

A Fast and Dedicated First-Order Differencing EPT Reconstruction Method

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Synopsis

A new method for reconstructing electrical properties from B_1^+ data based on Maxwell's equations in an E polarized field (found in the midplane of a birdcage coil) is presented. This first-order EPT (foEPT) method uses first order spatial derivatives as opposed to the second order Helmholtz based MR-EPT methods and is thus less susceptible to noise. Furthermore, the method does not rely on any homogeneity assumptions. The method is validated using an in-vivo phantom measurement and compared to an MR-EPT reconstruction. FoEPT conductivity reconstructions show less noise-amplification and less boundary artefacts compared to Helmholtz-based MR-EPT reconstructions.

Introduction

Precise knowledge about the dielectric properties of tissue (conductivity σ and permittivity ϵ) is an important prerequisite in many clinical applications. The accurate and efficient reconstruction of these properties from measured B_1^+ data is the main objective of Electrical Properties Tomography (EPT). Over the past decade various EPT methods have been developed, most often based on either the local Helmholtz equation^[1-3] or on global integral representations of the RF field.⁴ Helmholtz-based methods are typically very efficient, but suffer from reconstruction artefacts on tissue interfaces and noise amplification due to the second-order spatial derivatives that act on the B_1^+ data.⁵ Global methods on the other hand do not suffer from these effects, but are computationally more complex and therefore less efficient.

We propose a new hybrid reconstruction method dubbed first-order EPT (foEPT) which combines the speed of local methods with the accuracy of global methods for tissue interfaces. It requires the E-polarized field structure in the mid-plane of a birdcage coil⁶ to do so, and uses a first-order spatial derivative as opposed to the second-order Laplacian operator in Helmholtz-based reconstructions and is, therefore, less sensitive to noise. Furthermore, the method can handle inhomogeneous structures and does not rely on any homogeneity assumptions.

Methods

The method works in two steps. First, the induced currents in the sample are determined from the measured B_1^+ field. In particular, for an RF field with an E-polarized field structure,⁶ the induced current density follows from the B_1^+ data as

$$J_{\text{ind}} = \frac{2}{\mu_0 j} (\partial_x - j \partial_y) B_1^+,$$

where μ_0 is the background permeability, j is the imaginary unit, and $\partial_{x/y}$ are the spatial derivatives in the x - and y -directions. Second, we relate this current density to the electric field using the global integral representation

$$E + G\{E\} = E_{\text{inc}} - G\{J_{\text{ind}}\} \quad (1)$$

where $G\{\cdot\}$ is the electric vector potential and E_{inc} is the known electric field in an empty coil, which can be simulated using a realistic coil model.

Equation (1) can be solved for the electric field using an iterative solver such as GMRES⁷ and typically requires only a small number of iterations (≤ 10) to reach a satisfactory error level. Having solved Equation (1), the total electric field is known and since the induced electric currents have already been determined, the electrical properties follow as

$$\sigma = \frac{\mathcal{I}(E^* J_{\text{ind}})}{|E|^2}$$

$$\epsilon_r = -\frac{\mathcal{R}(E^* J_{\text{ind}})}{\omega |E|^2}$$

where the asterisk denotes complex conjugation and ω is the frequency. To validate our foEPT method, measurements at 3T (Ingenia, Philips) of an agar based conductive phantom (see Figure 1 for image and properties of the phantom) are used. The foEPT conductivity reconstruction is also compared to a standard MR-EPT reconstruction.

The B_1^+ amplitude (see Figure 2.) of the field was measured using an AFI sequence,⁸ while the RF transceive phase was measured using two single Spin Echo sequences, thus compensating for eddy currents.⁹ Both sequences were carried out with an NSA equal to 10, and the mid-plane slice was used. Furthermore, the phantom was placed at the centre and a body coil was used for transmitting, while a head coil (Philips, Eindhoven) was used for reception. The receive array data was phase-referenced to the body coil. Furthermore, to investigate the potential of foEPT for in-vivo permittivity and conductivity mapping of a human brain, simulations of a human head model (ELLA, virtual family¹⁰) were performed.

Results and Discussion

In Figure 3, the reconstruction of the conductivity using the Helmholtz approach (left) as well as the new foEPT method (right) can be seen side by side. We can observe that foEPT enables accurate reconstructions of phantom conductivity without severe artefacts around compartment boundaries compared to the Helmholtz-based reconstruction (Figure 3-left). The reconstructed conductivity also shows a local bias pattern near the center and towards the lower right. Current hypotheses for the origin of this artefact point towards either incident field calibration errors, or at the transceive phase assumption. From the simulated reconstructions (Figures 4 and 5) clear advantages in terms of noise amplification and boundary errors for the foEPT method are visible, compared to standard MR-EPT. The noise in the center of the image domain is due to the lower E-field there.

