

# $\Sigma\Delta$ INTERFERENCE CANCELING ADC'S FOR ANTENNA ARRAYS

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## ABSTRACT

This paper examines a beamforming approach towards reducing the power consumption in oversampling sigma delta ( $\Sigma\Delta$ ) analog to digital converters (ADC's) for multi-user antenna array communication systems. We propose a setup where the output of the ADC's are fed-back to the input of a  $\Sigma\Delta$  modulator through a beamformer to cancel the contributions of interfering signals. We examine the feasibility of such ADC's and propose algorithms to estimate the beamformer and cancel the interfering users. Simulation results show that introducing such algorithms inside the  $\Sigma\Delta$  modulators results in reducing interferer power up to a factor of four for a scenario with 2 users and 4 antennas.

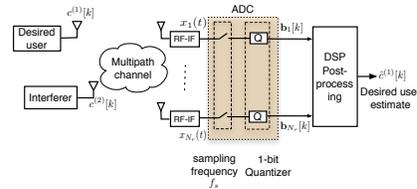
**Keywords:** multi-antenna systems,  $\Sigma\Delta$  modulators, source separation, filter banks, channel identification.

## 1. INTRODUCTION

Antenna arrays at the receiver exploit spatial diversity to achieve reliable communications at low signal energies and in the presence of interferers. However multiple antennas lead to multiple RF and ADC chains, and increase the circuit area and power consumed. The ADC's consume nearly 40 % of the power in the receiver. In a multi-user scenario, even though interfering user signals are usually canceled out in the baseband, the ADC's spend most of their power in processing the interferers. Thus the need to preprocess signals and reduce the power ADC consumption.

The ADC power consumption depends on the sampling frequency  $f_s$  and resolution  $res$  (in bits), approximated as  $P_{adc} \propto f_s 2^{2res}$ . The resolution of the ADC is increased either by amplitude quantization i.e. reducing the quantization step size, or by oversampling at rates much higher than the Nyquist rate. One subclass of oversampling ADC's is a  $\Sigma\Delta$  modulator [1]. The feedback in the  $\Sigma\Delta$  modulator reduces the in-band quantization noise and forces the average value of the oversampled quantized signals to follow that of the input signal.

The advantage of using  $\Sigma\Delta$  ADC's is that the sophisticated post-processing (converting oversampled low res-



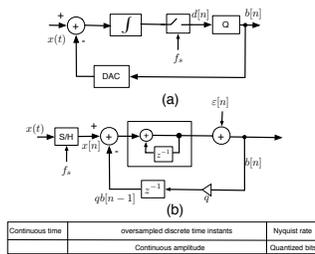
**Fig. 1:** Antenna array configuration with desired and interfering users and oversampled ADC's

olution signals to a Nyquist rate high resolution signal) is performed digitally, which is more robust and consumes less power. One such post-processing approach is shown in [2], where the oversampled signal estimates are improved by convex projections of quantizer outputs.

We consider a setup where the desired and interfering users transmit over a common wireless channel as shown in Fig. 1. These signals are received by an antenna array at the receiver and digitized using  $\Sigma\Delta$  ADC's. The input signal at each  $\Sigma\Delta$  modulator contains contributions from both the desired and interfering users. We reduce ADC power consumption through source separation. Our approach is to insert an additional spatial or temporal beamformer in the feedback block of each ADC to cancel the contributions of interfering signals. Canceling the interfering signals allows a more faithful representation of desired user signals and reduces the ADC power consumption. We refer to our setup with  $\Sigma\Delta$  modulators operating on the antenna array as *multi-channel ADC's*.

We propose an ADC architecture, where the delayed multi-channel ADC outputs are fed-back through a beamformer to the input of the  $\Sigma\Delta$  modulator to cancel the interfering user signals. At each oversampling instant (OSI), the beamformer operates on the multi-channel outputs from the previous OSI's to obtain an estimate of the interfering user. These interfering user estimates are successively canceled before the integrator, inside the  $\Sigma\Delta$  modulators. To facilitate interference cancelation, we exploit the oversampling nature of the  $\Sigma\Delta$  modulator, and combine the wireless channel with the  $\Sigma\Delta$  modulator as a set of polyphase filters [3]. The combined *wireless channel -  $\Sigma\Delta$  modulator* is represented with (a) an analysis filter bank (FB) made of the wireless channel and integrator and (b) a synthesis FB to perform interference

