SI-JADE: AN ALGORITHM FOR JOINT ANGLE AND DELAY ESTIMATION USING SHIFT-INVARIANCE PROPERTIES

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In a multipath communication scenario, it is often relevant to estimate the directions and relative delays of each multipath ray. We derive a closed-form subspace-based algorithm for the joint high-resolution estimation of both angles and delays from measured impulse response data, assuming knowledge of the modulation pulse shape function. The algorithm uses a 2-D ESPRIT-like shift-invariance technique to separate and estimate the phase shifts due to delay and direction-of-incidence, with automatic pairing of the two parameter sets.

1. INTRODUCTION

Source localization is one of the recurring problems in electrical engineering. In mobile communications, source localization by the base station is of interest for advanced handover schemes, emergency localization, and potentially many user services for which a GPS receiver is impractical. In a multipath scenario, this involves the estimation of the directions and relative delays of each multipath ray. It is often assumed that the directions and delays of the paths do not change quickly, as fading affects only their powers, so that it makes sense to estimate these parameters. The parameters are essential for space-time selective transmission in the downlink, especially in FDD systems.

In this paper, we derive an algorithm for the joint high-resolution estimation of multipath angles and delays, assuming linearly modulated sources with a known pulse shape function and no appreciable doppler shifts. Specifically, we work under the following conditions:

1. The number of sources is small. For convenience, we consider only one source here.
2. The multipath consists of discrete rays, each parameterized by a delay, complex amplitude (fading), and angle.
3. A channel estimate is available.
4. Doppler shifts and residual carriers of sources are neglected.
5. The source signals are received by a uniform linear antenna array consisting of at least two antennas spaced at half-wavelength or closer. (Extensions are possible.)
6. The data received by the antennas is sampled at or above the Nyquist rate.

The method is based on a transformation of the impulse response data by a DFT and a deconvolution by the known pulse shape function, which transforms temporal shifts into phase shifts in the frequency domain. This is of course a classical approach and has been

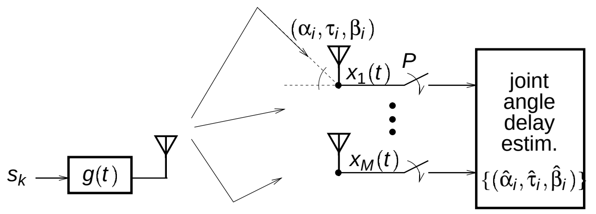

Figure 1. Multiray propagation channel

considered e.g., in [1, 2] as well. New in this paper is the observation that by stacking the result into a Hankel matrix, the problem is reduced to one that can be solved using 2-D ESPRIT techniques [3, 4], which was developed for joint azimuth-elevation estimation. Thus, the algorithm is closed-form and computationally attractive, and angles and delays are jointly estimated and automatically paired. The number of rays may be larger than the number of antennas.

2. DATA MODEL

Assume we transmit a digital sequence $\{s_k\}$ over a channel, and measure the response using M antennas. The noiseless received data in general has the form $\mathbf{x}(t) = \sum_{k=1}^{N} s_k \mathbf{h}(t-kT)$, where T is the symbol rate, which will be normalized to $T = 1$ from now on. A commonly used multiray propagation model, for specular multipath, writes the $M\times 1$ channel impulse response as

$$\mathbf{h}(t) = \sum_{i=1}^{r} \mathbf{a}(\alpha_i)\beta_i g(t-\tau_i)$$

where $g(t)$ is a known pulse shape function by which $\{s_k\}$ is modulated. In this model, there are r distinct propagation paths, each parameterized by $(\alpha_i, \tau_i, \beta_i)$, where α_i is the direction-of-arrival (DOA), τ_i is the path delay, and $\beta_i \in \mathbb{C}$ is the complex path attenuation (fading). The vector-valued function $\mathbf{a}(\alpha)$ is the array response vector to a signal from direction α.