Conclusion

These results indicate the potential of the method to reliably reconstructs the conductivity profiles without boundary artefacts, with a better noise tolerance than Helmholtz based methods. Furthermore reconstructions are typically obtained in less than a second on a standard laptop or PC which is a significant speed up compared to global methods. This indicates that foEPT is a promising alternative for in-vivo electrical properties reconstruction.

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Figures

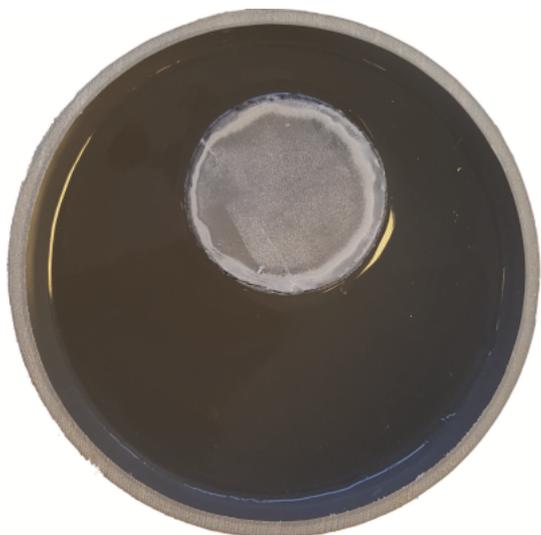


Figure 1: Photograph of the agar based phantom. It consists of an inner and an outer cylinder with conductivities given by 0.95 S/m and 0.45 S/m, respectively. These conductivities were independently obtained using the Stogryn equation.¹¹

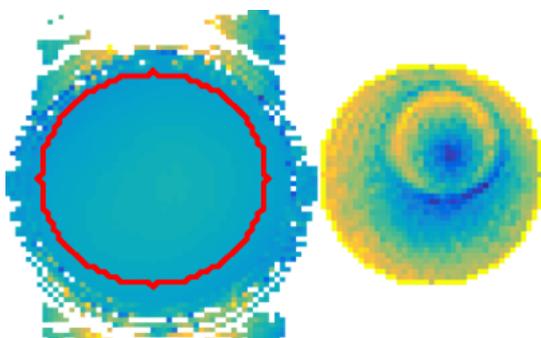


Figure 2: Measured B_1^+ magnitude (left); red line indicates area used for reconstruction. Reconstructed induced currents J_{ind} (right) [normalised].

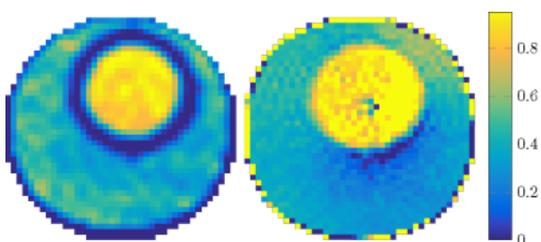


Figure 3: Reconstruction results for the conductivity [S/m]. Helmholtz reconstruction (left) and foEPT (right).

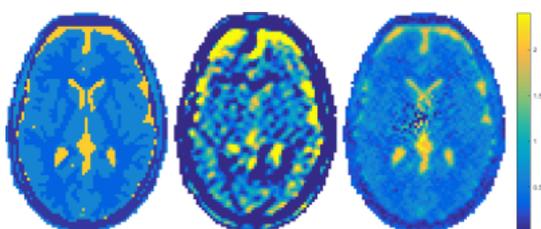


Figure 4 Simulated conductivity profile of a head slice. Left is original profile, center the Helmholtz reconstruction using a smoothing kernel and right the foEPT reconstruction all in [S/m]. Signal to noise level for the simulation is 40dB.

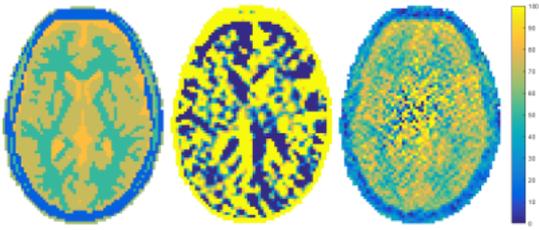


Figure 5: Simulated relative permittivity profile of a head slice. Left is original profile, center the Helmholtz reconstruction using a smoothing kernel and right the foEPT reconstruction. Signal to noise level for the simulation is 40dB.