Matched Subspace Based Correction of Large Errors in UWB Localization

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Abstract— **Localization using Ultra Wide Band (UWB) signals can show large errors. Due to Non-Line-of-Sight (NLOS) propagation and low signal to nosie ratio (SNR), it is possible to expect that the first arriving path and the strongest path are not equal to each other. This problem can be modeled as a multihypothesis problem. We base our decision on matched subspace detectors. The case where we deal both with Time of Arrivals (TOA) and Angle of Arrivals (AOA) is also studied.**

I. INTRODUCTION

Ultra Wide Band (UWB) technology has a strong potential for positioning in indoor environments [**?**]. Positioning is a mature field with variety of techniques available. However, indoor environments and UWB technology pose new challenges and require development of new positioning techniques. Some problems that indoor positioning brings can be outlined as non-line-of-sight (NLOS) propagation, power limitations due to FCC, and difficulties caused by high sampling rates [?]. Also a far field assumption may not be valid anymore.

Typically, positioning is based on two steps. The first step gives estimates of angle-of-arrival (AOA), time-of-arrival (TOA), time-difference-of-arrivals (TDOA), or signal-strength (SS) from channel measurements. The second step uses these parameters to obtain the final position. Nevertheless, a direct approach like the one in [**?**] is possible among other variations.

One important aspect of UWB systems is that due to NLOS and low SNR we may not always have that the strongest arriving path is also the earliest arriving path. In such a case a decision has to be made between these two parameters. The inequality of the first arriving path and the strongest path can either be due to NLOS or low SNR. If the problem is addressed only at a single channel then there is not much room for making the right decision. However when we put these pairs of estimates from every channel together then we show in this paper that it is possible to improve the positioning performace. The idea is based on the fact that estimated parameters at every base station should correspond to a consistent location. As long as we assume that either the early arriving or the strongest path is correct or relatively more accurate, then it is possible to form a multihypothesis decision problem. Since the test statistic of Matched Subspace Detectors (MSD) [**?**] is a good estimate of goodness of fit, we base our decision on matched subspace detectors. If in addition to time delay estimation,
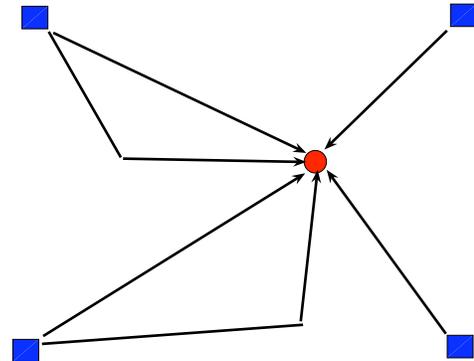
Fig. 1.   Four base stations with multiple ranges towards the mobile station

we have the associated AOAs then again a decision has to be made for every early arriving and strongest path so that the correct or the more accurate parameter pairs are chosen.

Large error performance of UWB ranging systems has been studied in detail in [**?**], [**?**]. It is mentioned in [?] that due to low SNR, the Cramer Rao Lower Bound (CRLB) gives very optimistic bounds and hence large errors should be added into the performance study. However no solution was proposed in [?] for reducing the errors due to problems caused by large errors. Depending on the correlator threshold it is shown that there can be three types of large errors: early delay estimation, late delay estimation and a miss. Since missing a signal is not desirable we may choose to set the threshold of the correlator at a low level. In such a case we will have, due to noise many false estimates. The aim of this paper is to detect these false estimates.

The organization of this paper is as follows. In the following section we introduce the channel models and the correlating receiver. In the next section we develop the idea of matched TOA detection. Later we extend the results to joint angle and delay estimation. Finally we provide the computer simulations.

II. CHANNEL MODELS AND THE RECEIVER

Consider the following multipath channel model

$$x(t) = \sum_{\ell=1}^{L} \alpha_\ell s(t - \tau_\ell) + n(t) \qquad (1)$$

where $x(t)$ denotes the received signal at the base station and $n(t)$ denotes thermal noise, multiuser interference and narrowband interferences. We assume that the signal $s(t)$ is known. $R_{rs}(\tau)$ denotes crosscorrelation of signal template with the multipath channel and is given as

$$R_{xs}(\tau) = \int x(t)s(t - \tau)dt. \qquad (2)$$

In such a case, a detector can be constructed

$$\hat{\tau} = \arg\max_{\tau} |R_{xs}(\tau)| \qquad s.t., \ |R_{xs}(\tau)| > \gamma \qquad (3)$$

which picks the strongest peak above the threshold. Instead we can also pick the earliest path that is above the threshold. Note that if, due to noise, the early arriving peak and the strongest peak are not equal to each other then we need to choose between them. In most high SNR and LOS propagation scenarios the first arriving path and the strongest path will be the same.