Suppose $\mathbf{h}(t)$ has finite duration and is zero outside an interval $[0, L)$, where L is the (integer) channel length. We assume that the received data $\mathbf{x}(t)$ is sampled at a rate P times the symbol rate. Using either training sequences (known $\{s_k\}$) or perhaps blind channel estimation techniques, it is possible to estimate $\mathbf{h}(k)$, $k = 0, \frac{1}{P}, \cdots, L-\frac{1}{P}$, at least up to a scalar.

Collect the samples of the known waveform $g(t)$ into a row vector $\mathbf{g} = [g(0)\;\; g(\frac{1}{P})\;\; \cdots\;\; g(L-\frac{1}{P})]$. The data model can be written as

$$\begin{aligned} H &:= [\mathbf{h}(0) \cdots \mathbf{h}(L-\tfrac{1}{P})] \\ &= [\mathbf{a}_1 \cdots \mathbf{a}_r] \begin{bmatrix} \beta_1 & & \\ & \ddots & \\ & & \beta_r \end{bmatrix} \begin{bmatrix} \mathbf{g}_1 \\ \vdots \\ \mathbf{g}_r \end{bmatrix} =: ABG \end{aligned} \quad (1)$$

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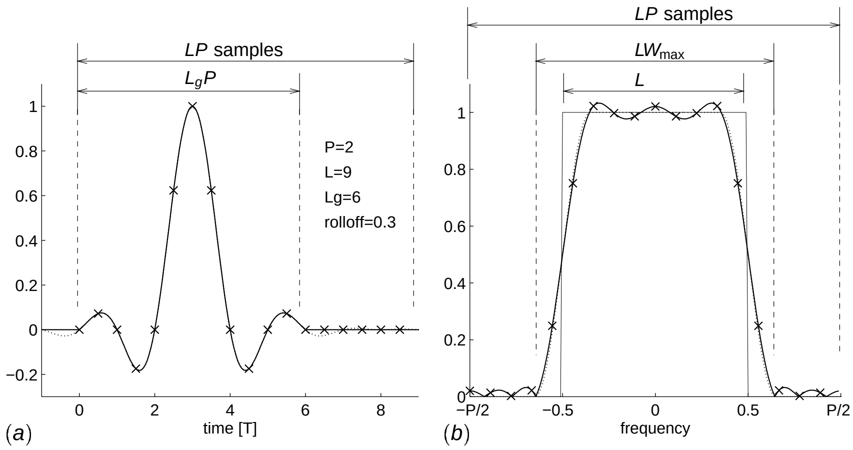

Figure 2. Definition of parameters. (*a*) time domain, (*b*) frequency domain.

where $\mathbf{a}_i = \mathbf{a}(\alpha_i)$, and $\mathbf{g}_i = [g(t-\tau_i)]_{k=0,1/P,\cdots;L-1/P}$ is a row vector containing the samples of $g(t-\tau)$.

The delay estimation algorithm is based on the property that the Fourier transform maps a delay to a phase shift. Thus let $\tilde{\mathbf{g}} = \mathbf{gF}$ where $\mathbf{F}$ denotes the DFT matrix of size $LP \times LP$, defined by

$$\mathbf{F} := \begin{bmatrix} 1 & 1 & \cdots & 1 \\ 1 & \phi & \cdots & \phi^{LP-1} \\ \vdots & \vdots & & \vdots \\ 1 & \phi^{LP-1} & \cdots & \phi^{(LP-1)^2} \end{bmatrix}, \qquad \phi = e^{-j\frac{2\pi}{LP}}.$$

