BLIND ESTIMATION OF MULTIPLE DIGITAL SIGNALS TRANSMITTED OVER MULTIPATH CHANNELS

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Abstract

A number of authors have recently proposed algorithms for the blind separation and equalization of multiple co-channel digital signals transmitted through wireless environments with long delay spreads. These algorithms compute the coefficients of a joint space-time equalizer in which the outputs of multiple antennas are oversampled and linearly combined to produce estimates of the transmitted finite alphabet symbols. In this paper, we present improvements in three directions, namely (1) a more efficient way to do the inherent subspace intersections, (2) a way to solve the problem for channels with largely differing and ill-defined delay spreads, and (3) algorithm-independent lower bounds for the number of antennas and amount of oversampling to provide sufficient resolution in the case of bandlimited signals. The improved algorithm is tested on experimental data.

1. INTRODUCTION

A challenging signal processing problem is the blind joint space-time equalization of multiple digital signals transmitted over multipath channels. This problem is an abstraction of a PCS wireless communication scenario (see fig. 1) in which a number of users broadcast co-channel digitally modulated signals towards a central base station in a multipath propagation environment. The sources are unsynchronized and interfere with each other. Moreover, the multipath is diffuse with long delay spread, causing intersymbol interference of up to 10–15 symbols. The objective of the base station is to separate and equalize the signals.

We assume that the base station has an array of M antennas, and that the received signal at each antenna is sampled faster than the symbol rate by a factor P. Our goal is to derive a block algorithm for computing the coefficients of a joint space-time equalizer. Once a solution is obtained, the equalizer can switch to decision-directed mode to track slow changes. Analysis of the block problem allows us to address resolution issues such as minimal values for M, P and other parameters that are required for a given scenario.

There are several leverages for solving the resulting blind FIR-MIMO (multi-input multi-output) identification problem. E.g., the *fixed symbol rate* of digital signals in combination with multiple antennas and oversampling allows to blindly synchronize and equalize (but not separate) such signals. This was originally shown in the FIR-SISO case by Tong, Xu and Kailath [1]. More recent algorithms are in essence based on subspace intersections [2–4]. Another useful property is the *finite alphabet* structure of digital signals, which can be used both for equalization and signal separation [5]. The two properties are readily combined into one algorithm to solve the FIR-MIMO problem [6, 7]. Many other properties could also be used, for example high-order statistics or constant modulus properties.

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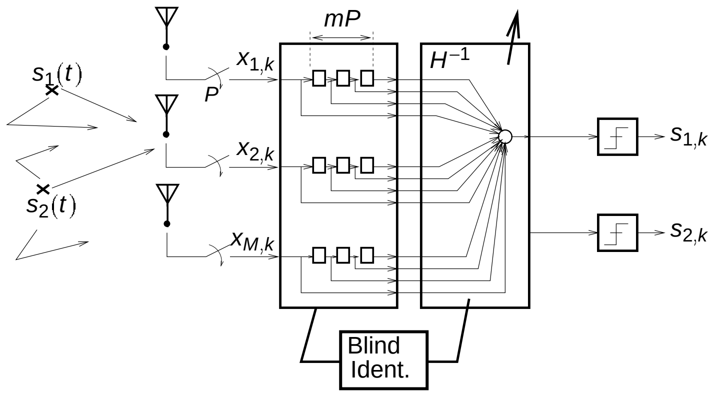

Figure 1. Channel and filter model

2. DATA MODEL

The data model is the same as we used in [7, 8]. A digital signal $s(t)$ is written as a series of dirac pulses, $s(t) = \sum_{-\infty}^{\infty} s_k \delta(t-k)$, where s_k are transmitted symbols, and the symbol rate is normalized to $T = 1$. An array of M sensors, with outputs $x_1(t), \cdots, x_M(t)$, receives d digital signals $s_1(t), \cdots, s_d(t)$ through independent channels $h_{ij}(t)$. Each impulse response $h_{ij}(t)$ is a convolution of the shaping filter of the i-th signal and the actual channel from the i-th input to $x_j(t)$, including any propagation delays. This data model is written compactly as $\mathbf{x}(t) = H(t) * \mathbf{s}(t)$, where

$$\mathbf{x}(t) = \begin{bmatrix} x_1(t) \\ \vdots \\ x_M(t) \end{bmatrix}, \; H(t) = \begin{bmatrix} h_{11}(t) \cdots h_{1d}(t) \\ \vdots \qquad \vdots \\ h_{M1}(t) \cdots h_{Md}(t) \end{bmatrix}, \; \mathbf{s}(t) = \begin{bmatrix} s_1(t) \\ \vdots \\ s_d(t) \end{bmatrix}.$$