This research was supported in part by the Dutch Min. Econ. Affairs under the IOP-Gencom program (IGC0502B)



**Fig. 2:** (a) First order continuous time  $\Sigma\Delta$  modulator with 1-bit output (b) Discrete time equivalent model

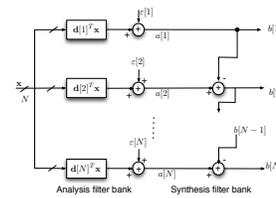
cancelation with the feedback beamformer. We show that the analysis filters of the multi-channel  $\Sigma\Delta$  modulators satisfy the sufficient conditions for channel identification [4, 5] and allow the separation of interfering users. The synthesis filters perform interference cancelation and reconstruct desired user signals. Interference cancelation significantly improves the desired signal to interference plus noise ratio (SINR) at the ADC's. We show through simulation results that our setup reduces the power caused by interfering signals by a factor of 4 for a 2-user and flat fading scenario.

*Notation:*  $(\bar{\cdot})$ ,  $(\cdot)^T$ ,  $(\cdot)^H$ ,  $\odot$  and  $\|\cdot\|$  represent conjugate, transpose, Hermitian, pointwise multiplication and Frobenius norm. Vectors and matrices are represented in lower case and upper case bold letters. Continuous time and sampled signals are indexed as  $(\cdot)$  and  $[\cdot]$  respectively.

## 2. FILTER BANK REPRESENTATION OF $\Sigma\Delta$ MODULATOR

Before we explain the antenna array setup, let us briefly revisit the  $\Sigma\Delta$  modulation. Consider a first order  $\Sigma\Delta$  modulator as shown in Fig. 2(a) and its discrete-time equivalent in Fig. 2(b). The input signal  $x(t)$  is bandlimited by a frequency  $f_0$  and sampled at  $f_s$ , much higher than the Nyquist frequency of  $2f_0$ . The oversampling ratio (OSR)  $N$  is defined as:  $N \geq \lceil \frac{f_s}{2f_0} \rceil + 1$ .  $x(t)$  is oversampled  $N$  times in the interval  $[(k-1)T, kT)$ , where  $T = \frac{N}{f_s}$  is the observation period of  $x(t)$  to obtain a  $N \times 1$  vector  $\mathbf{x}[k]$ . For simplicity, the time index  $k$  is dropped from our notations i.e  $\mathbf{x}[k]$  is rewritten  $\mathbf{x} = [x[1], \dots, x[N]]^T$ .

The oversampled signal  $x[n]$   $n \in [1 \dots N]$  are assumed to be present in the no overload range  $[-q : q]$ .  $x[n]$ , usually scaled by  $\frac{1}{q}$  feeds an integrator, followed by a quantizer. For sake of simplicity consider a 1-bit quantizer  $Q\{\cdot\}$ , operating on the integrator output  $d[n]$ . The quantizer output has possible values  $+1$  or  $-1$  depending on the positive or the negative sign of the integrator output  $b[n] = Q\{d[n]\}$ . The quantizer output  $b[n]$  is fed back and subtracted from the input through a digital to analog converter (DAC):  $\text{DAC}(b[n]) = qb[n-1]$ .



**Fig. 3:**  $\Sigma\Delta$  modulator represented as an analysis filter bank

The discrete-time integrator yields a difference equation  $d[n] = \frac{1}{q}x[n] + d[n-1] - qb[n-1]$ , where  $d[0]$  is zero or a constant. For observation period  $[(k-1)T, kT)$ , the  $\Sigma\Delta$  modulator outputs a binary sequence  $\mathbf{b}[k] = [b[1], \dots, b[N]]^T$ .

