Fast 3D Design of High-Permittivity Pads for Dielectric Shimming using Model Order Reduction and Non-linear Optimization

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Purpose: Analyzing high-permittivity pads to improve B_1^+ homogeneity and intensity is normally accomplished by using full-wave solvers.[1] The "optimal" design is found by evaluating a large number of pad realizations, typically, covering only a subset of the design parameters such as the permittivity, dimension, and location of the pad.[2,3] This optimization process is computationally intensive, taking hours or even days to complete. We have shown previously that the process of evaluating such pad realizations can be accelerated by means of domain decomposition.[4] This decomposition allows us to generate a pad-independent library for a pad design domain into which the dielectric pad can subsequently be positioned. In the current work we present further speed improvements by compressing this library significantly, such that the optimal pad can be found within 30 seconds using nonlinear optimization techniques.

Methods: A spatial resolution of 5 mm^3 is used for modeling the human head and corresponding RF fields for a 7T (298 MHz) neuroimaging application. The virtual family member "Duke" is used as body model[5] combined with a 16-rung high-pass birdcage coil as shown in Figure 1(a). For the domain decomposition we consider a 1 cm thick pad design domain surrounding the head, as shown in Figure 1(b). Computations are performed with Remcom XFdtd (v.7.5.0.3) and Matlab (R2014b) on a Windows 64-bit machine (Intel Xeon CPU X5660@2.80 GHz, 48 GB memory, two NVIDIA Tesla K40c GPU's).

Since we do not need a 5 mm^3 resolution for designing dielectric pads, we artificially decrease the resolution by subdividing the pad design domain into 400 subdomains of approximately 2×2×1 cm^3 as is shown in Figure 1(c). We then parameterize the design domain to model a rectangular pad with a single permittivity by using a superposition of Heaviside step functions. In such a way, we formulate the design problem as an optimization of five variables only, as illustrated in Figure 1(d).

Within the range of pads that can be generated using this parameterization and subdomains, we can now compress the size of the original full order library by dispensing with information that does not contribute to the associated range of B_1^+ solutions. This compression is obtained by evaluating a series of pad realizations or "snapshots" where randomized values are assigned to our five design parameters. The resulting electric current density in the pad design domain is stored for each realization. Hereafter, the 500 most significant contributions are extracted by using a singular value decomposition, which forms a projection matrix.[6] Finally, we project the library using this projection matrix on a new reduced order library giving us a Reduced Order Model (ROM). This process is illustrated in Figure 2.

For the optimization we define a cost function that is to be minimized as

$$C(\boldsymbol{p}) = \frac{\left\|\boldsymbol{b}_1^+(\boldsymbol{p}) - \boldsymbol{b}_1^{+;\text{desired}}\right\|_2^2}{\left\|\boldsymbol{b}_1^{+;\text{desired}}\right\|_2^2},$$

where $\boldsymbol{b}_1^+(\boldsymbol{p})$ is the B_1^+ field computed by the ROM and $\boldsymbol{b}_1^{+;\text{desired}}$ is the desired B_1^+ field intensity which is set to 1.2 µT. Finally, vector $\boldsymbol{p}$ contains the parameters of the pad. The nonlinear optimization problem is solved using a Gauss-Newton algorithm combined with backtracking line search to determine the stepsize.

The optimized pads have been implemented using a stabilized suspension of barium and calcium titanate[7] such that the B_1^+ maps can be compared with measured data obtained in vivo using a DREAM B_1^+ mapping sequence.[8]

Results: The model order reduction procedure compresses the library for the pad design domain, from 29 GB to 1 GB. Evaluating the B_1^+ field using the full order model completes in about 90 seconds when using GMRES (tolerance=10^{-4}), whereas the ROM completes in 0.35 seconds when using a direct solver (~250x speedup). A comparison of the B_1^+ fields for different dielectric pads obtained using the full order model and ROM is shown in Figure 3, where the global error in B_1^+ magnitude is around 5%.

The evolution of the optimization method for two scenarios is shown in Figure 4, where the region of interests are indicated by black squares. Within 10 iterations and 30 seconds the method converges to a dielectric pad with a relative permittivity of 200 and 220, respectively, with corresponding dimensions of 13×16×1 cm^3 and 13×22×1 cm^3. The in vivo results are in good agreement as shown in Figure 5.

Discussion and conclusion: Practical dielectric pads can be designed in 3D within 30 seconds using the parameterized ROM and the optimization scheme. With the ROM, the B_1^+ field in the entire head can be obtained in 0.35 seconds for any dielectric pad. This approach can facilitate efficient 3D design of high-permittivity pads for dielectric shimming in high-field MRI.

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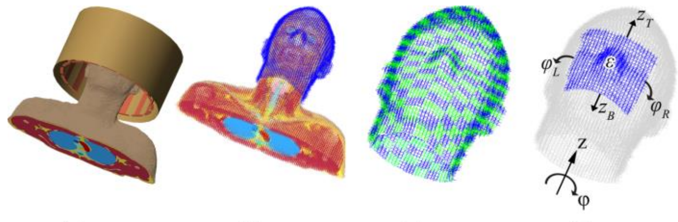

Figure 1: Head model Duke in a 7T quadrature birdcage coil (a). The pad design domain is defined to be 1 cm thick surrounding the head (b). The design domain is divided into 400 subdomains (c). The model is parameterized such that a rectangular pad can easily be defined (d).