We sample each $x_i(t)$ over N symbol periods at a rate $P \in \mathbb{N}$, where P is the oversampling factor, and that we put the MP samples of each symbol period in a data vector

$$\mathbf{x}_k := \begin{bmatrix} \mathbf{x}(k) \\ \mathbf{x}(k+\frac{1}{P}) \\ \vdots \\ \mathbf{x}(k+\frac{P-1}{P}) \end{bmatrix}, \quad k = 0, \cdots, N-1.$$

If we assume that all $h_{ij}(t)$ are FIR filters of length at most $L \in \mathbb{N}$, *i.e.*, $h_{ij}(t) = 0$, $t \notin [0, L)$, then at most L consecutive symbols of each signal play a role in $\mathbf{x}(t)$ at any given moment, *i.e.*,

$$
\begin{aligned}
\mathbf{x}_k &= \mathbf{H}\mathbf{s}_{k:k-L+1} \\
&=: \begin{bmatrix} H(0) & H(1) \cdots & H(L-1) \\ H(\frac{1}{P}) & \cdot & \cdot \\ \vdots & & \vdots \\ H(\frac{P-1}{P}) & \cdot & \cdots H(L-1+\frac{P-1}{P}) \end{bmatrix} \begin{bmatrix} \mathbf{s}_k \\ \vdots \\ \mathbf{s}_{k-L+1} \end{bmatrix} \\
&\quad \mathbf{H}:\ MP \times dL\,.
\end{aligned}
$$

We collect all data in a block-Hankel matrix by left-shifting and stacking m times, where m is the equalizer length (measured in symbol periods),

$$
\mathcal{X} := \begin{bmatrix} \mathbf{x}_0 & \mathbf{x}_1 & \iddots & \mathbf{x}_{N-m} \\ \mathbf{x}_1 & \mathbf{x}_2 & \iddots & \iddots \\ \iddots & \iddots & \iddots & \mathbf{x}_{N-2} \\ \mathbf{x}_{m-1} & \iddots & \mathbf{x}_{N-2} & \mathbf{x}_{N-1} \end{bmatrix} : mMP \times (N-m+1)\,.
$$

It is seen that $\mathcal{X}$ has a factorization

$$
\begin{aligned}
\mathcal{X} &= \mathcal{H}\mathcal{S} \\
&= \begin{bmatrix} \mathbf{0} & \mathbf{H} \\ & \iddots \\ \mathbf{H} & \\ \mathbf{H} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{s}_{m-1} & \ddots & \mathbf{s}_{N-2} & \mathbf{s}_{N-1} \\ \ddots & \ddots & \ddots & \mathbf{s}_{N-2} \\ \mathbf{s}_{-L+2} & \mathbf{s}_{-L+3} & \ddots & \ddots \\ \mathbf{s}_{-L+1} & \mathbf{s}_{-L+2} & \ddots & \mathbf{s}_{N-L-m+1} \end{bmatrix} \\
&\quad \mathcal{H} : mMP \times d(L+m-1) : \text{block-Hankel}, \\
&\quad \mathcal{S} : d(m+L-1) \times (N-m+1) : \text{block-Toeplitz}.
\end{aligned}
$$

Identification is possible if this is a minimal rank factorization. *Necessary* conditions for $\mathcal{X}$ to have a minimal-rank factorization are that $\mathcal{H}$ is a 'tall' matrix and $\mathcal{S}$ is a 'wide' matrix, which for $L > 1$ leads to

$$
\begin{aligned}
MP &> d \\
m &\geq \frac{dL-d}{MP-d} \\
N &> dL + (d+1)(m-1)\,.
\end{aligned}
\tag{1}
$$

3. BLIND IDENTIFICATION

Suppose that the conditions (1) are satisfied, and that $\mathcal{H}$ has full column rank $d(L+m-1)$. Then $\text{row}(\mathcal{X}) = \text{row}(\mathcal{S})$, so that we can determine the row span of $\mathcal{S}$ from that of $\mathcal{X}$. The first step of the algorithm is to compute an orthonormal basis $\hat{V}$ of $\text{row}(\mathcal{X})$. The next step is to find linear combinations of the rows of $\hat{V}$ such that the result both belongs to the finite alphabet (FA) and has a Toeplitz structure.