**Proposition 1:** In the no overload region, the one-bit  $\Sigma\Delta$  modulator yields

$$b[n] = Q \left\{ \frac{1}{q} \sum_{j=1}^n x[j] - \sum_{j=1}^{n-1} b[j] \right\} \quad (1)$$

We show the proposition by induction on  $n$ . The initial value  $b[0]$  is a constant (usually made 0), and  $b[1]$  follows the quantized output  $Q\{\frac{1}{q}x[1]\}$ . We assume that (1) holds true for certain oversampled instants  $1, \dots, n$ .

$$\begin{aligned} b[n+1] &= Q \left\{ \frac{1}{q} \left[ x[n+1] + \sum_{j=1}^n x[j] - \sum_{j=0}^n qb[j] \right] \right\} \\ &= Q \left\{ \frac{1}{q} \sum_{j=1}^{n+1} x[j] - \sum_{j=0}^n b[j] \right\}. \end{aligned}$$

The  $\Sigma\Delta$  modulation can be simplified by modeling  $Q\{\cdot\}$  as a noise source  $\varepsilon[n]$  independent of the signal (Fig. 2(b)) [1]. Replacing  $Q\{\cdot\}$  in proposition 1 by  $\varepsilon[n]$  and assuming  $b[0] = 0$ ,  $b[n] = \frac{1}{q} \sum_{j=1}^n x[j] + \varepsilon[n] - \sum_{j=1}^{n-1} b[j]$ . The result from proposition 1 enables us to separate the  $\Sigma\Delta$  modulator as linear model with noise:  $a[n] = \sum_{j=1}^n x[j] + \varepsilon[n]$ , subtracted by the feedback term:  $\sum_{j=1}^{n-1} b[j]$ .  $a[n]$  is the integrator output, without the feedback term. Stacking  $a[n]$  for  $n \in [1 \dots, N]$

$$\begin{aligned} \begin{bmatrix} a[1] \\ \vdots \\ a[N] \end{bmatrix} &= \begin{bmatrix} 1 & & \\ & \ddots & \\ 1 & \dots & 1 \end{bmatrix} \begin{bmatrix} x[1] \\ \vdots \\ x[N] \end{bmatrix} + \begin{bmatrix} \varepsilon[1] \\ \vdots \\ \varepsilon[N] \end{bmatrix} \\ \Leftrightarrow \mathbf{a} &= \mathbf{U}^T \mathbf{x} + \boldsymbol{\varepsilon}. \end{aligned} \quad (2)$$

The  $N \times N$  upper triangular matrix  $\mathbf{U}$  can also be viewed as an analysis FB  $\mathbf{U} = [\mathbf{d}[1], \dots, \mathbf{d}[N]]$  operating on  $\mathbf{x}$  i.e.  $a[n] = \mathbf{d}[n]^T \mathbf{x} + \varepsilon$  (refer Fig. 3). The  $\Sigma\Delta$  modulator output is obtained from the synthesis FB as

$$b[n] = \begin{cases} a[n] & n = 1 \\ a[n] - b[n-1] & n \in \{2, \dots, N\}. \end{cases}$$

This approach is somewhat similar to [2], where the authors show that the  $\Sigma\Delta$  operation can be seen as a vector quantization of the input signal ( $\mathbf{x}$ ). Our perspective is to look at  $\mathbf{a}$  as a polyphase representation [3] of  $\mathbf{x}$  using a filter bank  $\mathbf{U}$ . Our objective is to combine the polyphase representation of  $\mathbf{x}$  with multi-user fractionally spaced wireless channel and cancel the interfering users as seen in the next section.

### 3. SOURCE SEPARATION WITH ANTENNA ARRAYS

#### 3.1. Data model

We consider a multi-user scenario where the desired and interfering users occupy a common wireless channel. For sake of simplicity we assume  $N_t = 2$  with user 1 being the desired user and user 2 the interfering user, transmitting  $c^{(1)}[k]$  and  $c^{(2)}[k]$  respectively. These signals are received as shown in Fig. 1 by an array of  $N_r$  antennas. For the time being we assume a narrowband channel and only one receive antenna, later we will extend to the array of  $N_r$  antennas.

$x(t)$  denotes a continuous time signal at the antenna, obtained from wireless channel  $g^{(j)}$  operating on the transmitting signal from user  $j$ :  $c^{(j)}(t)$ ,  $\mathbf{c}(t) = [c^{(1)}(t), c^{(2)}(t)]^T$  and  $\mathbf{x}$  is sampled version of  $x(t)$ .

$$\begin{aligned} x(t) &= [g^{(1)} \quad g^{(2)}] \mathbf{c}(t) + v(t) \\ \Rightarrow \mathbf{x} &= [\mathbf{g}^{(1)} \quad \mathbf{g}^{(2)}] \mathbf{c} + \mathbf{v}. \end{aligned} \quad (3)$$

$\mathbf{c}$  denotes its discrete time equivalent of  $\mathbf{c}(t)$  and  $\mathbf{x}$ ,  $\mathbf{g}^{(1)}$ ,  $\mathbf{g}^{(2)}$  and  $\mathbf{v}$  are discrete time  $N \times 1$  vectors corresponding respectively to the channel output, channel responses of the desired user, interfering user and noise in Fig. 4(a).