For simplicity we assume that either the strongest path or the early arriving path is correct. The problem is making the right decision. Then a solution to this problem is as follows. We use the fact that all the solutions should correspond to a consistent location. We could very well use a GLRT approach however it requires the computation of MLs several times hence it is not practical. Instead we use the linearized versions of ranges.

III. PROPOSED SOLUTION

Every TOA estimate is directly mapped to range estimates by using the propagation speed at the medium. Let us assume we have L base stations, (x_k, y_k) denote the coordinates of the kth base station while x_m, y_m denote the unknown coordinates of the mobile station. r_k denotes the range from the mobile station to base station k. For the kth base station we have the following equation:

$$r_k^2 = (x_m - x_k)^2 + (y_m - y_k)^2. \qquad (4)$$

By substracting the first equation from the kth we obtain the following well known set of equations

$$\mathbf{b} = \mathbf{H}\boldsymbol{\theta} + \mathbf{w} \qquad (5)$$

$$\mathbf{H} = 2\begin{bmatrix} x_{21} & y_{21} \\ x_{31} & y_{31} \\ . & . \\ x_{L1} & y_{L1} \end{bmatrix} \qquad \boldsymbol{\theta} = \begin{bmatrix} x_m \\ y_m \end{bmatrix} \qquad (6)$$

$$\mathbf{b} = \begin{bmatrix} x_2^2 + y_2^2 - x_1^2 - y_1^2 - r_2^2 + r_1^2 \\ x_3^2 + y_3^2 - x_1^2 - y_1^2 - r_3^2 + r_1^2 \\ ... \\ x_L^2 + y_L^2 - x_1^2 - y_1^2 - r_L^2 + r_1^2 \end{bmatrix} \qquad (7)$$

where $x_{kl} = x_k - x_l$. The estimate of the true location is given as:

$$\hat{\boldsymbol{\theta}} = (\mathbf{H}^T\mathbf{H})^{-1}\mathbf{H}^T\mathbf{b} \qquad (8)$$

Note that in general $\mathbf{b} \notin \mathcal{R}(\mathbf{H})$. The idea is to choose the hypothesis for which $\mathbf{b}$ is closest to $\mathcal{R}(\mathbf{H})$. For this purpose we propose the use of matched subspace detectors [?]. From here the matched subspace detector can be written as:

$$\mathcal{L} = \frac{\|\mathbf{P_H b}\|}{\|\mathbf{P_H^\perp b}\|} > \gamma \qquad (9)$$

where

$$\mathbf{P_H} = \mathbf{H}(\mathbf{H}^T\mathbf{H})^{-1}\mathbf{H}^T \qquad (10)$$
$$\mathbf{P_H^\perp} = \mathbf{I} - \mathbf{H}(\mathbf{H}^T\mathbf{H})^{-1}\mathbf{H}^T \qquad (11)$$

The idea behind the use of a matched subspace detector is simple. If a signal is present then there must be a strong component of b in $\mathcal{R}(\mathbf{H})$ and we simply check the SNR for the signal subspcae and the noise subspace. However this can only be used for detecting a single signal. In order to extend it to multiple cases, then our detector chooses between the strongest dissimilarity. In other words, if there is large misalginment of the vector $\mathbf{b}$ from the space $\mathcal{R}(\mathbf{H})$ then it will not be preferred. Matched subspace detectors measure the dissimilarity between the signal and the model.

Let us for simplicity assume that we have the early-strongest ambiguity in two base stations only, then

$$\hat{r}_1 \in \{c_1, d_1\} \qquad (12)$$
$$\hat{r}_2 \in \{c_2, d_2\} \qquad (13)$$

the range estimates at each of these base stations take two possible values.