If τ is an integer multiple of $\frac{1}{P}$, or if $g(t)$ is bandlimited* and we sample at or above the Nyquist rate, then it is straightforward to see that the Fourier transform $\tilde{\mathbf{g}}_\tau$ of the sampled version of $g(t-\tau)$ is given by $\tilde{\mathbf{g}}_\tau = [1 \;\; \phi^{\tau P} \;\; (\phi^{\tau P})^2 \; \cdots \; (\phi^{\tau P})^{LP-1}] \, \mathrm{diag}(\tilde{\mathbf{g}})$. The same holds approximately true if τ is not an integer multiple of $\frac{1}{P}$, depending on the bandwidth of $g(t)$ and the number of samples LP. Thus we can write the Fourier-transformed data model $\tilde{H} := H\mathbf{F}$ as $\tilde{H} = ABF\,\mathrm{diag}(\tilde{\mathbf{g}})$, where

$$F_{LP} := \begin{bmatrix} 1 & \phi_1 & \phi_1^2 & \cdots & \phi_1^{LP-1} \\ \vdots & \vdots & & & \vdots \\ 1 & \phi_r & \phi_r^2 & \cdots & \phi_r^{LP-1} \end{bmatrix}, \quad \phi_i := e^{-j\frac{2\pi}{L}\tau_i}$$

(we usually omit the size index of F). The next step is to do a deconvolution of $g(t)$ by dividing $\tilde{H}$ by $\mathrm{diag}(\tilde{\mathbf{g}})$. Obviously, this can be done only on intervals where $\tilde{\mathbf{g}}$ is nonzero. To be specific, assume that $g(t)$ is bandlimited with bandwidth $W_{\max}$ (that is, its Fourier transform is nonzero only for frequencies $|f| \le \frac{1}{2}W_{\max}$), and that $P > W_{\max}$. Then $\tilde{\mathbf{g}}$ has at most $LW_{\max}$ nonzero entries, and we can limit our attention to this interval. For a raised-cosine pulse shape with roll-off factor (excess bandwidth) ρ, we have $W_{\max} = 1+\rho$, see figure 2. Usually, however, we would select a somewhat smaller number, W say, since the entries at the border can be relatively small as well, and their inversion can blow up the noise. Indeed, in the case of a raised-cosine pulse, we advise to set $W = 1$ and select only the L center frequency samples.

Let $J_{\tilde{\mathbf{g}}} : LP \times LW$ be the corresponding selection matrix for $\tilde{\mathbf{g}}$. (It should be such that the selected frequencies appear in increasing order.) If there are no other (intermittent) zeros, we can factor $\tilde{\mathbf{g}}J_{\tilde{\mathbf{g}}}$ out of $\tilde{H}J_{\tilde{\mathbf{g}}}$ and obtain

$$\bar{H} := H\mathbf{F}J_{\tilde{\mathbf{g}}}\{\mathrm{diag}(\tilde{\mathbf{g}}J_{\tilde{\mathbf{g}}})\}^{-1}, \qquad (M \times LW).$$

which (up to a possible phase shift in B) satisfies the model

$$\bar{H} = ABF. \tag{2}$$

If $r \le M$, then it is possible to estimate the ϕ_i's and hence the τ_i's from the shift-invariance structure of F, independent of the structure of A, which is essentially the ESPRIT algorithm. To estimate the DOAs as well, we need to know the array manifold structure. For simplicity, we will assume a uniform linear array (ULA) consisting of omnidirectional elements with interelement spacing of Δ wavelengths, but other configurations are possible. The correct pairing of the τ_i's to the α_i's requires the use of ideas from 2-D DOA estimation (viz. [3,4]).

In general, the number of antennas is not large enough to satisfy $M > r$. We can avoid this problem by constructing a Hankel matrix out of $\bar{H}$.