The channel  $\mathbf{G} = [\mathbf{g}^{(1)} \quad \mathbf{g}^{(2)}]$  is combined with the analysis FB of the  $\Sigma\Delta$  modulator as shown in Fig. 4(b). The transmitted signals  $c^{(j)}$  are modulated by the *fractionally spaced channel-integrator* block to obtain  $\mathbf{a}$ :

$$a[n] = \sum_{l=1}^{N_t=2} \mathbf{d}[n]^T \mathbf{g}^{(l)} c^{(l)} + \varepsilon[n] = \sum_{l=1}^{N_t} h^{(l)}[n] c^{(l)} + \varepsilon[n] \quad (4)$$

where  $\varepsilon[n] = v[n] + \varepsilon[n]$  and  $v[n]$  is the thermal noise for OSI  $n$ . The fractionally spaced wireless channel and its relationship with antenna signals and the integrator output i.e.  $\mathbf{x} \rightarrow \mathbf{a}$  allows  $\mathbf{a}$  to be represented as a combination of desired and interfering user signals. Note that the elements of  $\mathbf{a}$  are low resolution signals (usually 1-bit).  $a[n]$  can be seen as different phases of  $\mathbf{g}_i$  operating on  $c^{(1)}$ , hence the polyphase representation.

The low resolution signal  $a[n]$  can be viewed as the output of the fractionally spaced  $h^{(l)}[n] = \mathbf{d}[n]^T \mathbf{g}^{(l)}$  whose input is  $c^{(l)}$  for  $l = \{1, 2\}$  as shown in Fig. 4(b).

Our objective is to cancel the interfering user signals in (4) before the integrator. (4) contains  $N_t$  unknowns, and one approach to extract/cancel user signals at each OSI is through exploiting the antenna array setup. Neglecting the thermal and quantization noise, the integrator output of the antenna array and OSI  $n$  can be stacked as

$$\begin{aligned} \begin{bmatrix} a_1[n] \\ \vdots \\ a_{N_r}[n] \end{bmatrix} &= \sum_{l=1}^{N_t} \begin{bmatrix} \mathbf{g}_1^{(l)T} \\ \vdots \\ \mathbf{g}_{N_r}^{(l)T} \end{bmatrix} \mathbf{d}[n] c^{(l)} \\ \Leftrightarrow \mathbf{a}[n] &= \sum_{l=1}^{N_t} \mathbf{h}^{(l)}[n] c^{(l)} \end{aligned} \quad (5)$$

where  $\mathbf{g}_i^{(l)}$  is a  $N \times 1$  channel response from user  $l$  to antenna  $i$ ,  $\mathbf{h}^{(l)}$  is the antenna array response from user  $l$  and  $a_i[n]$  is the integrator output at antenna  $i$ .

#### 3.2. Baseband source separation

For antenna  $i$  and OSI  $n$ , the input signals  $x_i[n]$  can be separated into contributions from desired user  $\hat{x}_i^{(1)}[n]$  and interfering user  $\hat{x}_i^{(2)}[n]$ :  $x_i[n] = \hat{x}_i^{(1)}[n] + \hat{x}_i^{(2)}[n]$ . An estimate of  $\hat{x}_i^{(2)}[n]$  is computed using a  $N_r \times 1$  beamformer  $\mathbf{w}_i[n]$  operating on the antenna array signals  $\mathbf{a}[n]$ :

$$\hat{x}_i^{(2)}[n] = \mathbf{w}_i[n]^H \mathbf{a}[n] \quad (6)$$

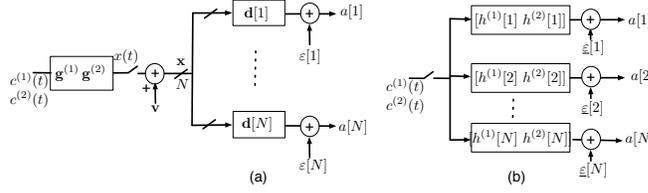
An estimate of desired signal  $\hat{x}_i^{(1)}[n]$  is given by  $\hat{x}_i^{(1)}[n] = x_i[n] - \mathbf{w}_i[n]^H \mathbf{a}[n]$ .