We desire to choose the correct range difference measurement. In such a case we need to choose between 4 hypotheses:

$$\mathcal{H}_{11} : \text{Pair } c_1, c_2 \text{ correct} \qquad (14)$$
$$\mathcal{H}_{12} : \text{Pair } c_1, d_2 \text{ correct} \qquad (15)$$
$$\mathcal{H}_{21} : \text{Pair } d_1, c_2 \text{ correct} \qquad (16)$$
$$\mathcal{H}_{22} : \text{Pair } d_1, d_2 \text{ correct} \qquad (17)$$

whre $\mathcal{H}_{ij}$ denotes that the ith estimate is chosen from the first base station and the jth estimate is chosen from the second base station. In a similar manner, since the range differences enter only the vector $\mathbf{b}$, we label the vector depending on the enumeration: $\mathbf{b}_{ij}$ means we constructed the vector by choosing the ith estimate from the first base station and the jth estimate from the second base station.

Then the dissimilarity measure for the parameter is

$$\mathcal{L}_{ij} = \frac{\|\mathbf{P_H b}_{ij}\|}{\|\mathbf{P_H^\perp b}_{ij}\|} \qquad (18)$$

We base the decision on the following criterion

$$\{\hat{i}, \hat{j}\} = \arg\max_{i,j} \mathcal{L}_{ij} \qquad (19)$$

Hence the true hypothesis is $\mathcal{H}_{\hat{i}\hat{j}}$

In general if we have K base stations the there will be 2^K possible variations. Testing for such large number of hypotheses with ML is intractable. However by using the closed form solution and the proposed matched subspace detectors the problem becomes more feasible.

IV. LINEAR ARRAY

Until now we assumed that the matrix $\mathbf{H}$ is full rank, however the case where we have a linear array is also relevant. In the case of a linear array the matrix $\mathbf{H}$ is not full rank. Let the SVD of $\mathbf{H}$ be given as:

$$\mathbf{H} = \mathbf{U}\boldsymbol{\Sigma}\mathbf{V}^T \tag{20}$$

and $\mathbf{U} = [\mathbf{u}_1\mathbf{u}_2]$. Then the following projection operator gives the projection operators

$$\mathbf{P_H} = \mathbf{U}_1\mathbf{U}_1^T \tag{21}$$
$$\mathbf{P_H^\perp} = \mathbf{U}_2\mathbf{U}_2^T \tag{22}$$

V. AOA+TOA CASE

In this section we consider the case where we want to choose between the right pair of direction of arrivals and time of arrivals belonging seperately to the strongest and first arriving paths. This is an extension to the previous section where we had only TOA information.

From the following we need to develop an algorithm that can incorporate all the knowledge. A simple combination of various estimates of TOA and AOA, is the following [?]:

$$\mathbf{b} = \mathbf{H}\boldsymbol{\theta} + \mathbf{w} \tag{23}$$

where

$$\mathbf{H} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \\ 0 & 1 \\ . & . \\ . & . \\ . & . \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \quad \boldsymbol{\theta} = \begin{bmatrix} x_m \\ y_m \end{bmatrix} \quad \mathbf{b} = \begin{bmatrix} x_1 + r_1\cos(\theta_1) \\ y_1 + r_1\sin(\theta_1) \\ x_2 + r_2\cos(\theta_2) \\ y_2 + r_2\sin(\theta_2) \\ . \\ . \\ . \\ x_L + r_L\cos(\theta_L) \\ y_L + r_L\sin(\theta_L) \end{bmatrix} \tag{24}$$

In the above notation θ_k is the angle of arrival estimation of the kth base station and r_k is the corresponding range estimate.

The estimate is a simple LS and can be obtained as:

$$\hat{\boldsymbol{\theta}} = (\mathbf{H}^T\mathbf{H})^{-1}\mathbf{H}^T\mathbf{b} \tag{25}$$

We assume that there are available pairs from the first base station $(c_1,\alpha_1),(c_2,\alpha_2)$ and for the second base station $(d_1,\beta_1),\{d_2,\beta_2\}$. The first terms c_1,c_2,d_1,d_2 are estimated ranges and $\alpha_1,\alpha_2,\beta_1,\beta_2$ are the corresponding angles.