3. JOINT DELAY AND ANGLE ESTIMATION

3.1. Algorithm outline

Our objective is to estimate $\{(\alpha_i, \tau_i)\}$ from the shift-invariance properties present in the data model $\bar{H} = ABF$. Let us assume that our antenna array is a uniform linear array consisting of M omnidirectional antennas spaced at a distance of Δ wavelengths. For integers $2 \le m_1 \le LW$, $1 \le m_2 \le M-1$, define

$$\Theta = \mathrm{diag}[\theta_1 \cdots \theta_r], \quad A_\theta = \begin{bmatrix} 1 & \cdots & 1 \\ \theta_1 & \cdots & \theta_r \\ \vdots & & \vdots \\ \theta_1^{M-m_2} & \cdots & \theta_r^{M-m_2} \end{bmatrix}, \quad \theta_i := e^{j2\pi\Delta\sin\alpha_i}.$$

$$\Phi = \mathrm{diag}[\phi_1 \cdots \phi_r], \quad A_\phi = \begin{bmatrix} 1 & \cdots & 1 \\ \phi_1 & \cdots & \phi_r \\ \vdots & & \vdots \\ \phi_1^{m_1-1} & \cdots & \phi_r^{m_1-1} \end{bmatrix}, \quad \phi_i = e^{-j\frac{2\pi}{L}\tau_i}.$$

*This is not in full agreement with the FIR assumption. The truncation widens the pulse bandwidth, introducing a small bias due to aliasing.

Also define the following equal-sized submatrices of $\bar{H}$:

$$\bar{H}^{(i,j)} = \begin{bmatrix} \bar{H}_{i,j} & \cdots & \bar{H}_{i,LW-m_1+j} \\ \vdots & & \vdots \\ \bar{H}_{M-m_2+i,j} & \cdots & \bar{H}_{M-m_2+i,LW-m_1+j} \end{bmatrix}, \qquad \begin{array}{l} 1 \le i \le m_2, \\ 1 \le j \le m_1 . \end{array}$$

Using (2), it is straightforward to show that $\bar{H}^{(i,j)}$ has a factorization $\bar{H}^{(i,j)} = A_\theta B \Theta^{i-1} \Phi^{j-1} F$. If we now construct a Hankel-like matrix $\mathcal{H}$ as

$$\mathcal{H} = \begin{bmatrix} \bar{H}^{(1,1)} & \cdots & \bar{H}^{(m_2,1)} \\ \vdots & & \vdots \\ \bar{H}^{(1,m_1)} & \cdots & \bar{H}^{(m_2,m_1)} \end{bmatrix}, \quad m_1(M-m_2+1) \times m_2(LW-m_1+1) .$$

then $\mathcal{H}$ has a factorization

$$\mathcal{H} = \mathcal{A} B \mathcal{F} := \begin{bmatrix} A_\theta \\ A_\theta \Phi \\ \vdots \\ A_\theta \Phi^{m_1-1} \end{bmatrix} B [F \quad \Theta F \quad \cdots \quad \Theta^{m_2-1} F]$$

The parameters m_1 and m_2 should be used to ensure that this is a rank-deficient matrix, if possible (this puts a limit on the number of rays that can be estimated). Usually the number of antennas is limited and we would select $m_2 = 1$, but $m_2 > 1$ is needed to ensure that $\mathcal{F}$ is full rank in case two sources have the same delay.

The idea is to estimate the column span of $\mathcal{H}$, which is equal to the column span of $\mathcal{A}$ provided $\mathcal{F}$ is full rank. Note that $\mathcal{A} = (A_\phi \diamond A_\theta)$, where $\diamond$ denotes a column-wise Kronecker product. The estimation of Φ and Θ from the column span of $\mathcal{H}$ is based on exploiting the various shift-invariant structures present in $A_\phi \diamond A_\theta$. Define selection matrices

$$\begin{array}{ll} J_{x\phi} := [I_{m_1-1} \quad 0_1] \otimes I_{M-m_2+1} , & J_{x\theta} := I_{m_1} \otimes [I_{M-m_2} \quad 0_1] , \\ J_{y\phi} := [0_1 \quad I_{m_1-1}] \otimes I_{M-m_2+1} , & J_{y\theta} := I_{m_1} \otimes [0_1 \quad I_{M-m_2}] , \end{array}$$