### 4. INTERFERENCE CANCELATION WITH $\Sigma\Delta$ MODULATION

Our objective is to predict  $\hat{x}_i^{(2)}[n]$  using  $\mathbf{w}_i[n]^H \mathbf{a}[n-1]$ . This allows us to successively subtract the interfering user signals, from the input signal  $x_i[n]$ , thus the  $\Sigma\Delta$  ADC operates only on the desired signal and consumes less power.

#### 4.1. Conditions for source separation in fractionally spaced channels

The integrator output for an antenna  $i$ ,  $i \in \{1, \dots, N_r\}$  is rewritten from (2), and neglecting the noise terms as  $\mathbf{a}_i = \mathbf{U}^T \mathbf{x}_i$ . Given the input signal  $\mathbf{x}_i$  and the channel response  $\mathbf{G}_i = [\mathbf{g}_i^{(1)}, \mathbf{g}_i^{(2)}]$ , their relationship with integrator output  $\mathbf{a}_i$  follows (3) and (4).  $\mathbf{a}_i = \mathbf{U}^T \mathbf{x}_i$  can be interpreted as a set of low pass filters (LPF's) operating on the fractionally spaced channel output  $\mathbf{x}_i$ .  $a_i[1]$ , obtained from  $[h_i^{(1)}[1] \quad h_i^{(2)}[1]]$ , is an all pass filter operation on the wireless channel:  $\mathbf{d}[1]^T \mathbf{G}_i$ . For  $n \in \{2, \dots, N\}$ ,



**Fig. 4:** (a) Oversampled wireless channel with analysis filter (b) equivalent *wireless channel-integrator*  $[h_i^{(1)}[n] h_i^{(2)}[n]]$  is  $\mathbf{d}[n]^T \mathbf{G}_i$ . The structure of  $\mathbf{d}[n]$  suggests that  $\mathbf{a}[n]$  can be seen as a moving average operation with increasing for increasing of  $n$ .

The LPF relationship is more visible in the frequency domain. The  $Z$  transform of the channel response  $[h_i^{(1)}[n] h_i^{(2)}[n]]$  at oversampling rate  $\frac{N}{T}$  and antenna  $i$  is

$$H_{i,n}(z) = \sum_{l=1}^{N_t} \left[ \mathbf{d}[n] \odot \mathbf{g}_i^{(l)} \right]^T \begin{bmatrix} 1 \\ \vdots \\ z^{-(N-1)} \end{bmatrix}. \quad (7)$$

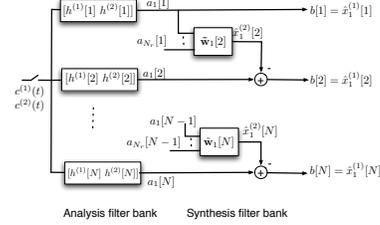
From (7), at OSI  $n$  and antenna  $i$ , the  $H_{i,n}(z)$  is obtained by a LPF operation on  $H_{i,n-1}(z)$ . A sufficient condition for channel identification from polyphase components is that there needs to be aliasing or overlap between the spectrum of adjacent polyphase components [5]. The LPF interpretation suggests that there is considerable overlap between the adjacent oversampled phases of  $H_{i,n}(z)$  and  $H_{i,n-1}(z)$ . To estimate  $h_i^{(2)}[n]$  and cancel the contributions of  $c^{(2)}$ , we exploit spatial diversity of the multi-channel  $\Sigma\Delta$  response and use  $H_{j,n-1}(z) \forall j \in \{1, \dots, N_r\}$  to identify  $H_{i,n}(z)$ . Interference cancellation is accomplished by multiple spatial channels, hence we refer our setup as *multi-channel  $\Sigma\Delta$  ADC's*.

#### 4.2. Multi-channel $\Sigma\Delta$ ADC's

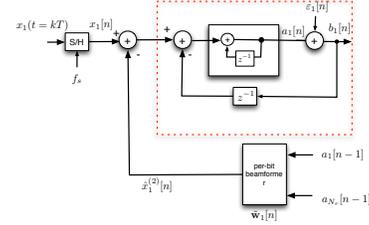
Our approach is to first estimate the interfering user contributions at antenna  $\hat{x}_i^{(2)}[n]$  using  $\mathbf{a}[n-1] = [a_1[n-1], \dots, a_{N_r}[n-1]]^T$ . Canceling the interfering user signals through feedback in the  $\Sigma\Delta$  modulator allows representation the desired user signals in reduced resolution.

