LOW-DELAY SCHEDULING FOR GRASSMANNIAN BEAMFORMING WITH A SINR CONSTRAINT

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ABSTRACT

We are presenting an algorithm for scheduling users in a single-cell broadcast scenario. The presented algorithm aims to minimize the number of transmissions that are necessary to serve all the users in the cell a single time, while the different users still fulfill a strict SINR constraint. Depending on the individual channel characteristics, the algorithm adapts the number of users scheduled for transmission on the fly, and dynamically allocates the transmit power to the scheduled users. A high-performance and a low-complexity variant of the algorithm are presented and their performance is evaluated through simulations.

Index Terms— Array signal processing, broadcast channels, scheduling, multiuser channels

1. INTRODUCTION

An important scenario in modern communications is the wireless broadcast channel. It covers the transmission from a single base station to multiple users. Transmission schemes for broadcast channels are generally designed to maximize the data rate from the base station to the users. The multiuser diversity in a broadcast channel allows high data rates by simultaneously transmitting to a set of users, i.e., spatial division multiple access (SDMA) [1].

A popular low-complexity joint-beamforming-and-scheduling algorithm is opportunistic SDMA (OSDMA) [2]. OSDMA uses a random set of orthogonal beamforming vectors. The users calculate the individual signal-to-interference-plus-noise ratios (SINR) for the beamforming vectors and feed back the SINR and the index of the beamforming vector with the highest SINR. OSDMA is optimal for scenarios with a large number of users in the cell. The OSDMA algorithm was extended to the use of multiple sets of orthogonal beamforming vectors in [3]. The transmission to a number of users that is higher than the number of antennas at the base station was presented in [4]. There, Grassmannian codebooks [5] were used as beamforming vectors. An algorithm that switches from TDMA to SDMA based on statistical assumptions has been proposed in [6] .

These systems focus mainly on opportunistic or proportional-fair scheduling. This allows high data rates and achieves long-term fairness. However, improving the data-rate is not the only problem in practical systems. More important is that modern applications, e.g., audio and video communications systems, have strict delay requirements. Further, communication systems have strict SINR constraints, and improving the SINR over the minimum requirement does not yield any benefits. Even though opportunistic scheduling of the users has received much attention lately, scheduling schemes that minimize the maximal delay time, i.e., the time until all the users in the cell have been scheduled for transmission once, are not so well investigated, despite being of high practical relevance [7].

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An exception is [8], where the problem of minimizing the frame duration in a MAC channel by dynamically switching between SDMA and TDMA is adressed. The presented algorithm, i.e., the Best Fit algorithm [8], explicitly aims to minimize the number of transmissions needed by the users to address the base station once. Further, all the scheduled users fulfill a strict SINR constraint.

We adapt the Best Fit algorithm for the broadcast channel with Grassmannian beamforming and imperfect channel state information (CSI) at the base station. The adapted algorithm further fulfills a strict transmit power constraint imposed by the base station. We present two variants of the algorithm, i.e., a high-performance variant and a low-complexity variant. The performance of the two variants is demonstrated through simulations, and it is shown that the adapted algorithm reduces the necessary transmission time while still guaranteeing the SINR constraint.

Notation: We use capital boldface letters to denote matrices, e.g., $\mathbf{A}$, and small boldface letters to denote vectors, e.g., $\mathbf{a}$. The $L2$-norm of a vector $\mathbf{a}$ is denoted as $|\mathbf{a}|$. $\mathcal{E}(\cdot)$ denotes expectation, and the set $\mathcal{X}$ contains $|\mathcal{X}|$ elements.

2. SYSTEM MODEL

We assume a narrowband single-cell system. The base station has M antennas, and there are N single-antenna users in the cell. The N users are indexed by decimal numbers and the set that contains all the indices is denoted $\mathcal{U}_{\text{all}} = \{i \in \mathbb{N} \mid i \leq N\}$. The system equation for the transmission from the base station to the user i is [2]

$$y_i = \mathbf{h}_i^T \mathbf{x} + n_i \qquad \forall i \in \mathcal{U}_{\text{all}} \tag{1}$$

where $y_i \in \mathbb{C}$ is the data received by user i, $\mathbf{h}_i \in \mathbb{C}^{M\times 1}$ is the channel between the base station and the user i, $\mathbf{x} \in \mathbb{C}^{M\times 1}$ is the data transmitted over the M antennas by the base station, and $n_i \in \mathbb{C}$ is the noise experienced by user i. The elements of the user channel are independent and identically distributed complex Gaussian with zero mean and unit variance. The noise is also independent and identically distributed complex Gaussian with zero mean, but with variance N_0. The channel is block-fading, i.e., it remains constant throughout a block of length L. The maximal transmit energy of the data vector $\mathbf{x}$ is limited to E_T, i.e., $\mathcal{E}\left\{\mathbf{x}^H\mathbf{x}\right\} \leq E_T$. All the users in the cell experience the same transmitted signal-to-noise ratio (SNR) of $\frac{E_T}{N_0}$.

