## COMBINATIONAL LOGIC

## Combinational Logic - Outline

- Conventional Static CMOS basic principles
- Complementary static CMOS

■Complex Logic Gates
■VTC, Delay and Sizing
$\square$ Ratioed logic
■ Pass transistor logic
■ Dynamic CMOS gates

## Combinational vs. Sequential Logic


(a) Combinational
(b) Sequential
§ $6.2 \quad$ Output $=f(\ln )$
Output = $\boldsymbol{f}$ (In, History)

# Complementary static CMOS 

■ Complex Logic Gates
■VTC, Delay and Sizing

## Complementary Static CMOS



- Conduction of PDN and PUN must be mutually exclusive (Why?)
- Pull-up network (PUN) and pull-down network (PDN) are dual


## 2-input Nand/Nor



## Mutual Exclusive PDN and PUN


$\left.\begin{array}{|ccc|cc|c|}\hline & & & P & P & \\ C & B & A & \mathrm{~N} & \mathrm{~N} & \text { Out } \\ \hline 0 & 0 & 0 & ? & 1 & 1 \\ 0 & 0 & 1 & ? & 1 & 1 \\ 0 & 1 & 0 & ? & 1 & 1 \\ 0 & 1 & 1 & 0 & ? & 0 \\ 1 & 0 & 0 & 0 & ? & 0 \\ 1 & 0 & 1 & 0 & ? & 0 \\ 1 & 1 & 0 & 0 & ? & 0 \\ 1 & 1 & 1 & 0 & ? & 0 \\ \hline\end{array}\right\}$

PDN Off
PUN On

PUN Off PDN On

For all Complementary Static CMOS Gates, either the PUN or the PDN is conducting, but never both.

## Complementary Static CMOS (2)

■ Conduction of PUN and PDN must be mutually exclusive
■ PUN is dual (complement) network of PDN series $\Leftrightarrow$ parallel nmos $\Leftrightarrow$ pmos
■ Complementary gate is inverting

- No static power dissipation
- Very robust
- Wide noise margin
- Need 2N transistors for $\mathbf{N}$-input gate


## NMOS vs. PMOS, pull-down vs. pull-up


pull-up

$\square$ PMOS is better pull-up
$\square$ NMOS is better pull-down

## Bad Idea



Exercise: Determine logic function
Determine $\mathrm{V}_{\text {out }}$
for $V_{\text {in }}=V_{D D}$ and $V_{\text {in }}=V_{S s}$
Why is this a bad circuit?

## CMOS Gate is Inverting

Assume full-swing inputs (high $=\mathrm{V}_{\mathrm{DD}}$, low $=\mathrm{V}_{\mathrm{SS}}$ )
■ Highest output voltage of NMOS is

$$
V_{G S}-V_{T n}=V_{D D}-V_{T n}
$$

- An 1 on NMOS gate can produce a strong 0 at the drain, but not a strong 1
■ Lowest output voltage of PMOS is

$$
\begin{aligned}
& V_{D D}+V_{G S}-V_{T p}=\left|V_{T p}\right| \\
& \text { (with } V_{G S}, V_{T p}<0 \text { for PMOS) }
\end{aligned}
$$

■ An 0 on PMOS gate can produce a strong 1 at the drain, but not a strong 0

- Need NMOS for pull-down, PMOS for pull-up

A 1 at input can pull-down, 0 at input can pull-up

## Inverting behavior

For a non-inverting Complementary CMOS Gate, you can only use 2 inverting gates

## Implementation of Combinational Logic

■ How van we construct an arbitrary combinational logic network in general, using NMOS and PMOS transistors (using Complementary static CMOS)?

■ Example: $\quad \mathrm{Y}=\overline{(\mathrm{A}+\mathrm{BC}) \mathrm{D}}$
■ Remember: only inverting gates available


## Implementation of Combinational Logic

- Example: $\quad Y=(A+B C) D$
- Remember: only inverting gates available

■ Logic depth: number of gates in longest path $\Rightarrow$ DELAY

\# transistors $\square$ logic depth


■ \{TPS\}: Can this be improved? If so, how?

## Improved Gate Level Implementation

$\square$ Using DeMorgan $\quad A+B C=\overline{\bar{A} \cdot \overline{B C}}$

$\square$ \{TPS\}: Can this be further improved?