From (6) we know to obtain  $\hat{x}_i^{(2)}[n]$  from  $\mathbf{a}[n]$ . At the start of data packet, training signals are used to design the beamformer  $\mathbf{w}_i[n]$  as explained in the next section. Exploiting the properties of section 4.1, we estimate  $\mathbf{a}[n]$  from  $\mathbf{a}[n-1]$  using a  $N \times N$  fitting matrix  $\mathbf{F}[n]: \hat{\mathbf{a}}[n] = \mathbf{F}[n]\mathbf{a}[n-1]$ . The estimate  $\hat{\mathbf{a}}[n]$  is plugged into (6) to compute  $\hat{x}_i^{(2)}[n] = \tilde{\mathbf{w}}_i[n]^H \mathbf{a}[n-1]$ , where  $\tilde{\mathbf{w}}_i[n] = \mathbf{F}[n]^H \mathbf{w}_i[n-1]$ .  $\tilde{\mathbf{w}}_i[n]$  is appropriately scaled using the interfering user channel response at OSI  $n$  and antenna  $i$  i.e.  $\tilde{\mathbf{w}}_i[n] = \tilde{\mathbf{w}}_i[n] h_i^{(2)}[n]$ . The interfering user contribution is fed-back and subtracted from  $x_i[n]$ .

The interference cancellation for symbol period  $T$  is explained with analysis and synthesis FB in Fig. 5. The



**Fig. 5:** Filter bank interpretation of multi-channel  $\Sigma\Delta$  modulator at antenna 1



**Fig. 6:** Interference cancellation with multi-channel  $\Sigma\Delta$  ADC

analysis filters consists of the combined channel given by (4) and the synthesis filters consists of the interference cancellation and quantizer. The sequential operation of the multi-channel  $\Sigma\Delta$  modulator at oversampling rate with a beamformer in the feedback block at antenna 1:  $x_1[n]$  is illustrated in Fig. 6. The 1-bit modulator output from the antenna array  $\mathbf{a}[n-1]$  is fed-back through a beamformer and subtracted with the input signal  $x_1[n]$  to cancel interfering users. The operations are detailed in Table 1.

**Objective:** To cancel interference with multi-channel  $\Sigma\Delta$ .  
**Given:** Input signal  $\mathbf{x}_i$  for  $i \in \{1, \dots, N_r\}$  antennas.

- Initialize:  $\mathbf{a}[0] = \mathbf{0}; \mathbf{b}[0] = \mathbf{0}$
- For OSI  $n = 1$  to  $N$  and antennas  $i = 1$  to  $N_r$ 
  - $\hat{x}_i^{(1)}[n] = x_i[n] - \tilde{\mathbf{w}}_i[n]^H \mathbf{a}[n-1]$
  - $\hat{x}_i^{(1)}[n] = \hat{x}_i^{(1)}[n] - b_i[n-1]$
  - Integrator output  $a_i[n] = \sum_{m=1}^n \hat{x}_i^{(1)}[m]$
  - $b_i[n] = Q\{a_i[n]\}$

**Table 1:** Interference cancellation with multi-channel  $\Sigma\Delta$

## 5. $\Sigma\Delta$ BEAMFORMER DESIGN

In the previous section, we introduced a multi-channel  $\Sigma\Delta$  ADC setup to perform interference cancellation successively through feedback. In this section we design the beamformer  $\tilde{\mathbf{w}}_i[n]$ . The desired and interfering user signals as well as the thermal & quantization noise are assumed to be independent of each other and identically distributed with zero mean and unit variance [A1]. For the time being, we assume knowledge of the training sequence of the interfering user<sup>1</sup>.