We assume linear beamforming throughout the paper. The base station simultaneously transmits data to the users in a set $\mathcal{T} \subseteq \mathcal{U}_{\text{all}}$ that contains $|\mathcal{T}|$ different users. The data symbol s_i, that is transmitted to user i, is picked from a PSK constellation with average unit-energy, e.g., QPSK. The different data symbols for the users in $\mathcal{T}$ are multiplied by a unit-norm beamforming vector $\mathbf{w}_i \in \mathbb{C}^{M\times 1}$

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before transmission. The beamforming vectors are restricted to the elements of a codebook $\mathcal{W}$ with P elements which is known to the base station and all the users. We denote the mapping between each user and the index of its associated beamformer as $g : \mathcal{U}_{\text{all}} \rightarrow \{1, \ldots, P\}$. Thus the transmitted symbol vector is

$$\mathbf{x} = \sum_{i \in \mathcal{T}} \sqrt{E_{S,i}} \mathbf{w}_{g(i)} s_i \tag{2}$$

where $E_{S,i}$ is a power normalization factor. Due to the overall transmit power restriction the individual powers have to fulfill the constraint $\sum_{i \in \mathcal{T}} E_{S,i} \leq E_T$.

We assume perfect channel knowledge at the user side. Assuming a closed-loop scenario, there exists a feedback link between the users and the base station. The feedback link is only used for improving the downstream transmission, i.e., the transmission from the base station to the users. The link itself is assumed to be error-free and instantaneous.

3. MODIFIED GRASSMANNIAN BEAMFORMING

One of the differences between our approach and opportunistic schemes such as Grassmannian beamforming is that a user i does not feed back the resulting SINR of the strongest beam and the corresponding index, but the composite channel energy $\rho_{i,p} = |\mathbf{h}_i^T \mathbf{w}_p|^2$ for all the beamforming vectors $p = 1, \ldots, P$. The values of the composite channel energy for all the beamforming vectors are necessary in the scheduling step to evaluate the interference produced by the other scheduled users, as explained in Section 5.

3.1. Beamformer Codebooks

The beamforming vectors are selected from a Grassmannian codebook $\mathcal{W} = \{\mathbf{w}_1, \ldots, \mathbf{w}_P\}$ [5]. The unit-norm entries in the codebook are representing 1-dimensional subspaces in $\mathbb{C}^M$, i.e., points in the complex Grassmannian space $\mathcal{G}(M, 1)$ [9]. The codebook is designed to maximize the smallest chordal distance between two unit-norm entries in the codebook

$$\mathcal{W} = \underset{\substack{\{\mathbf{w}_1,\ldots,\mathbf{w}_P\} \\ |\mathbf{w}_p|=1,\ p=1\ldots P}}{\arg\max} \min_{1 \leq k < l \leq P} \sqrt{1 - |\mathbf{w}_k^H \mathbf{w}_l|^2}. \tag{3}$$

We design the codebook through a Monte-Carlo codebook design [10].

3.2. Training

The algorithm starts by estimating the individual user channels through training. Once every user has perfect CSI, he can calculate the matching ρ's for all the beamforming vectors, i.e., user i calculates

$$\rho_{i,p} = |\mathbf{h}_i^T \mathbf{w}_p|^2 \tag{4}$$

for $p = 1, \ldots, P$. These P ρ's are then transmitted to the base station. Note that we assume that $P < 2M$, and thus it is more efficient to feed back the P ρ's than feeding back the M complex channel coefficients.

If the ρ's are assumed to be noisy, due to erroneous CSI or due to quantization errors, then the users in $\mathcal{U}_{\text{sched}}$ are no longer guaranteed to fulfill the SINR constraint. Note that the implications of considering erroneous CSI and a data-rate limited feedback link are currently investigated by the authors.

3.3. Beamformer Selection

The base station selects the beamforming vector that results in the highest composite channel energy ratio

$$g(i) = \underset{j=1,\ldots,P}{\arg\max}\, \rho_{i,j} \qquad \forall i \in \mathcal{U}_{\text{all}}. \tag{5}$$

The beamforming has to rely on the fed back ρ's, and thus, beamforming techniques that select the beamforming vector based on the CSI of the other scheduled users, e.g., ZF or MMSE beamforming, cannot be used.