## Complex CMOS Logic Gates

■ Restriction to basic NAND, NOR etc. not necessary
■ Easy to synthesize complex gates


## How to Synthesize Complex Gates

$$
Y=\overline{(A+B C) D}
$$

■ Using tree representation of Boolean function


## Complex Gate Synthesis Example



|  | PDN | PUN |
| :---: | :--- | :--- |
| AND | Series | Parallel |
| OR | Parallel | Series |



## And-Or-Invert Gate



## And-Or-Invert Example

- From a Truth-Table: take 0-outputs

| $A$ | $B$ | $C$ | $Y$ |
| :--- | :--- | :--- | :--- |
| 0 | 0 | 0 | 1 |
| 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 1 |
| 0 | 1 | 1 | 0 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 1 | 0 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 |

$\bar{Y}=\bar{A} B C+A \bar{B} C$
$\overline{\mathrm{A}}, \overline{\mathrm{B}}$ to be created with extra inverters (or by restructuring previous circuits)

## And-Or-Invert Improvement


$Y=\overline{\bar{A} B C+A \bar{B} C}$
12 transistors

$Y=\overline{(\bar{A} B+A \bar{B}) C}$ 10 transistors

2-level logic minimization: boolean algebra technique

## CMOS Complex Gate Sizing



2 trans. in series

- Function of gate independent of transistor sizes: ratioless
■ But current-drive capability (timing) depends on transistor sizes

■ Worst-case currentdrive depends on number of transistors in series

## CMOS Complex Gate Sizing

■ Assume all transistors will have mininum length $L$
■ Determine $W_{n}$ for PDN transistor of inverter that would give the desired 'drive strength'
■ For each transistor in PDN of complex gate do the following:

- Determine the length I of the longest PDN chain in which it participates
- Set $W=I W_{n}$
- Repeat this procedure for PUN, using $W_{p}$ for PUN transistor of inverter.


## Gate Sizing



■ W/L ratios
$\square$ what are the W/L of 2-input NAND for the same drive strength?

0 -th order calculation

## Gate Sizing



## Exercise


(a)

## Exercise:


$\square$ Perform gate sizing of (a) for nominal drive strength equal to that of min size inverter, assume PU/PD = 3
■ Determine PUN of (b)

- Perform gate sizing of (b) for same drive strength (same PU/PD)
- Compare sum of gate areas in (a) and (b). Note: area ~ width


## Avoid Large Fan-In



C linear in N
$\mathbf{R}$ linear in $\mathbf{N}$
Delay $\propto$ RC quadratic in $\mathbf{N}$
C -Transistor Sizing
-Progressive Transistor Sizing
-Input Re-Ordering
-Logic Restructuring
Empirical
Delay $=\mathrm{a}_{1} \mathrm{FI}+\mathrm{a}_{2} \mathrm{FI}^{2}+\mathrm{a}_{3} \mathrm{FO}$

## Data-Dependent Timing



You should be able to identify the transistor paths that charge or discharge $C_{L}$, and calculate resulting RC delay model, including effects of wires and fan-out


Series connection
One input goes low
Two inputs go low, parallel connection

## $2^{\text {nd }}$ Order Effects

- Much more to say about performance of static gates
- Simulator can give accurate answer

■ Understanding needed to make design decisions

■ Data-dependent VTC

- Data-dependent Timing


## Data-dependent VTC: 2nd order effects




- Charge at 'int'
- Body effect in $\mathbf{M}_{2}$
\{TPS\}:
Explain VTC difference between I and II


## Data-dependent Timing



| Input Data <br> Pattern | Delay <br> $(\mathbf{p S})$ |
| :---: | :---: |
| $\boldsymbol{A}=\boldsymbol{B}=\mathbf{0} \rightarrow \mathbf{1}$ | 69 |
| $\boldsymbol{A}=\mathbf{1}, \boldsymbol{B}=\mathbf{0} \rightarrow \mathbf{1}$ | 62 |
| $\mathbf{A}=\mathbf{0} \rightarrow \mathbf{1}, \boldsymbol{B}=\mathbf{1}$ | 50 |
| $\boldsymbol{A}=\boldsymbol{B}=\mathbf{1} \rightarrow \mathbf{0}$ | 35 |
| $\boldsymbol{A}=\mathbf{1}, \boldsymbol{B}=\mathbf{1} \rightarrow \mathbf{0}$ | 76 |
| $\boldsymbol{A}=\mathbf{1} \rightarrow \mathbf{0}, \boldsymbol{B}=\mathbf{1}$ | 57 |