and let $X_\phi = J_{x\phi}\mathcal{H}$, $Y_\phi = J_{y\phi}\mathcal{H}$, $X_\theta = J_{x\theta}\mathcal{H}$, $Y_\theta = J_{y\theta}\mathcal{H}$. These data matrices have the structure

$$\left\{ \begin{array}{lcl} X_\phi & = & A' B \mathcal{F} \\ Y_\phi & = & A' \Phi B \mathcal{F} \end{array} \right. \qquad \left\{ \begin{array}{lcl} X_\theta & = & A'' B \mathcal{F} \\ Y_\theta & = & A'' \Theta B \mathcal{F} \end{array} \right. \tag{3}$$

where $A' = J_{x\phi}\mathcal{A}$, $A'' = J_{x\theta}\mathcal{A}$. If dimensions are such that these are low-rank factorizations, then we can apply the 2-D ESPRIT algorithm [3, 4] to estimate Φ and Θ. In particular, since

$$\begin{array}{lcl} Y_\phi - \lambda X_\phi & = & A'[\Phi - \lambda I_r] B \mathcal{F} \\ Y_\theta - \lambda X_\theta & = & A''[\Theta - \lambda I_r] B \mathcal{F} \end{array}$$

the ϕ_i are given by the rank reducing numbers of the pencil (Y_ϕ, X_ϕ), whereas the θ_i are the rank reducing numbers of (Y_θ, X_θ). These are the same as the nonzero eigenvalues of $X_\phi^\dagger Y_\phi$ and $X_\theta^\dagger Y_\theta$. († denotes the Moore-Penrose pseudo-inverse.)

The correct pairing of the ϕ_i with the θ_i follows from the fact that $X_\phi^\dagger Y_\phi$ and $X_\theta^\dagger Y_\theta$ have the same eigenvectors, which is caused by the common factor $\mathcal{F}$. In particular, there is an invertible matrix V which diagonalizes both $X_\phi^\dagger Y_\phi$ and $X_\theta^\dagger Y_\theta$. Various algorithms have been derived to compute such joint diagonalizations. Omitting further details, we propose to use the diagonalization method in [3], although the algorithm in [4] can be used as well. As in ESPRIT, the actual algorithm has an intermediate step in which $\mathcal{H}$ is reduced to its r-dimensional principal column span, and this step will form the main computational bottleneck.

3.2. Data extension

Since the eigenvalues (ϕ_i, θ_i) are on the unit circle, we can double the dimension of $\mathcal{H}$ by forward-backward averaging. In particular, let J denote the exchange matrix which reverses the ordering of rows, and define

$$\mathcal{H}_e = [\mathcal{H} \quad J\mathcal{H}^{(c)}] , \qquad (m_1(M-m_2+1) \times 2m_2(LW-m_1+1)) ,$$

where (c) indicates taking the complex conjugate. Since $J\mathcal{A}^{(c)} = \mathcal{A}\Phi^{-(m_1-1)}\Theta^{-(M-m_2)}$, it follows that $\mathcal{H}_e$ has a factorization

$$\mathcal{H}_e = \mathcal{A} B_e \mathcal{F}_e = \mathcal{A}[B\mathcal{F}, \quad \Phi^{-m_1+1}\Theta^{-M+m_2} B^{(c)} \mathcal{F}^{(c)}] .$$

The computation of Φ and Θ from $\mathcal{H}_e$ proceeds as before. It is at this point possible to do a simple transformation to map $\mathcal{H}_e$ to a real matrix, which will keep all subsequent matrix operations real as well. This has numerical and computational advantages and is detailed in [4].

