } \\
& \text {-Threshold and temperature } \\
& \text { - Leakage power } \\
& \text {-Switching activity }
\end{array}
$$

Power dissipation is data dependent depends on the switching probability $p_{0 \rightarrow 1}$ $\square$ Switched capacitance $C_{\text {switched }}=p_{0 \rightarrow 1} C_{L}=\alpha C_{L}$ ( $\alpha$ is called the switching activity)

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## Transition Probability

$p_{A=1}=p_{A}$ : given probability of value of signal $A$ being 1 in any clock cycle
$p_{A=0}=p_{A}$ : given probability of value of signal $A$ being 0 in any clock cycle
Note the ' prime (for inversion of signal)

Transition probability:
Probability of 1 to 0 or 0 to 1 transition at clock edge: $\alpha=p_{A}\left(1-p_{A}\right)=p_{A} p_{A^{\prime}}$

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## Impact of Logic Function

Example: Static 2-input XOR Gate

| $A$ | $B$ | Out |
| :---: | :---: | :---: |
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |

Assume signal probabilities
$p_{A=1}=1 / 2$
$p_{B=1}=1 / 2$
Then transition probability
$p_{0 \rightarrow 1}=p_{\text {Out }=0} \times p_{\text {Out }=1}$
$=1 / 2 \times 1 / 2=1 / 4$
If inputs switch in every cycle

$$
P_{0 \rightarrow 1}=1 / 4
$$

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## Impact of Logic Function

Example: Static 2-input NAND gate

| $A$ | $B$ | Out |
| :---: | :---: | :---: |
| 0 | 0 | 1 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |

## Assume signal probabilities

$$
\begin{aligned}
& p_{A=1}=1 / 2 \\
& p_{B=1}=1 / 2
\end{aligned}
$$

Then transition probability $\alpha=p_{0 \rightarrow 1}=p_{\text {Out }=0} \times p_{\text {Out }=1}$

If inputs switch every cycle

$$
=1 / 4 \times 3 / 4=3 / 16
$$

NOR gate yields similar result

Signal and Transition Probabilities OR Gate
$p_{A}$ : Probability of $A$ being 1 $p_{B}$ : Probability of $B$ being 1

Probability of Output being 0 : $\left(1-p_{A}\right)\left(1-p_{B}\right)$


Probability of Output being 1: $\left(1-\left(1-p_{A}\right)\left(1-p_{B}\right)\right)$
Transition probability: $\alpha=\left(1-p_{A}\right)\left(1-p_{B}\right)\left(1-\left(1-p_{A}\right)\left(1-p_{B}\right)\right)$

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Transition Probabilities for Basic Gates

As a function of the input probabilities

|  | $\mathrm{A}=p_{0 \rightarrow 1}$ |
| :---: | :---: |
| AND | $\left(1-p_{A} p_{B}\right) p_{A} p_{B}$ |
| OR | $\left(1-p_{A}\right)\left(1-p_{B}\right)\left(1-\left(1-p_{A}\right)\left(1-p_{B}\right)\right)$ |
| XOR | $\left(1-\left(p_{A}+p_{B}-2 p_{A} p_{B}\right)\right)\left(p_{A}+p_{B}-2 p_{A} p_{B}\right)$ |

Activity for static CMOS gates: $\alpha=p_{0} p_{1}$
Because of symmetry: AND $\Leftrightarrow$ NAND, OR $\Leftrightarrow$ NOR

Reconvergent Fanout (Spatial Correlation)
Inputs to gate can be interdependent (correlated)

reconvergent

$$
p_{Z}=1-\left(1-p_{A}\right) p_{A} ?
$$

NO!

$$
p_{z}=1
$$

Must use conditional probabilities

$$
p_{\mathrm{Z}}=1-p_{\mathrm{A}} \cdot p(X \mid A)=1
$$

$$
\text { probability that } \mathrm{X}=1 \text { given that } \mathrm{A}=1
$$

Becomes complex and intractable real fast
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Example: Chain of NAND Gates

## Complimentary CMOS Gates - Summary

■ Full rail-to-rail swing; high noise margins
■ Logic levels not dependent upon the relative device sizes; ratioless

- Always a path to Vdd or Gnd in steady state; low output impedance

■ Extremely high input resistance; nearly zero steadystate input current

- No direct path steady state between power and ground; no static power dissipation
- Need 2N transistors for N -input gate




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Solution: balancing delay paths!
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## RATIOED LOGIC

Ignore, because approximately equal
$\left.v_{O L} \approx \frac{k_{p}}{k_{n}}\left|V_{D S A T}\right| \approx \frac{\mu_{p} W_{p}}{\mu_{n} W_{n}} V_{D S A T} \right\rvert\,$


Pseudo NMOS Ratioed Logic
Performance of a pseudo-NMOS inverter

| Pseudo NMOS Ratioed Logic |  |  |  |
| :---: | :---: | :---: | :---: |
| Performance of a pseudo-NMOS inverter |  |  |  |
| Size | $\mathrm{V}_{\text {OL }}[\mathrm{V}]$ | Power [ $\mu \mathrm{W}$ ] | $\mathrm{t}_{\mathrm{pLH}}[\mathrm{ps}]$ |
| 4 | 0.693 | 564 | 14 |
| 2 | 0.273 | 298 | 56 |
| 1 | 0.133 | 160 | 123 |
| 0.5 | 0.064 | 80 | 268 |
| 0.25 | 0.031 | 41 | 569 |





Level restoring circuit
(-) Rail-to-rail swing
() No static dissipation
( 8 Rationed - use with care
(2) Increased capacitance

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## Pass Gates (Transmission Gates)

■ use an N-MOS and a P-MOS in parallel



## Pass Transistor Logic

- Most typical use: for multiplexing, or path selecting
- Assume in circuit below it is required to either connect $A$ or $B$ to $Y$, under control by $S$
- $Y=A S+B S^{\prime}\left(S^{\prime}\right.$ is easier notation for S-bar $=$ S-inverse $\left.=\bar{S}\right)$

■ $Y=\left((A S)^{\prime}(B S)^{\prime}\right)$ ' allows realization with 3 NAND-2 and 1 INV: 14 transistors

- Pass gate needs only 6 (or 8 ) transistors



## Dynamic CMOS gates

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Properties of Dynamic Gates (1)
■ Logic function is implemented by the PDN only

- Number of transistors is $\mathrm{N}+2$ (versus 2N for static complementary CMOS)
- Full swing outputs $\left(\mathrm{V}_{\mathrm{OL}}=\mathrm{V}_{\mathrm{SS}}\right.$ and $\mathrm{V}_{\mathrm{OH}}=\mathrm{V}_{\mathrm{DD}}$ )
$\square$ Nonratioed - sizing of the devices is not important for proper functioning




## Properties of Dynamic Gates (3)



- Needs a precharge clock
- \{TPS\} compare power of dynamic vs static CMOS: higher or lower
- Overall power dissipation usually significantly higher than static CMOS
() Reduced capacitance
() no static current path ever exists between $V_{D D}$ and GND (including $P_{s c}$ )
() no glitching
(2) higher transition probabilities (*) extra load on CLK

Issues in Dynamic Design (2)


Issues in Dynamic Design (3)
■ Backgate Coupling


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