$A=1, B=\star$ : need to charge int
$A=\downarrow, B=1$ : int does not need to be charged
$A=\downarrow, B=\downarrow$ : twice the pull-up strength
\{TPS\}:
Explain differences in $\mathrm{t}_{\mathrm{pLH}}$

## LOGICAL EFFORT

## Combinational Path Sizing for Timing



Given: $\mathrm{C}_{\mathrm{L}}, \mathrm{S}_{1}$
Determine $S_{2}, S_{3}, S_{4}$ to minimize delay
We know how to optimally size string of inverters:
make equal stage delays
\{TPS\}: What is different in comparison to string of inverters?

## Recap: Inverter Delay



S: relative size of inverter

$R_{0}, C_{g}, \gamma C_{g}$ : output res, input cap and output cap of min size inverter

## Recap: Inverter Delay


$R_{0}, C_{g}, C_{g}$ : output res, input cap and output cap of min size inverter

$$
\begin{aligned}
& t_{p}=\frac{R_{0}}{S}\left(\gamma C_{g} S+C_{e x t}\right) \\
& =\gamma R_{0} C_{g}\left(1+\frac{C_{e x t}}{\gamma S C_{g}}\right)=t_{p o}\left(1+\frac{C_{e x t}}{\gamma C_{i n}}\right) \quad \text { with } C_{i n}=S C_{g} \\
& =t_{p o}\left(1+\frac{f}{\gamma}\right) \quad \text { with } t_{p o}=\gamma R_{0} C_{g} \quad f=\frac{C_{e x t}}{C_{i n}}
\end{aligned}
$$

$t_{\mathrm{p} 0}$ : Delay of unloaded inverter, independent of sizing

## Inverter Delay Summary

$$
t_{p}=\frac{R_{0}}{S}\left(\gamma C_{g} S+C_{e x t}\right)=t_{p o}\left(1+\frac{f}{\gamma}\right) \quad \text { with } t_{p o}=\gamma R_{0} C_{g}, f=\frac{C_{e x t}}{C_{i n}}
$$

$$
\boldsymbol{d}=\mathbf{1}+\boldsymbol{f} / \boldsymbol{\gamma} \quad \text { In units of } \mathrm{t}_{\mathrm{po}}
$$

$R_{0} \quad$ Equivalent output resistance of min size inverter
$C_{g} \quad$ Input cap of min size inverter
$C_{0}=\gamma C_{g}$ Drain (and Miller) cap of min size inverter
$S \quad$ Size of inverter (relative to min inverter)
$f \quad$ electrical effort - ratio between $\mathrm{C}_{\text {load }}$ and $\mathrm{C}_{\text {in }}$
$\gamma \quad$ ratio of drain cap to gate cap
$t_{p o} \quad$ intrinsic delay-delay of unloaded inverter $t_{p 0} \approx 20 \mathrm{ps}$ for a 250 nm process, $t_{p 0} \approx 5 \mathrm{ps}$ for a 45 nm process
d normalized delay $=t_{p} / t_{p o}$


Path delay is minimized if all stage delays are equal
For string of inverters:
when ratio of load cap over input cap is identical for each stage
If $C_{g}=$ input cap of inverter with size 1 (minimum size):

$$
\frac{S_{2} C_{g}}{C_{i n}}=\frac{S_{2} C_{g}}{S_{1} C_{g}}=\frac{S_{2}}{S_{1}}=\frac{S_{3}}{S_{2}}=\frac{S_{4}}{S_{3}}=\frac{C_{\text {load }}}{S_{4} C_{i n v}}
$$

## Logical Effort Methodology

Inverter delay: $d_{\text {inv }}=1+f / \gamma$
In units of $t_{p 0}$
Gate delay: $\quad d_{\text {gate }}=p+g f / \gamma=p+h / \gamma$