3.3. Identifiability

To identify Φ and Θ from $\mathcal{H}_e$ and (3), necessary conditions are that the submatrices $A_{x\phi}$ and $A_{x\theta}$ of $\mathcal{A}$ are "tall", while $\mathcal{F}_e$ is "wide", i.e.,

$$\begin{array}{llll} (a) & r & \le & (m_1-1)(M-m_2+1) \\ (b) & r & \le & m_1(M-m_2) \\ (c) & r & \le & 2m_2(LW-m+1) . \end{array}$$

Subject to these conditions, we can try to maximize the number of rays that can be identified for given M and LW, by optimizing over m_1 and m_2. This is analytically feasible only if we assume continuous parameters, which after some calculations then produces

$$\begin{array}{ll} \text{if } LW \ge M + \frac{1}{2}\sqrt{2} : & \left\{ \begin{array}{lcl} r_{\max} & = & (LW+1)M(2-\sqrt{2})^2 \\ m_{1,\text{opt}} & = & (LW+1)(2-\sqrt{2}) \\ m_{2,\text{opt}} & = & M(\sqrt{2}-1) \end{array} \right. \\ \text{if } LW \le M - \frac{1}{2}\sqrt{2} : & \left\{ \begin{array}{lcl} r_{\max} & = & LW(M+1)(2-\sqrt{2})^2 \\ m_{1,\text{opt}} & = & 1 + LW(2-\sqrt{2}) \\ m_{2,\text{opt}} & = & (M+1)(\sqrt{2}-1) \end{array} \right. \end{array} \tag{4}$$

Additional conditions should hold true if multiple rays can have equal delays or directions. In particular, we can show that the following is necessary and (almost) sufficient for identifiability of r rays with at most d equal delays: (4) holds, and

$$m_2 \ge \tfrac{1}{2} d , \quad M \ge \tfrac{3}{2} d$$

Similarly, r rays with at most d equal angles are identifiable if and (almost) only if (4) holds, and

$$m_1 \ge d + 1 , \quad LW \ge \tfrac{3}{2} d$$

3.4. Cramer-Rao bound

The Cramer-Rao bound (CRB) provides a lower bound on the variance of any unbiased estimator. The bound for DOA estimation (without delay spread) was derived in [5], and is readily adapted to the present situation. Assuming the path fadings to be deterministic but unknown, we obtain for the model in (1) that

$$CRB(\alpha, \tau) = \frac{\sigma_h^2}{2} \left\{ \text{real} \left(\mathcal{B}^* D^* P_U^\perp D \mathcal{B} \right) \right\}^{-1} \tag{5}$$

where σ_h^2 is the variance of the noise on the entries of $\mathbf{h}(t)$ (assumed to be i.i.d. white Gaussian noise), $\mathcal{B} = I_2 \otimes B$, $U = A(\alpha) \diamond G^T(\tau)$, $P_U^\perp = U(U^*U)^{-1}U^*$, and $D = [A'(\alpha) \diamond G^T(\tau), A(\alpha) \diamond G'(\tau)^T]$ (prime denotes differentiation, where each column is differentiated with respect to the corresponding parameter and all matrices are evaluated at the true parameter values).