For each OSI  $n$ , the training symbols from the interfering user are used to design a beamformer  $\tilde{\mathbf{w}}_i[n]$  and minimize the mean squared error between the interfering user and its estimate [6] (chapter 8). The polyphase version of the  $\Sigma\Delta$  modulator is exploited to design  $\tilde{\mathbf{w}}_i[n]$ :

$$\underset{\tilde{\mathbf{w}}_i[n]}{\operatorname{argmin}} E \|\hat{x}_i^{(2)}[n] - \tilde{\mathbf{w}}_i[n]^H \mathbf{b}[n-1]\|^2 \quad \forall n \in \{1, \dots, N\} \quad (8)$$

where expectation  $E$  is taken over the entire training packet. The symbol period of  $c^{(2)}[n]$  is  $T$ , and is slowly varying in oversampled domain, allowing the approximation  $c^{(2)}[n] = c^{(2)}, \forall n \in \{1, \dots, N\}$ .  $g_i^{(2)}[n]$  at antenna  $i$  and OSI  $n$  is estimated as:

$$\begin{aligned} E\{\mathbf{x}_i \bar{c}^{(2)}\} &= E\{([\mathbf{g}_i^{(1)} \mathbf{g}_i^{(2)}] \mathbf{c} + \boldsymbol{\varepsilon}_i) \bar{c}^{(2)}\} \\ &\Rightarrow \hat{x}_i[n] \bar{c}^{(2)} = \hat{g}_i^{(2)}[n] E\{c^{(2)}\}^2. \end{aligned}$$

The beamformer  $\tilde{\mathbf{w}}_i[n]$  can also be obtained from other techniques well documented in the literature [7].  $\tilde{\mathbf{w}}_i[n]$  is scaled as  $\tilde{\mathbf{w}}_i[n] = \tilde{\mathbf{w}}_i[n] \hat{h}_i^{(2)}[n]$ . The sequence of operations to estimate beamformer is detailed in Table 2. This approach can be extended to frequency selective fading channels and is current research. The multi-channel  $\Sigma\Delta$  ADC outputs are postprocessed (baseband source separation [7]) to get a high resolution estimate of the desired user.

Objective: Beamformer design for interference cancellation in  $\Sigma\Delta$  modulators.

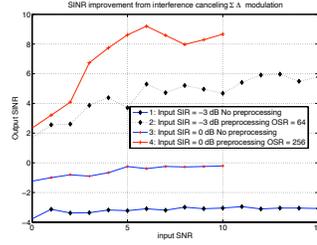
Given:  $\mathbf{x}_i$ ,  $\mathbf{b}[n]$  and  $c^{(2)}$  for the entire training packet.

- For  $n = 1 : N$  and antennas  $1 : N_r$ 
  - $\operatorname{argmin}_{\tilde{\mathbf{w}}_i[n]} E \|\bar{c}^{(2)} - \mathbf{w}[n]^H \mathbf{b}[n-1]\|^2$
  - $\hat{g}_i^{(2)}[n] = \frac{E\{\mathbf{x}_i[n] \bar{c}^{(2)}\}}{E\{|c^{(2)}\}^2}$
  - $\tilde{\mathbf{w}}_i[n] = \tilde{\mathbf{w}}[n] \hat{g}_i^{(2)}[n]$

## 6. SIMULATION RESULTS

The performance of multi-channel  $\Sigma\Delta$  ADC is observed for a  $N_t = 2, N_r = 4$  system. All users transmit BPSK

<sup>1</sup>This is current research and will be lifted in future work



**Fig. 7:** Performance of multi-channel  $\Sigma\Delta$  modulator signals over a randomly generated Rayleigh fading channel. At the start of each data packet, a training sequence is transmitted. The antenna array signals are sampled with a multi-channel  $\Sigma\Delta$  modulator operating on each antenna with OSR =64 or 256. Fig. 7 compares the power savings of  $\Sigma\Delta$  modulator having interference cancellation with a  $\Sigma\Delta$  modulator without interference cancellation. In each experiment, preprocessing refers to interference cancellation before the integrator inside  $\Sigma\Delta$  modulator (refer Fig. 6). The curves 1 and 2 show the SINR measured after the  $\Sigma\Delta$  modulation for a transmitted SINR of -3 dB. Interference cancellation results in an increase of 6 dB signal energy, which suggests that the power consumed due to interfering users in the ADC circuit can be reduced nearly by a factor of 4. Similar experiments for SINR=0 dB are seen from curves 3 & 4.

These results do not take the latency caused by the interference canceler into account. However, considering that ADC's consume most power in a receiver these results provide significant insight into translating the theoretical performance of MIMO systems in reality.

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