Logical Effort Methodology Definitions:
p parasitic delay

- ratio of intrinsic delay compared to inverter
- ratio of output cap for same drive strength
$g \quad$ logical effort
- how much more load the gate creates
- ratio of input cap for same drive strength
$h \quad$ gate effort, $h=\boldsymbol{g} f$
[Logical Effort - Designing Fast CMOS Circuits, Sutherland, Sproul, Harris]
Beware: compared to most texts, incl. Sutherland, Rabaey swaps definition of $f$ and $h$


## Intrinsic, Parasitic Delay


p parasitic delay - ratio of intrinsic delay compared to inverter
$p$ is ratio of output capacitances if gate is sized for identical drive strength
$p_{\text {nand }}$ is $(2+2+2) /(2+1)=2$

$$
d_{\text {gate }}=p+\boldsymbol{g f} / \gamma
$$

## Logical Effort


$g$ logical effort: how much load a gate provides relative to inverter for same drive strength
g ratio of input cap (per pin) if gate is sized for identical drive strength
$g_{\text {nand }}$
is $(2+2) /(2+1)=4 / 3$

$$
d_{\text {gate }}=p+\boldsymbol{g} / \gamma
$$

## Logical Effort



$$
p=1, g=1
$$

$p=2, g=4 / 3$
$\mathrm{p}=2, \mathrm{~g}=5 / 3$
$p$ ratio of intrinsic delay compared to inverter
$g$ logical effort - ratio of inp. cap for same strength $\mathrm{p}, \mathrm{g}$ independent of sizing, only topology of gate

## Delay vs Fan-Out



## Multistage Networks

$$
\text { Delay }=\sum_{i=1}^{N}\left(p_{i}+g_{i} \cdot f_{i}\right)
$$

Normalized w.r.t. unit delay, assume $\gamma=1$
Stage effort:

$$
h_{i}=g_{i} f_{i}
$$

Path electrical effort: $F=f_{1} f_{2} \ldots f_{N}=C_{\text {out }} / C_{\text {in }}$
Path logical effort: $\quad G=g_{1} g_{2} \ldots g_{N}$
Path effort: $\quad H=G F$
Path delay
$D=\Sigma d_{i}=\Sigma p_{i}+\Sigma h_{i}$

## Optimum Effort per Stage

When each stage bears the same effort, optimal effort $\mathrm{h}_{*}$ :

$$
\begin{gathered}
h_{*}^{N}=\boldsymbol{H} \\
h_{*}=\sqrt[N]{H}
\end{gathered}
$$

Stage efforts: $g_{1} f_{1}=g_{2} f_{2}=\ldots=g_{N} f_{N}=h$
Effective fanout of each stage: $\boldsymbol{f}_{\boldsymbol{i}}=\boldsymbol{h}_{\boldsymbol{*}} / \boldsymbol{g}_{\boldsymbol{i}}$
larger fanout for simpler stages
Minimum path delay

$$
\hat{D}=\sum\left(g_{i} f_{i}+p_{i}\right)=N H^{1 / N}+P
$$

## Combinational Path Sizing for Timing

$$
S_{1}=1, C_{L}=36.45
$$



| $\mathrm{g}_{1}=1$ | $\mathrm{~g}_{2}=4 / 3$ | $\mathrm{~g}_{3}=5 / 3$ | $\mathrm{~g}_{4}=1$ | $\mathrm{G}=\Pi \mathrm{g}_{\mathrm{i}}=20 / 9$ |
| :---: | :---: | :---: | :---: | :---: |
| $\mathrm{f}_{1}=\mathrm{S}_{2}$ | $\mathrm{f}_{2}=\mathrm{S}_{3} / \mathrm{S}_{2}$ | $\mathrm{f}_{3}=\mathrm{S}_{4} / \mathrm{S}_{3}$ | $\mathrm{f}_{4}=36.45 / \mathrm{S}_{4}$ | $\mathrm{~F}=\Pi \mathrm{f}_{\mathrm{i}}=36.45$ |

$$
\begin{aligned}
& \mathrm{H}=\mathrm{FG}=81 \quad h_{*}=\sqrt[N]{H}=\sqrt[4]{81}=3 \\
& \mathrm{f}_{1} \mathrm{~g}_{1}=3 \Rightarrow \mathrm{~S}_{2}=3 \\
& \mathrm{f}_{2} \mathrm{~g}_{2}=3 \Rightarrow \mathrm{~S}_{3}=27 / 4=6.75 \\
& \mathrm{f}_{3} \mathrm{~g}_{3}=3 \Rightarrow \mathrm{~S}_{4}=12.15
\end{aligned}
$$