3.1. Detection of d and L

If $\mathcal{H}$ and $\mathcal{S}$ have full column rank and row rank, respectively, then the rank of $\mathcal{X}$ is $d_{\mathcal{X}} := d(L+m-1)$. The number of signals d can be estimated by increasing the blocking factor m of $\mathcal{X}$ by one, and looking at the increase in rank of $\mathcal{X}$. This property provides a very effective detection mechanism even if the noise level is quite high since it is independent of the actual (observable) channel length $\hat{L}$. Furthermore, it still holds if all channels do not have equal lengths (see section 3.4 below). In case they do, then L can be estimated from the estimated rank of $\mathcal{X}$, $\hat{d}_{\mathcal{X}}$, and the estimated number of signals, $\hat{d}$, by $\hat{L} = \hat{d}_{\mathcal{X}}/\hat{d} - m + 1$.

3.2. Forcing the FA property: ILSP

For a given matrix X, the ILSP algorithm [5] solves the factorization

$$
(X = AS : A, S \text{ full rank}, [S]_{ij} \in \mathcal{FA})\,, \tag{2}
$$

where $\mathcal{FA}$ is a pre-specified finite alphabet, and A is any resulting non-singular matrix. The algorithm uses alternating projections to estimate A and S.

Suppose we already know an orthonormal basis $\hat{V}$ of $\text{row}(X)$. Then, a computationally more efficient version of ILSP proceeds by initializing $S^{(0)} = \hat{V}$, and iterating as follows:

- Project $S^{(k)}$ onto $\text{row}(X)$: $S^{(k)\prime} := S^{(k)}\hat{V}^*\hat{V}$,
- Project each entry $[S^{(k)\prime}]_{ij}$ onto the closest member of the alphabet,
- Check the independence of the rows, and modify duplicates, resulting in $S^{(k+1)}$.

The iteration converges rapidly and reliably when the number of rows of $\hat{V}$ is not too large.

Since the factorization $\mathcal{X} = \mathcal{H}\mathcal{S}$ is of the form (2), we could in principle use the ILSP algorithm directly on $\mathcal{X}$. However, $\mathcal{X}$ is generally a large matrix with many rows, limiting the performance of ILSP (mainly in the context of finding *all* independent signals). A second problem is that it doesn't force the Toeplitz structure of $\mathcal{S}$. After finding a candidate $\mathcal{S}$, we have to compare the rows and detect which rows are shifted copies (echos) of other rows, and permute the rows accordingly.

3.3. Forcing the Toeplitz property: subspace intersections

A standard procedure to find $\mathcal{S}$ as a block-Toeplitz matrix with $\text{row}(\mathcal{S}) = \text{row}(\mathcal{X})$ (but not forcing the FA property) is to rewrite this as

$$
\begin{aligned}
[\mathbf{s}_{m-1} \quad \mathbf{s}_{m} \quad \cdots \quad \mathbf{s}_{N-1}] &\in \text{row}(\mathcal{X}) \\
[\mathbf{s}_{m-2} \quad \mathbf{s}_{m-1} \quad \cdots \quad \mathbf{s}_{N-2}] &\in \text{row}(\mathcal{X}) \\
&\vdots \\
[\mathbf{s}_{-L+1} \quad \mathbf{s}_{-L+2} \quad \cdots \quad \mathbf{s}_{N-L-m+1}] &\in \text{row}(\mathcal{X})
\end{aligned}
\tag{3}
$$

These conditions can be aligned to apply to the same block-vector in several ways. We choose to work with

$$
S := [\mathbf{s}_{-L+1} \quad \mathbf{s}_{-L+2} \quad \cdots \quad \mathbf{s}_{N-1}]\,.
$$