Then we obtain the following hypothesis:

$$\mathcal{H}_{11} : \text{Pairs } (c_1,\alpha_1),(d_1,\beta_1) \text{ correct} \tag{26}$$
$$\mathcal{H}_{12} : \text{Pairs } (c_1,\alpha_1),(d_2,\beta_2) \text{ correct} \tag{27}$$
$$\mathcal{H}_{21} : \text{Pairs } (c_2,\alpha_2),(d_1,\beta_1) \text{ correct} \tag{28}$$
$$\mathcal{H}_{22} : \text{Pairs } (c_2,\alpha_2),(d_2,\beta_2) \text{ correct} \tag{29}$$

As before the detector for the right pairs is given as the part that maximizes the following SNR expression:

$$\mathcal{L}_{ij} = \frac{\|\mathbf{P_H}\mathbf{b}_{ij}\|}{\|\mathbf{P_H^\perp}\mathbf{b}_{ij}\|} \tag{30}$$

We base the decision on the following criterion

$$\{\hat{i},\hat{j}\} = \arg\max_{i,j} \mathcal{L}_{ij} \tag{31}$$

VI. SIMULATIONS

In order to test the performance of the multihypothesis detector we performed extensive computer simulations for the TOA only case. In the first set of simulations the locations of the five base stations were: $(0,0),(0,10),(10,0),(10,10),(17,12)$. The mobile station was put in near field point $(3,3)$. We choose the scales so that they can represent a realsitic indoor geometry. First we assumed that there is a path ambiguity only in two of the base stations and this large error is common in the sense that it is represented with the same parameter a, ($d_1 = c_1 + a$ and $d_2 = c_2 + a$). We changed this bias parameter and Fig. 2 shows the probability of detection for the right hypothesis. As can be seen from the figure, the probability of correct decision converges to 1 as we increase the value of the paramter from 1 to 10 (noise variance is set to 1). From here we can say that when the number of biased base stations is low good detection of large errors is possible.

To test the full capacity of the algorithm we performed another set of simulations where the base stations are located as $(0,0),(0,10),(10,0),(10,10)$. In this scenario every base station has an ambiguity on the early arriving and strongest path. We set the noise variance to one and the path offset on each base station is set to 10. In this case one may expect a good performance however a degeneracy occurs and as can be seen from Fig. 3 we do not have good probability of detection.

To test the non-degenerate case we set the noise variance to 0.1 and performed simulations for bias terms as $(1,2,3,4)$. In Fig.4 we see that again good probability of detection can be obtained. This result agrees with the first set of simulations. We have also tested the high noise variance case where the noise variance is set to 1. In Fig. 5 we see that now there is almost homogeneous distribution between the hypothesis and the probability of detection decreases. However we must note that in that case noise variance and bias terms are very close to each other hence it is expected not to have very accurate detection.

VII. CONCLUSION AND DISCUSSIONS

Large errors are an important deteriorating element for UWB localization. We have showed that in general it is possible to improve positioning accuracy by adopting a simple detector based on the fact that range estimates should point to consistent point as the estimate of mobile location. The decision is based on the best fitting pair of estimates that represent the ranges. In simulations we observed that for large
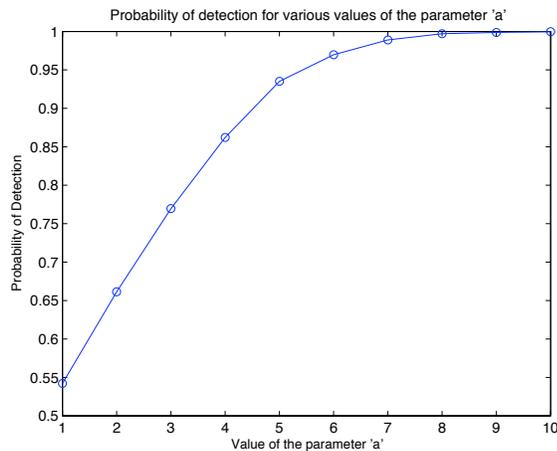
Fig. 2. Probability of choosing the right hypothesis with respect to the parameter 'a'
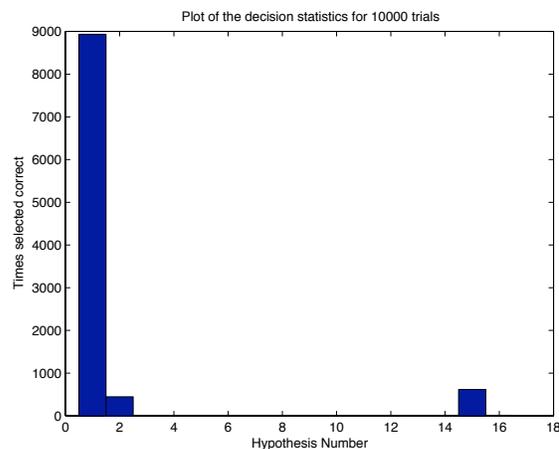