## Combinational Path Sizing for Timing

$$
S_{1}=1, C_{L}=36.45
$$



$$
\mathrm{f}_{1} \mathrm{~g}_{1}=3 \Rightarrow \mathrm{~S}_{2}=3 \quad \mathrm{f}_{2} \mathrm{~g}_{2}=3 \Rightarrow \mathrm{~S}_{3}=27 / 4=6.75 \quad \mathrm{f}_{3} \mathrm{~g}_{3}=3 \Rightarrow \mathrm{~S}_{4}=12.15
$$

|  | INV | NAND | NOR | INV | $\begin{aligned} & C_{L}= \\ & 36.45 C_{\text {in }} \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Width of N : | 1 | $3 \times 3 / 2=4.5$ | $3 \times 6.75 / 5=4.05$ | $3 \times 12.15 / 3=12.15$ |  |
| Width of P: | 2 | $3 \times 3 / 2=4.5$ | $3 \times 4 \times 6.75 / 5=16.2$ | $3 \times 2 \times 12.15 / 3=24.3$ |  |
| Nrmlzd Cin | 1 | $9 / 3=3$ | $20.25 / 3=6.75$ | $36.45 / 3=12.15$ |  |
|  |  |  |  |  |  |

## Logical Effort - Summary

- Numerical logical effort characterizes gates
- Inverters and NAND2 best for driving large caps
- NANDs are faster than NORs in CMOS
- Extension needed (see book) for branching
- Simplistic delay model
$\square$ Neglects input rise time effects
- Interconnect
- Iteration required in designs with wire
- Maximum speed only
$\square$ Not minimum area/power for constrained delay


## DYNAMIC POWER DISSIPATION

## Dynamic Power Dissipation

Power $=$ Energy/transition • Transition rate

$$
\begin{aligned}
& =C_{L} V_{D D}^{2} \cdot f_{0 \rightarrow 1} \\
& =C_{L} V_{D D}^{2} \cdot f \cdot p_{0 \rightarrow 1} \\
& =C_{\text {switched }} V_{D D}^{2} \cdot f
\end{aligned}
$$

-Transistor Sizing

- Physical capacitance
-Input and output rise/fall times
- Short-circuit power
-Threshold and temperature
- Leakage power
-Switching activity

■ Power dissipation is data dependent depends on the switching probability $p_{0 \rightarrow 1}$
$\square$ Switched capacitance $C_{\text {switched }}=p_{0 \rightarrow 1} C_{L}=\alpha C_{L}$ ( $\alpha$ is called the switching activity)

## Transition Probability

$p_{A=1}=p_{A}$ : given probability of value of signal $A$ being 1 in any clock cycle
$p_{A=0}=p_{A}$ : given probability of value of signal $A$ being 0 in any clock cycle
Note the ' prime (for inversion of signal)

Transition probability:
Probability of 1 to 0 or 0 to 1 transition at clock edge: $\alpha=p_{A}\left(1-p_{A}\right)=p_{A} p_{A^{\prime}}$

## Impact of Logic Function

Example: Static 2-input NAND gate
Assume signal probabilities

$$
\begin{aligned}
& p_{A=1}=1 / 2 \\
& p_{B=1}=1 / 2
\end{aligned}
$$

Then transition probability

$$
\alpha=p_{0 \rightarrow 1}=p_{\text {Out }=0} \times p_{\text {Out }=1}
$$

If inputs switch every cycle

$$
=1 / 4 \times 3 / 4=3 / 16
$$

$$
\alpha_{\text {NAND }}=3 / 16
$$

NOR gate yields similar result

## Impact of Logic Function

Example: Static 2-input XOR Gate

| $A$ | $B$ | Out |
| :---: | :---: | :---: |
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |

Assume signal probabilities

$$
\begin{aligned}
& p_{A=1}=1 / 2 \\
& p_{B=1}=1 / 2
\end{aligned}
$$

Then transition probability

$$
p_{0 \rightarrow 1}=p_{\text {Out }=0} \times p_{\text {Out }=1}
$$

If inputs switch in every cycle

$$
=1 / 2 \times 1 / 2=1 / 4
$$

$$
P_{0 \rightarrow 1}=1 / 4
$$

## Signal and Transition Probabilities OR Gate

$p_{A}$ : Probability of $A$ being 1 $p_{B}$ : Probability of $B$ being 1


Probability of Output being $1:\left(1-\left(1-p_{A}\right)\left(1-p_{B}\right)\right)$
Transition probability: $\alpha=\left(1-p_{A}\right)\left(1-p_{B}\right)\left(1-\left(1-p_{A}\right)\left(1-p_{B}\right)\right)$

## Signal and Transition Probabilities AND Gate

$p_{A}$ : Probability of $A$ being 1 $p_{B}$ : Probability of $B$ being 1


Probability of Output being $0:\left(1-p_{A}\right)\left(1-p_{B}\right)$
Probability of Output being 1: $p_{A} p_{B}$
Transition probability: $\left.\alpha=\left(1-p_{A}\right)\left(1-p_{B}\right) p_{A} p_{B}\right)$

## Transition Probabilities for Basic Gates

As a function of the input probabilities

|  | $\mathrm{A}=p_{0 \rightarrow 1}$ |
| :---: | :---: |
| AND | $\left(1-p_{A} p_{B}\right) p_{A} p_{B}$ |
| OR | $\left(1-p_{A}\right)\left(1-p_{B}\right)\left(1-\left(1-p_{A}\right)\left(1-p_{B}\right)\right)$ |
| XOR | $\left(1-\left(p_{A}+p_{B}-2 p_{A} p_{B}\right)\right)\left(p_{A}+p_{B}-2 p_{A} p_{B}\right)$ |

Activity for static CMOS gates: $\alpha=p_{0} p_{1}$ Because of symmetry: AND $\Leftrightarrow$ NAND, OR $\Leftrightarrow$ NOR

## Evaluating Power Dissipation of Complex Logic

Simple idea: start from inputs and propagate signal probabilities to outputs


## But:

Reconvergent fanout
Feedback and temporal/spatial correlations

## Reconvergent Fanout (Spatial Correlation) <br> Inputs to gate can be interdependent (correlated)


no reconvergence
$p_{Z}=1-p_{x} p_{B}=1-\left(1-p_{A}\right) p_{B}$

reconvergent

$$
\begin{gathered}
p_{Z}=1-\left(1-p_{A}\right) p_{A} ? \\
N O! \\
p_{Z}=1
\end{gathered}
$$

Must use conditional probabilities

$$
p_{\mathrm{Z}}=1-p_{\mathrm{A}} \cdot p(X \mid A)=1
$$

probability that $X=1$ given that $A=1$
Becomes complex and intractable real fast

## Glitching in Static CMOS

Analysis so far did not include timing effects


The result is correct, but extra power is dissipated

Also known as dynamic hazards: "A single input change causing multiple changes in the output"

## Example: Chain of NAND Gates



## What Causes Glitches?



Uneven arrival times of input signals of gate due to unbalanced delay paths
Solution: balancing delay paths!

## Complimentary CMOS Gates - Summary

■ Full rail-to-rail swing; high noise margins

- Logic levels not dependent upon the relative device sizes; ratioless

■ Always a path to Vdd or Gnd in steady state; low output impedance

■ Extremely high input resistance; nearly zero steadystate input current

■ No direct path steady state between power and ground; no static power dissipation

■ Need 2N transistors for $\mathbf{N}$-input gate

## RATIOED LOGIC

## Pseudo NMOS Ratioed Logic


(-) Reduced area
() Reduced capacitances
$*$ Increased $\mathbf{V}_{\mathbf{O L}}$
( $:$ Reduced noise margins
© Static dissipation


## Ratioed Logic $\mathrm{V}_{\text {oL }}$ Computation

$I_{D n}($ linear $)=I_{D p}$ (saturation) assumptions/steps
$k_{n}\left(\left(V_{D D}-V_{T n}\right) V_{O L}-\frac{K_{2}^{2}}{2}\right)=k_{p}\left(\left(-V_{D D}-V_{T p}\right) V_{D S A T}-\frac{V_{D}^{2} / A T}{2 \backslash}\right)$
Ignore quadratic terms (they are relatively small)
$k_{n}\left(V_{D D}<\nabla_{T_{n}}\right) V_{O L} \approx k_{p}\left(-V_{D D}<\nabla_{T p}\right) V_{D S A T}$
Ignore, because approximately equal
$V_{O L} \approx \frac{k_{p}}{k_{n}} V_{D S A T}\left|\approx \frac{\mu_{p} W_{p}}{\mu_{n} W_{n}} V_{D S A T}\right|$