Let $\hat{V}$ be a basis for $\text{row}(\mathcal{X})$. The conditions (3) are transformed into

$$
\begin{aligned}
S &\in \text{row}\hat{V}^{(1)}, & \hat{V}^{(1)} &:= \begin{bmatrix} \hat{V} & \mathbf{0} \\ \mathbf{0} & I_{L+m-2} \end{bmatrix}, \\
S &\in \text{row}\hat{V}^{(2)}, & \hat{V}^{(2)} &:= \begin{bmatrix} \mathbf{0} & \hat{V} & \mathbf{0} \\ 1 & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & I_{L+m-3} \end{bmatrix}, \\
&\vdots \\
S &\in \text{row}\hat{V}^{(L+m-1)}, & \hat{V}^{(L+m-1)} &:= \begin{bmatrix} \mathbf{0} & \hat{V} \\ I_{L+m-2} & \mathbf{0} \end{bmatrix}.
\end{aligned}
\tag{4}
$$

The identity matrices in each $V^{(k)}$ reflect the fact that, at that point, there are no range conditions on the corresponding columns of S.

Thus, S is in the intersection of the row spans of $\hat{V}^{(1)}$ till $\hat{V}^{(L+m-1)}$, and the problem is one of determining a basis for the intersection of a set of given subspaces. One order-insensitive way to do this is to compute the union of the complements of the subspaces, and take the complement again. This is the approach presented in [6, 7], but with a complexity of $\mathcal{O}(N^3(L+m))$, it is not attractive for large N. It is, however, possible to compute subspace intersections without forming complements. To this end, we use the fact that, for *orthonormal* bases $\hat{V}^{(k)}$ as we have in (4), precisely the same subspace intersection is obtained by computing the SVD of a stacking of all the basis vectors, or more conveniently, (for n intersections, $n = L+m-1$) by an SVD of

$$V_{T(n)} := \left[\begin{array}{cccc} \hat{V} & & & \mathbf{0} \\ & \hat{V} & & \\ & & \ddots & \\ \mathbf{0} & & \hat{V} & \\ \hline J_1 & & \mathbf{0} & \\ \mathbf{0} & & & J_2 \end{array}\right] \begin{array}{l} \Big\updownarrow n \\ \\ \\ \end{array} \qquad (5)$$

where the n copies of $\hat{V}$ are each shifted over 1 entry, and

$$J_1 = \begin{bmatrix} \sqrt{n-1} & & \mathbf{0} \\ & \ddots & \\ & & \sqrt{2} & \\ \mathbf{0} & & & 1 \end{bmatrix}, \quad J_2 = \begin{bmatrix} 1 & & & \mathbf{0} \\ & \sqrt{2} & & \\ & & \ddots & \\ \mathbf{0} & & & \sqrt{n-1} \end{bmatrix}.$$

The matrices J_1, J_2 summarize the identity matrices present in (4), which is possible because we are only interested in row spans. The estimated basis for the intersection (hence for S) is given by the right singular vectors of V_T that correspond to the *large* singular values of V_T: those that are close to $\sqrt{n}$. This subspace intersection algorithm has complexity $\mathcal{O}(d^2(L+m)^3N)$ and is linear in N. Incidentally, the structure of $V_{T(n)}$ also shows that the first and last $n-1$ columns of S are expected to have lower accuracy than the middle columns.

At this point, we have only obtained a *basis* for the row span of S. Ideally, it consists of d row vectors. To find S itself (hence $\mathcal{S}$ as well), we have to determine which linear combination of the basis vectors gives a finite alphabet structure. This is a problem of the form (2), and calls for the ILSP algorithm. Effectively, the subspace intersections perform a blind equalization jointly on all signals, but their separation is done based on the FA property.