4. PROBLEM FORMULATION

We consider the problem of minimizing the time necessary to successively schedule the users in a single cell once. The different scheduled users must sustain a minimum SINR, denoted SINR_{min}. This constraint allows to choose a fixed modulation and coding for all the users. Adapting these parameters would result in additional overhead, i.e., the base station would have to inform the users about the selected coding and modulation. However, due to the fading nature of the wireless channel, some user channels might be in a deep fade, and reliable communication is not possible to these users. Thus, we exclude the users that are not able to fulfill the SINR constraint even when there are no interfering users present and all available power is allocated to them, i.e., TDMA. The set of users in the cell that can fulfill the SINR constraint under TDMA is defined as

$$\mathcal{U}_{\text{sched}} = \left\{ u \in \mathcal{U}_{\text{all}} \mid \frac{E_T}{N_0} |\mathbf{h}_u^T \mathbf{w}_{g(u)}|^2 \geq \text{SINR}_{\text{min}} \right\}. \tag{6}$$

We want to find an algorithm that is able to address all the users in $\mathcal{U}_{\text{sched}}$ once and in a minimum of time, as presented in (7). The set of users scheduled for transmission at a time instant k is denoted $\mathcal{T}[k]$. The number of transmissions necessary to serve all the users in $\mathcal{U}_{\text{sched}}$ once is denoted K. We assume throughout the paper that $L = K$. If $L > K$ then the algorithm would restart scheduling all the users again in the same way until the end of the block, and if $L < K$ then the algorithm would simply address the remaining users in the next block.

The main problem is that the base station has to decide, based on the P ρ's that are fed back from the users, which users to schedule for transmission.

5. SCHEDULING

The base station decides based on the feedback which users are scheduled for transmission. We assume that the composite channel energies are perfectly known to the base station, i.e., no quantization on the feedback link and perfect channel knowledge at the user side. Thus, we can rewrite the SINR constraint, that all the users in the set $\mathcal{T}$ must fulfill, from (7) as

$$(\text{SINR}_{\text{min}})^{-1} E_{S,i} \rho_{i,g(i)} \geq \sum_{j \in \mathcal{T}[k], j \neq i} E_{S,j} \rho_{i,g(j)} + N_0 \tag{8}$$

where the term on the right side yields the interference penalty for choosing an additional user and the noise. Note, that we will omit the time index in the rest of the section.

We start by adapting the Best Fit algorithm to work with the composite channel energies, and then we present two variants to find a solution to (7). The two variants have different complexities and different performances. However, both variants can serve the same

$$\begin{aligned}
&\text{minimize} \quad K \\
&\text{subject to} \quad \frac{|\sqrt{E_{S,i}}\mathbf{h}_i^T\mathbf{w}_{g(i)}|^2}{\sum_{j\in\mathcal{T}[k],j\neq i}|\sqrt{E_{S,j}}\mathbf{h}_i^T\mathbf{w}_{g(j)}|^2+N_0} \geq \text{SINR}_{\text{min}},\ \forall i\in\mathcal{T}[k],\ k=1\ldots K \\
&\bigcup_{k=1}^{K}\mathcal{T}[k]=\mathcal{U}_{\text{sched}} \quad \text{and} \quad \sum_{i\in\mathcal{T}[k]}E_{S,i}\leq E_T,\ k=1\ldots K \quad \text{and} \quad g(i)\neq g(j),\ \forall i,j\in\mathcal{T}[k], i\neq j, k=1\ldots K
\end{aligned} \tag{7}$$

set of scheduled users $\mathcal{U}_{\text{sched}}$, i.e., both algorithms are using TDMA $\mathcal{T} = \{i\}$ with $E_{S,i} = E_T$ for a user i with adverse channel conditions. The main difference between the two variants is how they allocate the power to the beamforming vectors.