## Pseudo NMOS Ratioed Logic

## Performance of a pseudo-NMOS inverter

| Size | $\mathrm{V}_{\mathrm{OL}}[\mathrm{V}]$ | Power $[\mu \mathrm{W}]$ | $\mathrm{t}_{\mathrm{pLH}}[\mathrm{ps}]$ |
| :---: | :---: | :---: | :---: |
| 4 | 0.693 | 564 | 14 |
| 2 | 0.273 | 298 | 56 |
| 1 | 0.133 | 160 | 123 |
| 0.5 | 0.064 | 80 | 268 |
| 0.25 | 0.031 | 41 | 569 |

## Pseudo NMOS Ratioed Logic

Differential Cascode Voltage Switch Logic (DCVSL)

(:) Rail-to-rail swing
(:) No static dissipation
© Rationed
: Cross-over currents
© Wiring

## PASS TRANSISTOR LOGIC PASS GATE LOGIC

## Pass Transistor Logic

■ Save area, capacitances

- Need complementary inputs (extra inverters)



## But remember:



## Pass Transistor Logic

■ Save area, capacitances

- Need complementary inputs (might mean extra inverters)
- Reduced $\mathrm{V}_{\mathrm{OH}}$, noise margins

$\square V_{O H}=V_{D D}-\left(V_{T n o}+\gamma\left(\left(\sqrt{2 \phi_{f} \mid+V_{O H}}\right)-\sqrt{\mathbf{2} \phi_{f} \mid}\right)\right)$
■ Static dissipation in subsequent static inverter/buffer
■ Disadvantages (and advantages) may be reduced by complementary pass gates (NMOS + PMOS parallel)
\{TPS\}: Why is there static dissipation in next conventional gate?


## Pass Transistor Logic

■ Level restoring circuit



Level restoring circuit
(;) Rail-to-rail swing
() No static dissipation
(:) Rationed - use with care
( : Increased capacitance

## Exercise

■ Discuss what happens when you connect the output of a single pass-transistor (not a pass-gate) to the input of another pass-transistor stage (i.e. the gate of another pass-transistor). Why should you never use such a circuit?


## Pass Gates (Transmission Gates)

■ use an N -MOS and a P-MOS in parallel



■ Pass gates eliminate some of the disadvantages of simple pass-transistors

- Eliminates reduced noise margins \& static power consumption
$\square$ Disadvantages of pass gate:
■ Requires both NMOS and PMOS in different wells, both true and complemented polarities of the control signal needed, increases node capacitance
■ Design remains a trade-off!


## Pass Transistor Logic



$$
t_{p}\left(V_{n}\right)=0.69 \sum_{k=0}^{n} k C R_{e q}=0.69 C R_{e q} \frac{n(n+1)}{2}
$$


$\square$ Propagation delay is proportional to $\mathbf{n}^{2}$ !
■ Insert buffers

$$
m_{\text {opt }}=1.7 \sqrt{\frac{t_{\text {pupf }}}{C R_{\text {eq }}}}
$$

■ In current technologies, $\mathbf{m}_{\mathrm{opt}}$ is typically 3 or 4

## Pass Transistor Logic

- Most typical use: for multiplexing, or path selecting
- Assume in circuit below it is required to either connect $A$ or $B$ to $Y$, under control by $S$
- $Y=A S+B S^{\prime}\left(S^{\prime}\right.$ is easier notation for $S$-bar $=$ S-inverse $=\overline{\mathbf{S}}$ )

■ $Y=\left((A S)^{\prime}(B S)^{\prime}\right)$ ' allows realization with 3 NAND-2 and 1 INV: 14 transistors