3.4. Approach for unequal and ill-defined channel lengths

Usually, channels do not have the same well-defined channel length L. Multipath echos with a long delay generally have a smaller amplitude, so that the channel responses trail down to zero rather than filling out a sharply defined interval in time. In such cases, $\mathcal{H}$ is not of full column rank or is ill-conditioned, and the subspace intersection cannot be used to precisely cancel all echos. The problem is two-fold: (i) with too many intersections (say $L+m-1$), signals with channel lengths shorter than L are wiped out, and (ii) for channels with ill-defined lengths, the rows of $\hat{V}$ corresponding to the weaker echos have a large variance due to the ambient noise, which with full row span intersections translates to a large variance on the estimate of S. The first effect is actually exploited in [9] to separate the signals in a recursive scheme, but with ill-defined channel lengths that approach might be sensitive. We propose to take only the well-defined intersections, ideally $m + \min_j(L_j) - 1$ (where $L_j = \max_i(L_{ij})$) but without prior knowledge of channel lengths perhaps even only m, and then use the finite alphabet property to do the remaining equalization and the signal separation as well. This means that the Toeplitz structure in $\mathcal{S}$ is only partly enforced, and that the ILSP algorithm has a larger responsibility. A second improvement is to recognize that, with ill-defined channel lengths, the basis obtained from the intersections needs further equalization. To this end, the basis is extended with a number of shifted copies of itself, so that we obtain a Toeplitz matrix with m_1 block rows, where m_1 is some small number (a "secondary" equalizer length). After ILSP has found the candidate signals, we have to select a subset of d independent signals which are not shifts of each other and with minimal distance to the finite alphabet. The algorithm is summarized in figure 2. The significant improvement obtained by $m_1 > 1$ will be clear from the experiment in section 5.

In: $\mathcal{X}$, **out:** generator of $\mathcal{S}$ s.t. $\mathcal{X} = \mathcal{H}\mathcal{S}$, w. $\mathcal{S}$ Toeplitz+FA
1. *Estimate row*($\mathcal{X}$):
   a. Compute SVD($\mathcal{X}$): $\mathcal{X} =: U\Sigma V$
   b. Estimate $\hat{d}_{\mathcal{X}} = \text{rank}(\mathcal{X})$ from Σ
   c. $\hat{V}$ = first $\hat{d}_{\mathcal{X}}$ rows of V
   (d. Estimate d from another value of m)
2. *Partial time-equalization: do n subspace intersections*:
   a. Set $n = m+\hat{L}-1$, with $\hat{L} = 1$ or $\hat{L} = \min(L_j)$
   b. Construct $V_{T(n)}$ in equation (5)
   c. Compute SVD($V_{T(n)}$)
   d. Set $\hat{d}_S = \hat{d}_{\mathcal{X}} - dn + d$
   e. $\hat{S}$:= largest $\hat{d}_S$ right singular vectors
3. *Separate signals based on FA property*:
   a. Select m_1, *e.g.*, $m_1 = \lceil \frac{\hat{d}_S}{d} \rceil$
   b. Do ILSP on Hankel matrix from $\hat{S}$, with $m_1 - 1$ shifts
   c. Detect echos, keep d independent signals with lowest variance

Figure 2. Blind FIR-MIMO identification algorithm

4. BANDLIMITED SIGNALS

Another important case where $\mathcal{H}$ is ill-conditioned occurs for bandlimited signals. In view of Shannon's theorem, it would appear unlikely that it is possible to separate two bandlimited signals based on oversampling only. The confusion is due to some extent by the fact that, in Shannon's language, oversampling is measured with respect to the Nyquist rate, whereas in the fractionally-sampled literature, it is measured w.r.t. the symbol rate. The two are the same only if the pulse shape function is a pure sinc-function. In practice, other pulse shapes that occupy a larger bandwidth are often used, and these might indeed be separated by sampling faster than the symbol rate. Suppose that the normalized ($T = 1$) bandwidth of the pulse is $1+\beta$. Then sampling much faster than $1+\beta$ is not useful since it does not provide new information.* Hence, unlike as suggested in (1), antennas and oversamping are not equivalent in the bandlimited case: the leverage of P is limited, so that a larger M is required to compensate.

A second point is that, for bandlimited signals, the singular values of the H-matrix in general do not show a steep drop but trail down more gradually, making the problem ill-conditioned. For very small values of M, there is not enough resolution to observe changes in channel lengths or even changes in d. Relations between M, β, P, m, d and L, and the expected number of large singular values in $\mathcal{H}$ may be derived both theoretically and via simulations. This will be reported elsewhere, but some relevant results are listed below. Loosely speaking, one might say that sampling faster than Nyquist, $P > 1+\beta$, gives the same resolution as sampling at Nyquist rate, so that the condition $MP > d$ in (1) becomes $M > d/(1+\beta)$ in the bandlimited case. However, this only gives sufficient resolution for detection of d, not necessarily of L. To enable equalization of arbitrary long L, a certain correction factor $1+\varepsilon\beta$ $(0 < \varepsilon \leq 1)$ is in order, where ε is a quality parameter. Ideally, $\varepsilon = 1$, but small values are usually already good enough. A final point is that M should be sufficiently large such that extremely large equalizer lengths m are avoided: we will require $m < 2L$. In summary, algorithm-independent minimal values for M and m (taking $P > 1+\beta$), below which $\mathcal{H}$ will not have rank $L+m-1$, are provided by the following equations:

$$\begin{aligned} M &\geq 1\tfrac{1}{2} d \frac{1+\varepsilon\beta}{1+\beta} \quad (0 < \varepsilon \leq 1) \\ m &\geq \frac{d(L-1)}{M(1+\beta)-d} \end{aligned} \qquad (6)$$

The factor $1\frac{1}{2}$ in the condition for M ensures $m < 2L$. Figure 3 shows the relation for M, with $\varepsilon = 0.1$ and $\varepsilon = 0.2$ (dashed). The required number of antennas is linear in d. With these values for M, the resolution is high enough such that arbitrary long channel lengths L are allowed, provided the equalizer length m is large enough.

5. WIRELESS INDOOR EXPERIMENTS

In this section, we report on a test of the algorithm in an off-line experiment, in which we simulate the reception of a number of BPSK signals through an indoor wireless channel at 2.4 GHz. The channel impulse responses are derived from experimental data measured in an office at FEL-TNO (The Hague, The Netherlands) in 1992 [10].

The office is 5.6m × 5.0m, height 3.5m. The actual measurement set-up had a transmit antenna in the center of the room at a height of 3.0m, and a receiving antenna cluster located at varying positions, at a height of 1.5m. The cluster consisted of six wideband antennas spaced $\lambda/2$ in a circular array (radius 6.25cm). At each location, a batch of 801 equidistant samples in the range 2.15–2.65 GHz were measured, including the absolute phase at each cluster element.

*One has to be careful in interpreting the signal processing literature: reported positive effects of oversampling in simulations are sometimes exaggerated by the noise averaging. In reality, signals are tightly bandpass filtered and wideband SNRs have to be corrected by $10\log((1+\beta)/P)$ to reflect only the inband noise power.

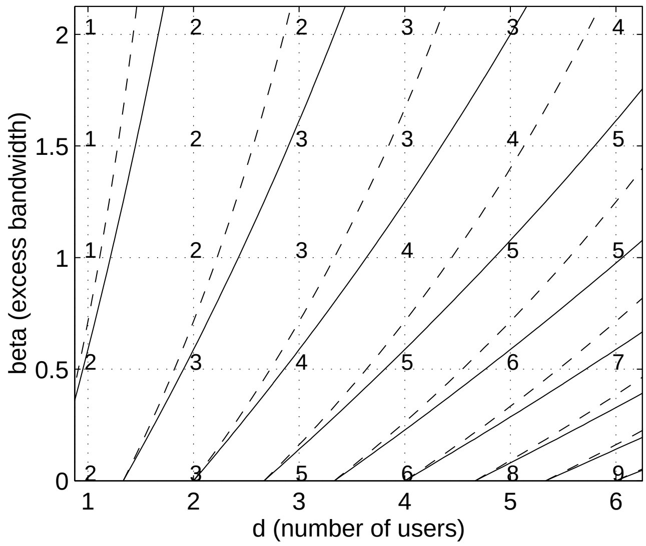

Figure 3. Minimal number of antennas M to enable equalization of bandlimited channels with arbitrary long delay spread (equation (6), $\varepsilon = 0.1$; dashed: $\varepsilon = 0.2$)

Assuming reciprocity (not quite true) we can pretend to simulate a central basestation antenna array of up to six elements, receiving a superposition of signals from a number of user locations. We have used data from two such locations, one with a direct line of sight (RMS delay spread = 7.3 ns) and one without LOS (RMS delay spread = 16.7 ns). The relative power in the frequency domain is plotted in figure 4(a); figure 4(b) shows the amplitude of the response to a raised-cosine pulse (T = 6ns, $\beta = 0.5$, demodulated to baseband from a carrier frequency of 2.4 GHz), each normalized to unit power. Such a pulse uses all of the measured bandwidth.