5.1. Adapted Best Fit Algorithm

The algorithm starts by eliminating all the users from the original user set $\mathcal{U}_{\text{all}}$ that can not even fulfill the desired SINR constraint SINR_{min} when they are scheduled alone, i.e., in the TDMA mode. The set of users that can be scheduled for transmission is calculated as

$$\mathcal{U}_{\text{sched}} = \left\{ u \in \mathcal{U}_{\text{all}} \mid \frac{E_T}{N_0}\rho_{u,g(u)} \geq \text{SINR}_{\text{min}} \right\}. \tag{9}$$

From the resulting set $\mathcal{U}_{\text{sched}}$ the user with the lowest composite channel energy is selected

$$u = \underset{i\in\mathcal{U}_{\text{sched}}}{\arg\min}\,\rho_{i,g(i)} \tag{10}$$

and the scheduled user set is initialized with $\mathcal{T} := \{u\}$. The algorithm now iteratively tries to add users to the set $\mathcal{T}$. How such an additional user u_{best} is selected varies on the possible power allocation for the beamforming vectors and is explained in the following two subsections. Once the best matching user u_{best} is found the set is updated $\mathcal{T} := \mathcal{T} \cup \{u_{\text{best}}\}$ and the user is removed from the set of scheduled users $\mathcal{U}_{\text{sched}} := \mathcal{U}_{\text{sched}} \setminus \{u_{\text{best}}\}$. This continues until the algorithm does not find an additional user anymore and the base station starts transmitting to the users in $\mathcal{T}$. Finally, the algorithm restarts with the updated set $\mathcal{U}_{\text{sched}}$ until $\mathcal{U}_{\text{sched}} = \emptyset$.

5.2. Low-Complexity Variant

The low-complexity variant equally distributes the available transmit energy E_T to the scheduled users

$$E_{S,i} = \frac{E_T}{|\mathcal{T}|}, \qquad i \in \mathcal{T}. \tag{11}$$

The user selection is done by

$$u_{\text{best}} = \underset{\{u\in\mathcal{U}_{\text{sched}}\setminus\mathcal{T}\ \mid\ g(u)\neq g(i), i\in\mathcal{T}\}}{\arg\max} \min_{v\in\mathcal{T}\bigcup\{u\}} (\text{SINR}_{\text{min}})^{-1}\rho_{v,g(v)} - \sum_{j\in\mathcal{T}}\rho_{v,g(j)} - N_0\frac{|\mathcal{T}|+1}{E_T}. \tag{12}$$

The penalty for adding an additional user, i.e., the negative terms in (12), consists of the interference produced by the other scheduled users and the reduction of the power allocated to the individual users since the transmission power is now distributed over more users. Note that the selection function guarantees that the different elements in the set have unique beamforming vectors.

After a candidate u_{best} has been determined, the algorithm then checks if the SINR constraint (8) is still fulfilled. The SINR constraint can also be checked simultaneously with the user selection by checking if the SINR term inside the user selection remains positive for all the members in the set $\mathcal{T} \bigcup \{u\}$.

5.3. High-Performance Variant

The following variant can assign different transmit powers to the beamforming vectors in order to balance the SINRs of the different users. An algorithm to maximize the smallest SINR in a set of users with fixed beamforming vectors by adapting the power allocation assigned to the different users was presented in [11].

The high-performance variant selects the additional users by solving

$$u_{\text{best}} = \underset{\{u\in\mathcal{U}_{\text{sched}}\setminus\mathcal{T}\ \mid\ g(u)\neq g(i), i\in\mathcal{T}\}}{\arg\max} \min_{v\in\mathcal{T}\bigcup\{u\}} (\text{SINR}_{\text{min}})^{-1}E_{S,v}\rho_{v,g(v)} - \sum_{j\in\mathcal{T}}E_{S,j}\rho_{v,g(j)} - N_0(|\mathcal{T}|+1) \tag{13}$$

under the constraint $\sum_{i\in\mathcal{T}\cup\{u\}} E_{S,i} = E_T$. Thus, for every considered set $\mathcal{T}\cup\{u\}$ the individual transmit powers $E_{S,i}$, $i\in\mathcal{T}\cup\{u\}$, have to be recalculated by using the algorithm in [11].

6. SIMULATION RESULTS

In this section the performance of the high-performance (HP) variant and the low-complexity (LC) variant are compared. We assume that the base station has $M = 3$ antennas and that there are $N = 100$ single-antenna users present in the cell. We consider a beamforming codebook with $P = 4$ entries, and all the users experience the same $\text{SNR} = 18$ dB.

Fig. 1 depicts the number of transmissions needed to serve all the users in $\mathcal{U}_{\text{sched}}$. We see how the two variants of the algorithm gradually require more transmissions to serve the $K = 100$ users. The two variants slowly switch from SDMA to a TDMA mode in order to fulfill the given SINR constraint (8). We see that the HP variant outperforms the LC variant over all possible SINR constraints. The HP variant performs better because it can allocate more energy to the weak users in the scheduled set, and thus, improve the smallest SINR in the scheduled set.