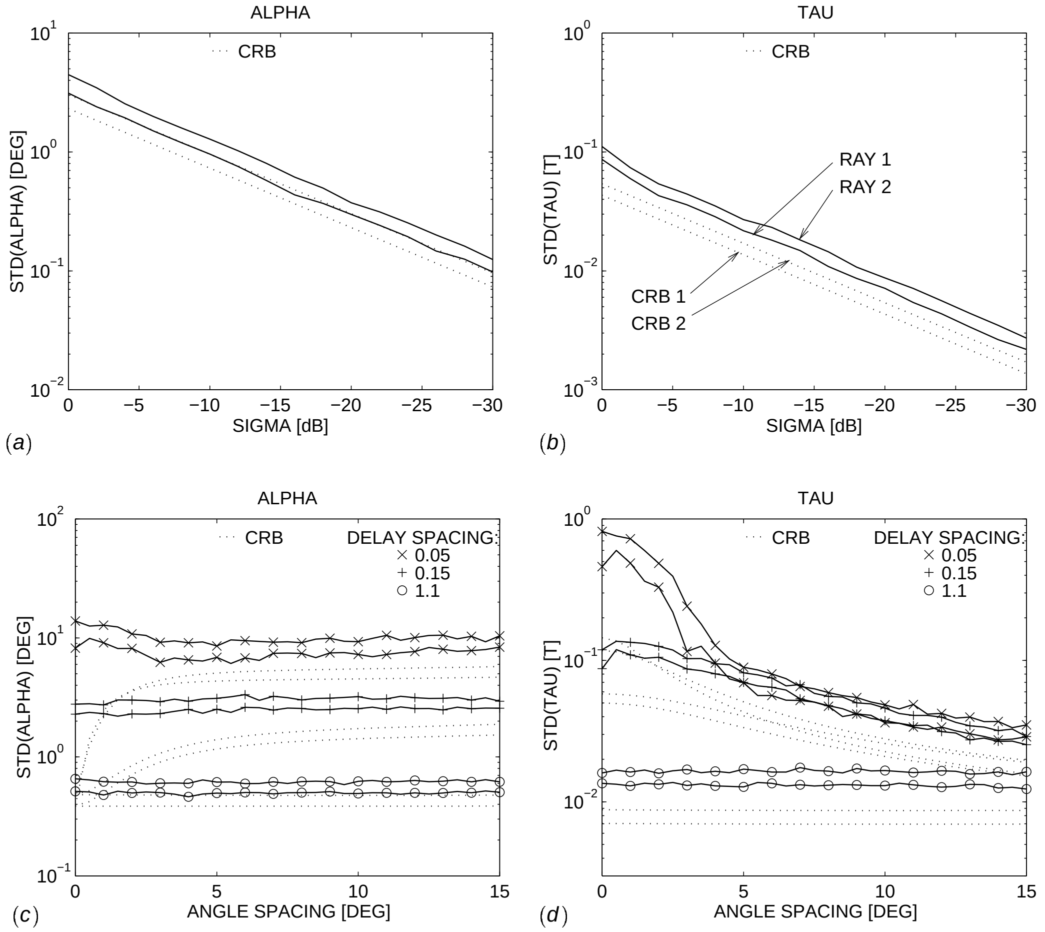

Figure 3. Standard deviation of estimates: (a,b) varying noise power, (c,d) varying angle and delay separation, with $\sigma_x = -15$ dB.

4. SIMULATION RESULTS

To illustrate the performance of the algorithm, we report some computer simulation results. Here, we assume one user and an array of $M = 2$ sensors. We also assume the communication protocol uses $N = 40$ training bits, from which the channel is estimated using least squares. The pulse shape function is a raised cosine with 0.35 excess bandwidth, truncated to a length of $L_g = 6$ symbols. We set $W = 1$. Figure 3 shows the experimental variance of the DOA and delay estimates as a function of standard deviation σ_x of the (i.i.d. white Gaussian) noise on the received data, for a scenario with $r = 2$ paths with angles $[-10, 20]^\circ$, delays $[0, 1.1]T$, fading amplitudes $[1, 0.8]$, a randomly selected but constant fading phase, stacking parameters $m_1 = 3$, $m_2 = 1$, and $P = 2$ times oversampling. It is seen that the difference in performance compared to the CRB is approximately 4 dB. The bias of the estimates was at least an order of magnitude smaller than their standard deviation.

The achievable resolution is demonstrated by varying the DOA and delay of the second ray, keeping the DOA and delay of the first ray fixed at $(-10^\circ,\ 0T)$. The same parameters as before were used, with noise power –15 dB. As expected, the performance in comparison to the CRB suffers when both τ's and α's are closely spaced, since with two antennas we cannot separate two rays with identical delays using ESPRIT.

5. REFERENCES

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