In the experiment, we took $d = 2$ BPSK sources, transmitted over the above channels, $M = 3$ antennas (in correspondence with figure 3), $P = 3$ times oversampling, $N = 300$ samples, and added white Gaussian noise with signal-to-noise ratio SNR = 15dB per antenna per sample per signal (inband SNR = 18 dB). The received power of both signals is equal. We set an equalizer length of $m = 10$ symbols. The singular values of $\mathcal{X}$ are plotted in figure 5, for a range of values of m. It is seen that the numerical rank of $\mathcal{X}$ $(= \hat{d}_{\mathcal{X}})$ cannot very well be estimated, but clearly, $d = 2$, as deduced from the horizontal shifts for increasing m. Table 1 lists the standard deviations of the symbol estimates (before classification as ±1) for a range of parameter settings m, $\hat{L}$, $\hat{d}_{\mathcal{X}}$, m_1. For $M = 3, P = 3, \beta = .5$, the optimal standard deviations in case there was no dispersive channel would be $\sigma\sqrt{(1+\beta)/(PM)} = 0.073$ (for $M = 6$, optimally 0.051). It is seen from the table that some parameter settings get us reasonably close to these optimal values. Precisely how to find these settings a priori is yet an open problem. However, it seems essential that the number of subspace intersections be small ($\hat{L} \leq 1$) and that ILSP should be used as equalizer as well ($m_1 > 1$).

Acknowledgement

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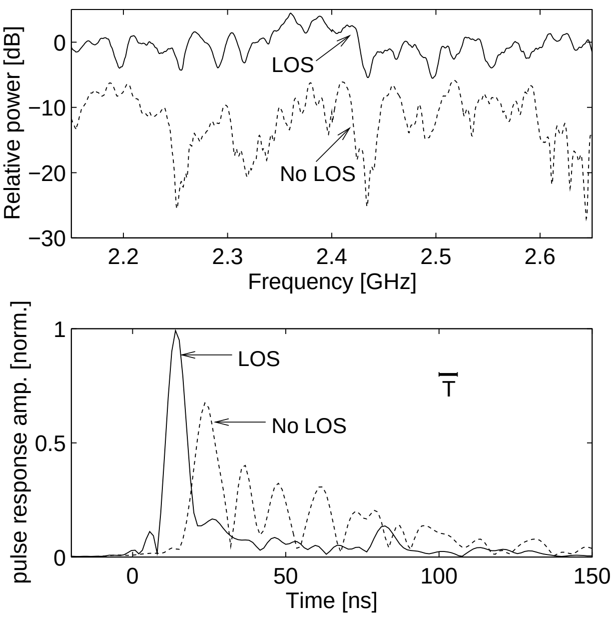

Figure 4. (a) Relative power, and (b) response to a raised-cosine pulse ($T = 6$ns, $\beta = 0.5$) of two measured indoor channels