In Fig. 2 we compare our algorithm to the traditional Grassmannian beamforming algorithm [4]. We depict the ratio of the number of users that are scheduled for transmission $|\mathcal{U}_{\text{sched}}|$ to the number of users in the cell $|\mathcal{U}_{\text{all}}|$ for different SINR constraints. The Grassmannian beamforming algorithm feeds back one scalar, i.e., the SINR

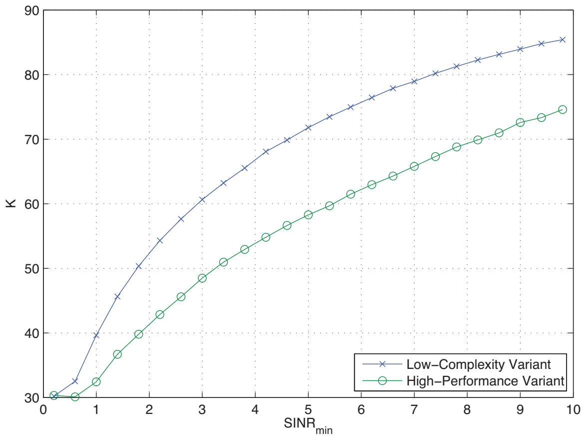

Fig. 1. Number of transmissions K as a function of SINR_{min}. ($M = 3$, $P = 4$, $N = 100$, and $\text{SNR} = 18$ dB)

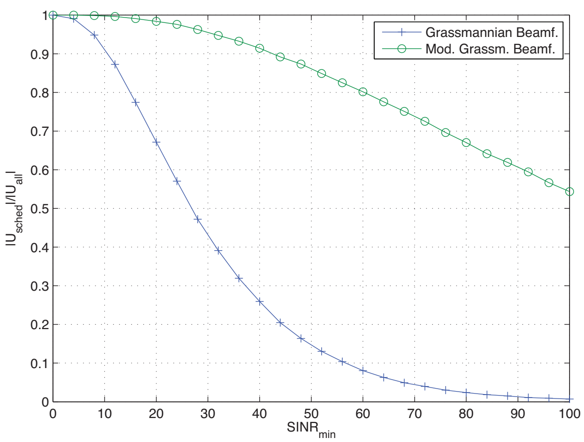

Fig. 2. Ratio between $|\mathcal{U}_{\text{sched}}|$ and $|\mathcal{U}_{\text{all}}|$ as a function of SINR_{min}. ($M = 3$, $P = 4$, $N = 100$, and $\text{SNR} = 18$ dB)

for the strongest beamformer, and one integer, i.e., the index of this beamformer. We see that feeding back the composite channel energies, and thus being able to adapt the number of scheduled users, allows a larger amount of users to be served for a given SINR constraint.

The next simulation depicts how the different variants group their users into sets. We see in Fig. 3 that the high-performance (HP) variant collects in general more users in a set than the low-complexity variant. For increased values of SINR_{min} both variants are no longer able to schedule sets with the maximum number of users P. The amount of incomplete sets increases until for very high values of SINR_{min} most scheduled sets consist of a single users.

7. CONCLUSIONS

We have proposed two variants of an adapted Best Fit algorithm to schedule all the users in a broadcast channel once. The two variants try to minimize the amount of transmissions needed to schedule the different users under strict SINR constraints. The variants differ in their computational complexity., and they can both switch between TDMA and SDMA depending on the channel conditions.

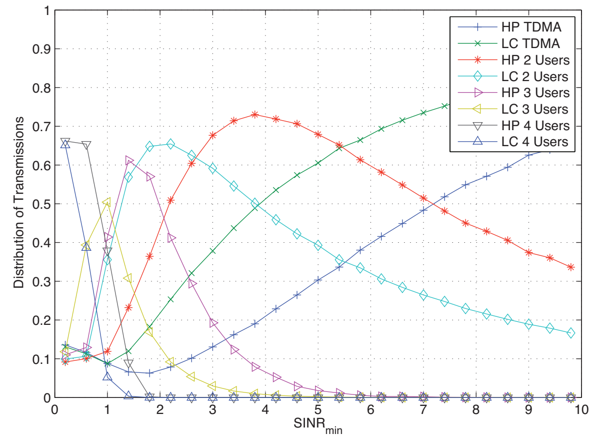

Fig. 3. Distribution of the transmissions with different number of users for the high-performance variant (HP) and the low-complexity variant (LC). ($M = 3$, $P = 4$, $N = 100$, and $\text{SNR} = 18$ dB)

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