- Pass gate needs only 6 (or 8) transistors



S

## Dynamic CMOS gates

## Static vs. Dynamic CMOS Circuits

## Static

- At every point in time (except during the switching transients) each gate output is connected to either Vdd or Vss via a low-resistive path.
- The outputs of the gates assume at all times the value of the Boolean function, implemented by the circuit (except during switching periods)
- Require $2 \mathbf{N}$ transistors for $\mathbf{N}$ inputs (fan-in of $\mathbf{N}$ )


## Dynamic

■ Output not permanently connected to Vdd or Vss

- Output value partly relies on storage of signal values on the capacitance of high impedance circuit nodes.
- Input only active when clock is active
- Requires N+2 transistors for $\mathbf{N}$ inputs


## Dynamic Gate



## Conditions on Output

■ Only one output transition per clock cycle, after CLK $\mathbf{0} \rightarrow \mathbf{1}$. It cannot be charged again until the next precharge operation


- Inputs to the gate can make at most one transition during evaluation
■ Output can be in the high impedance state during and after evaluation (PDN off), state is stored on $C_{L}$


## Properties of Dynamic Gates (1)

■ Logic function is implemented by the PDN only

- Number of transistors is $\mathbf{N + 2}$ (versus 2N for static complementary CMOS)
■ Full swing outputs $\left(\mathrm{V}_{\mathrm{OL}}=\mathrm{V}_{\mathrm{Ss}}\right.$ and $V_{\mathrm{OH}}=\mathrm{V}_{\mathrm{DD}}$ )
- Nonratioed - sizing of the devices is not important for proper functioning


## Properties of Dynamic Gates (2)

■ Faster switching speeds
$\square$ reduced capacitive load to predecessor (only PDN)
■ reduced internal capacitance
 (drain cap. of only one pullup)
■ Low noise margin ( $\mathrm{NM}_{\mathrm{L}}$ )
■ PDN starts to work as soon as the input signals exceed $\mathrm{V}_{\mathrm{TN}} \Rightarrow \mathrm{V}_{\mathrm{M}}$ and $\mathrm{V}_{\mathrm{IL}}$ equal to $\mathrm{V}_{\mathrm{TN}}$

## Properties of Dynamic Gates (3)

- Needs a precharge clock

- \{TPS\} compare power of dynamic vs static CMOS: higher or lower
- Overall power dissipation usually significantly higher than static CMOS
() Reduced capacitance
() no static current path ever exists between $V_{D D}$ and GND (including $P_{s c}$ )
() no glitching
( $\cdot$ higher transition probabilities
© extra load on CLK


## Issues in Dynamic Design (1)

■ Charge leakage - via reversed-biased



## Issues in Dynamic Design (2)



- Charge redistribution

Charge stored originally on $C_{L}$ is redistributed (shared) over $C_{L}$ and $C_{A}$ leading to reduced robustness

If $\Delta V_{\text {out }}>V_{\text {Tn }}$ then $V_{\text {out }}$ and $V_{x}$ reach the same value

$$
\Delta v_{\text {out }}=-v_{D D} \frac{\bar{c}_{a}}{c_{a}+c_{L}}
$$

Target is to keep $\Delta V_{\omega_{L}}<\left|V_{T N}\right|$ since output may drive a static gate

## Issues in Dynamic Design (2)



## Solution to charge redistribution

■ Pre-charge internal nodes using a clock-driven PMOS transistor (at the cost of increased area and power)

## Issues in Dynamic Design (3)

- Backgate Coupling


Dynamic NAND
Static NAND

## Issues in Dynamic Design (3)

■ Backgate Coupling


## Issues in Dynamic Design (4)

■ Clock Feedthrough

- Coupling between $\mathrm{V}_{\text {Out }}$ and $\mathrm{Clk}_{\text {in }}$ of the




## Issues in Dynamic Design (5)

■ Cascading Dynamic Gates


Problem when input of $2^{\text {nd }}$ gate not being 0 during precharge

## Delayed Clocks



Evaluation starts when Out1 is stable

## Domino Logic

Ensures all inputs to the Domino gate is set to 0 during precharge period


## NP Logic, aka NORA Logic



## Differential (Dual Rail) Domino Logic



Solves the problem of non-inverting logic

## Summary

■ Conventional Static CMOS basic principles
■ Complementary static CMOS
-Complex Logic Gates
■VTC, Delay and Sizing

- Ratioed logic

■ Pass transistor logic
■ Dynamic CMOS gates