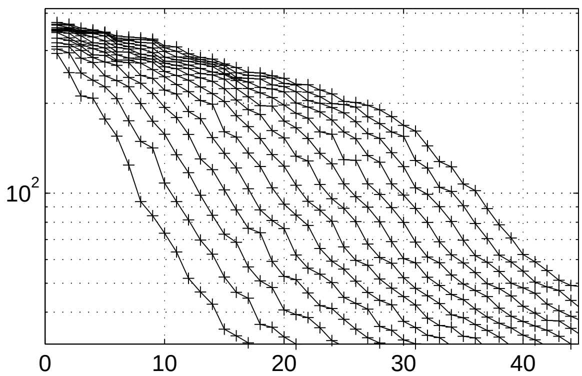

Figure 5. Singular values of $\mathcal{X}$, for $m = 2, \cdots, 15$

Table 1. Standard deviations of symbol estimates for varying parameter settings

$\hat{d}_{\mathcal{X}}$	$m_1 = 1$		2		4		6	
$m = 10, \hat{L} = 0$								
24	0.31	0.39	0.23	0.28	0.15	0.20	0.11	0.14
25	0.33	0.41	0.22	0.31	0.14	0.19	0.10	0.15
26	0.30	0.39	0.19	*0.23	0.12	0.16	**0.09**	**0.11**
27	0.29	0.36	0.21	0.22	0.12	0.16	0.09	0.12
28	0.29	0.36	0.21	0.25	0.11	0.38	0.32	0.37
$m = 10, \hat{L} = 1$								
24	0.34	0.42	0.24	0.31	0.16	0.24	0.12	0.18
25	0.33	0.42	0.23	0.30	0.14	0.21	0.11	0.18
26	0.32	0.45	0.22	0.25	0.14	0.17	**0.10**	**0.11**
27	0.32	0.42	0.22	0.22	0.13	0.15	0.10	0.12
28	0.32	0.41	0.21	0.29	0.13	0.15	0.09	0.14
$m = 10, \hat{L} = 2$								
24	0.46	0.46	0.39	0.40	0.25	0.32	0.19	0.27
25	0.45	*0.46	0.38	*0.43	0.27	*0.43	0.21	*0.40
26	0.42	0.44	0.30	0.33	0.17	0.22	**0.13**	**0.16**
27	0.41	0.44	0.28	0.36	0.16	0.22	0.14	0.18
28	0.41	0.43	0.24	0.32	0.19	*0.25	0.12	0.40
$m = 12, \hat{L} = 1$								
28	0.33	0.46	0.24	0.36	0.16	0.20	0.13	0.15
29	0.33	0.42	0.23	0.33	0.15	0.19	0.12	0.14
30	0.32	0.42	0.22	0.24	0.14	0.18	0.11	0.39
31	0.33	0.42	0.21	0.22	0.14	0.16	**0.10**	**0.12**
32	0.32	0.40	0.19	0.21	0.13	0.15	0.10	0.13
$m = 10, \hat{L} = 1$, $M = 6$ antennas								
24	0.35	0.41	0.23	0.24	0.13	0.17	0.09	0.10
25	0.33	0.40	0.22	0.22	0.10	0.15	0.08	0.09
26	0.33	0.37	0.20	0.21	0.10	0.14	0.08	0.09
27	0.29	0.34	0.15	0.19	0.09	0.11	0.07	0.07
28	0.21	*0.23	0.13	0.15	0.08	0.09	**0.06**	**0.07**

(*): signal 2 not recovered

References

[1] L. Tong, G. Xu, and T. Kailath, "Blind identification and equalization based on second-order statistics: A time domain approach," *IEEE Trans. Informat. Th.*, vol. 40, pp. 340–349, Mar. 1994.

[2] H. Liu, G. Xu, and L. Tong, "A deterministic approach to blind equalization," in *27-th Asilomar Conf. on Signals, Systems and Computers*, pp. 751–755, 1993.

[3] D. Slock, "Blind fractionally-spaced equalization, perfect-reconstruction filter banks and multichannel linear prediction," in *Proc. IEEE ICASSP*, pp. IV:585–588, 1994.

[4] E. Moulines, P. Duhamel, J.F. Cardoso, and S. Mayrargue, "Subspace methods for the blind identification of multichannel FIR filters," in *Proc. IEEE ICASSP*, pp. IV:573–576, 1994.

[5] S. Talwar, M. Viberg, and A. Paulraj, "Blind estimation of multiple co-channel digital signals using an antenna array," *IEEE Signal Processing Letters*, vol. 1, pp. 29–31, Feb. 1994.

[6] H. Liu and G. Xu, "A deterministic approach to blind symbol estimation," *IEEE Signal Processing Letters*, vol. 1, pp. 205–207, Dec. 1994.

[7] A.J. van der Veen, S. Talwar, and A. Paulraj, "Blind estimation of multiple digital signals transmitted over FIR channels," *IEEE Signal Processing Letters*, vol. 2, pp. 99–102, May 1995.

[8] A.J. van der Veen, S. Talwar, and A. Paulraj, "Blind identification of FIR channels carrying multiple finite alphabet signals," in *Proc. IEEE ICASSP*, pp. 1213–1216, 1995.

[9] H. Liu and G. Xu, "Multiuser blind channel estimation and spatial channel pre-equalization," in *Proc. IEEE ICASSP*, pp. 1756–1759, 1995.

[10] G.J.M. Janssen and R. Prasad, "Propagation measurements in an indoor radio environment at 2.4 GHz, 4.75 GHz and 11.5 GHz," in *42nd IEEE VTS Conference*, pp. 617–620, 1992.