Fig. 4. Histogram for decision statistics of hypothesis under 10000 trials, nondegenerate case
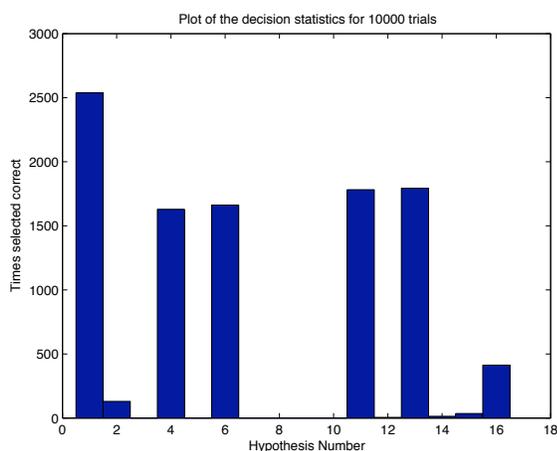

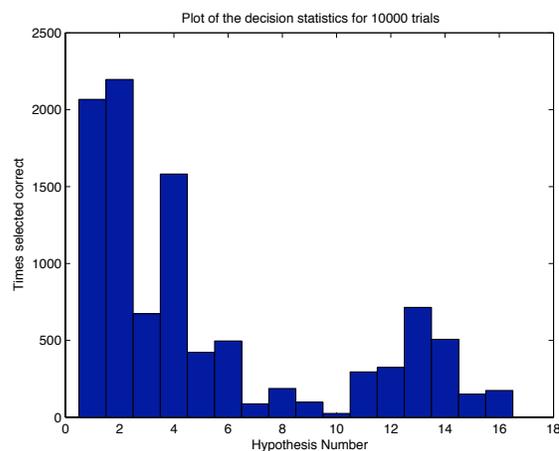Fig. 3. Histogram for decision statistics of hypothesis under 10000 trials, degenerate case

Fig. 5. Histogram for decision statistics of hypothesis under 10000 trials

errors it is very easy to distinguish between correct ranges and incorrect ranges. While when the errors are smaller and hence it is more difficult to make the right decision. However, as the errors are smaller, the errors in positioning will be smaller. In conlcusion large errors are easy to separate. We performed all the simulations in 2D for simplicity, however extension to 3D is easy.

We must note that we used ℓ_2 norm instead of ℓ_1 norm. It is possible to pose the same decision problem with ℓ_1 norm and as a parameter estimation problem to solve the problem. We can also use the extensions discued in [**?**].

REFERENCES

[1] S. Gezici, Z. Tian, G. B. Giannakis, H. Kobayashi, A. F. Molisch, H. V. Poor, and Z. Sahinoglu, "Localization via ultra-wideband radios," *IEEE Signal Processing Magazine*, pp. 70–84, July 2005.
[2] A. J. Weiss, "Direct position determination of narrowband radio frequency transmitters," *IEEE Signal Processing Letters*, vol. 11, no. 5, pp. 513–516, May 2004.
[3] L. L. Scharf and B. Friedlander, "Matched subspace detectors," *IEEE Transactions on Signal Processing*, vol. 42, no. 8, pp. 2146–2157, Aug. 1994.
[4] J. Y. Lee and S. Yoo, "Ranging performance of uwb radio in multipath and multiuser environments," *IEEE 16th International Symposium on Personal, Indoor and Mobile Communications*, pp. 496–500, 2005.
[5] J. Lee and R. A. Scholtz, "Ranging ina dense multipath environment using an uwb radio link," *IEEE Journal on Selcted Areas in Comminications*, vol. 20, no. 9, pp. 1677–1683, Dec. 2002.
[6] A. H. Sayed, A. Tarighat, and N. Khajehnouri, "Network-based wireless location," *IEEE Signal Processing Magazine*, pp. 24–40, July 2005.
[7] M. N. Desai and R. S. Mangoubi, "Robust gaussian and non-gaussian matched subspace detection," *IEEE Transactions on Signal Processing*, vol. 51, no. 12, pp. 3115–3127, Dec. 2003.