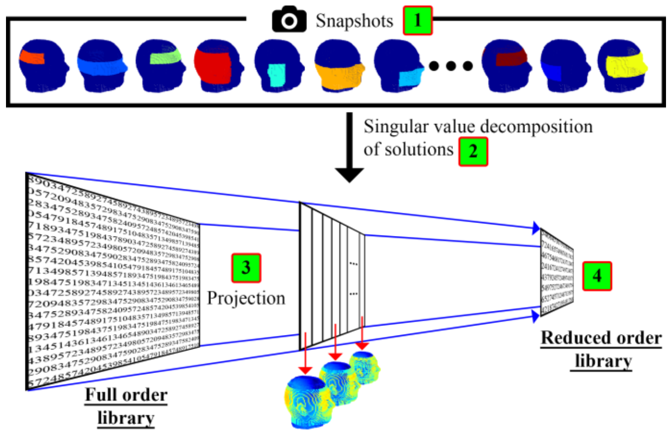

Figure 2: Series of snapshots is created by simulating a variety of pads (1). The solutions are stored in snapshot matrix, hereafter, a singular value decomposition is performed to create a projection matrix (2). The full order library is then projected (3) to obtain our desired reduced order library (4).

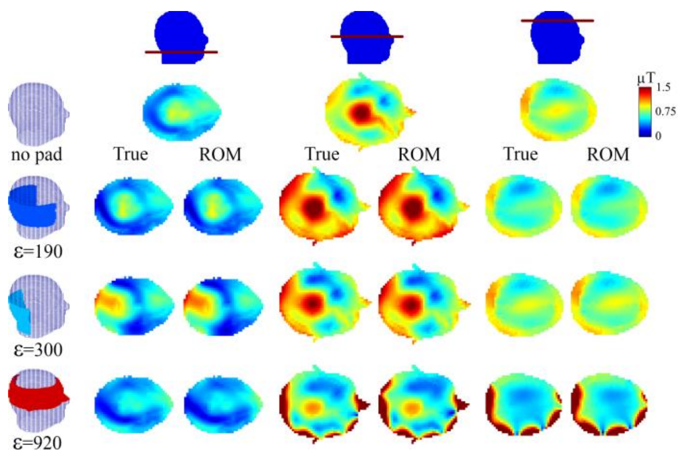

Figure 3: Comparison between "true" B_1^+ field and ROM solution. Top row indicates evaluated slices and left column the pads for comparison. The top row with B_1^+ fields is the result when no pad is used. The remaining plots are the true (left) and the reduced order solution (right).

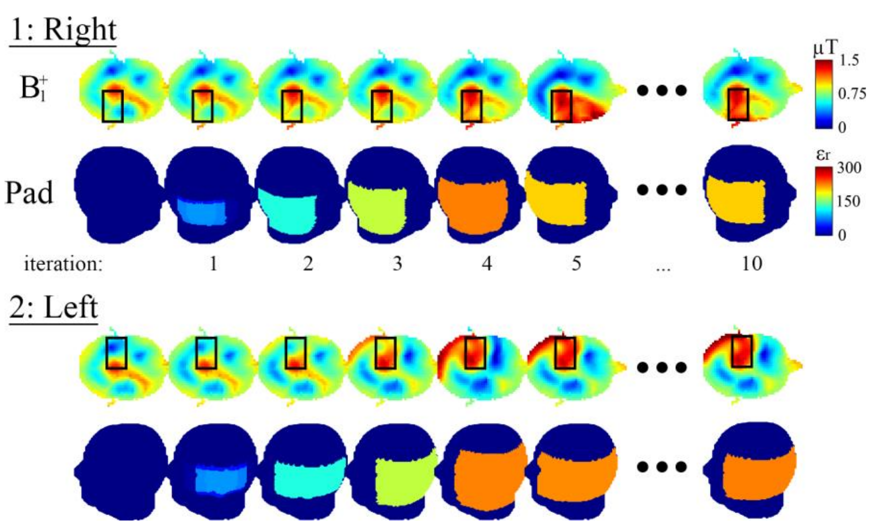

Figure 4: A region of interest is selected for two scenarios, indicated by black square for a transverse slice. The second and fourth row show the pad with corresponding permittivity. The optimal pad is found within 10 iterations and 30 seconds. The results are shown for most of the iterations.

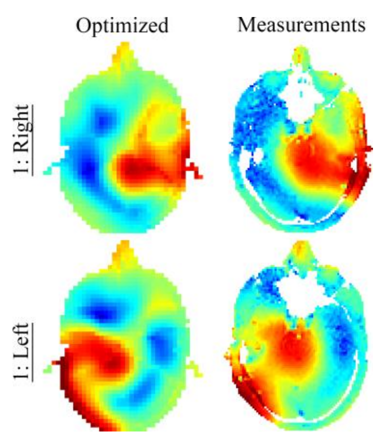

Figure 5: In vivo validation of the optimized dielectric pads. The optimized B_1^+ fields are shown in the left column and in vivo measurements in the right column. In vivo data is acquired using a DREAM B_1^+ mapping sequence (2.5 mm^2 resolution, 5 mm slice thickness, STEAM/imaging tip angle = 50°/10°).