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BALANCED CAPACITY OF WIRELINE MULTIPLE ACCESS CHANNELS WITH INDIVIDUAL POWER CONSTRAINTS

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ABSTRACT

This paper applies the concept of balanced capacity (a trade-off between performance and fairness) to the uplink of a frequency-selective multiuser channel. Individual power constraints are applied on each transmitter, which makes the computation of the balanced capacity very difficult. A sub-optimal solution, based on the Iterative Multiuser Water-Filling algorithm, is proposed and shown to be very close to the optimal solution.

1. INTRODUCTION

The powerline access channel is an example of a strongly frequency-selective multiuser channel. It provides simultaneous connections between the cable head-end and a number of subscribers spatially distributed along the common physical cable. The quality of the different user channels can be very heterogeneous, depending on the position of each remote terminal along the distribution cable. In this context, it is important to introduce fairness constraints in the design of the communication system. The capacity region, which is defined as the set of error-free achievable data rates $\{R_k\}$, reflects the trade-off among the individual data rates of the different users competing for the limited resources. For the Gaussian K-user channel, it is a convex region in the K-dimensional space. The balanced capacity of a multiuser channel, introduced in [1, 2], represents a nice compromise between global performance and fairness. It is defined as the distribution of maximum simultaneously achievable data rates that are proportional to the single user rates. It is a specific point of the boundary of the capacity region for which the coexistence with the other users has the same relative cost for every user. The above references provide algorithms for the computation of the balanced capacity associated with broadcast channels (downlink) and multiple access channels (uplink), and a constraint on the global transmitted power. In the case of multiple access channels, however, the usual constraint is rather on the individual transmitted powers. This paper demonstrates how the previous results can be extended to this scenario.

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2. OPTIMAL POWER ALLOCATION

The K-user multiple access channel is fully defined by the set of KN channel gains $\{h_{kn}^2\}$ and the set of N received noise powers $\{\sigma_n^2\}$ associated with the N frequency bins of width b. The total available bandwidth is $B = Nb$. The K individual power constraints are denoted by $\{\overline{P}_k\}$. Our objective is to find the optimal power allocation $\{P_{kn}\}$ providing maximum balanced rates $\{R_k\}$ and satisfying the power constraints $\sum_n P_{kn} = \overline{P}_k$ for every k. The capacity region associated with a given power allocation $\{P_{kn}\}$ is known to be a polyhedron with $K!$ vertices in the positive orthant [3, 4]. Each vertex is achievable by a successive decoding using one of the $K!$ possible decoding orders. The global capacity region is generated by the union of such polyhedrons, each one corresponding to a valid power allocation. The boundary of the resulting capacity region is curved, and can be traced out by means of a set of relative priorities $\{\alpha_k\}$: every boundary point maximizes a given aggregate rate $R_\alpha = \sum_k \alpha_k R_k$.

In [2], it is demonstrated that the maximum aggregate rate for a given spectral allocation $\{\overline{P}_n\}$ is

$$\max(R_\alpha) = b \sum_n \int_0^{\overline{P}_n} \max_k \left(\frac{\alpha_k h_{kn}^2/\sigma_n^2}{1 + P h_{kn}^2/\sigma_n^2} \right) dP. \quad (1)$$

This equation provides a way to obtain the optimal power allocation among the users in each frequency bin n, once the corresponding power budget $\overline{P}_n$ is known. A subset $S_J(n) = \{k_1, \cdots, k_J\}$ of $J(n) < K$ users is obtained, who get a non-zero fraction of the total power $\overline{P}_n$. The optimal spectral allocation $\{\overline{P}_n\}$ can be obtained by an extended water-filling algorithm [2].

Unfortunately, this solution is generally not compatible with the individual power constraints. The solution lies in the observation [5] that $R_\alpha = f\left(P_{kn}h_{kn}^2\right) = f\left(\mu_k P_{kn}\, \frac{h_{kn}^2}{\mu_k}\right)$, i.e. transmitting a signal with power P_{kn} in a channel with gain h_{kn}^2 is equivalent to transmitting a signal with power $\mu_k P_{kn}$ in a channel with gain $\frac{h_{kn}^2}{\mu_k}$ as far as the resulting capacity is concerned. The product $\mu_k P_{kn}$ appears as an 'equivalent power' allocated to user k for transmission on the 'equivalent channel' $\frac{h_{kn}^2}{\mu_k}$. For a given set $\{\mu_k\}$, we define the equivalent power sum as $\overline{P}_n^\mu = \sum_k \mu_k P_{kn}$. The result in (1) can then be generalized as follows:

$$\max(R_\alpha) = b\sum_n \int_0^{\overline{P}_n^\mu} \max_k \left(\frac{\alpha_k h_{kn}^2/\mu_k\sigma_n^2}{1+P^\mu h_{kn}^2/\mu_k\sigma_n^2}\right) dP^\mu. \tag{2}$$

The optimal spectral allocation of the equivalent power sum $\{\overline{P}_n^\mu\}$ can be obtained as follows :

$$\overline{P}_n^\mu = \max_{k\in[1,K]} \left(1-\left[\frac{\mu_k}{h_{kn}^2/\sigma^2}+(1-\alpha_k)\right]\right)_+ . \tag{3}$$

The power allocation algorithm is then applied separately in the N frequency bins. Combining the optimal spectral allocation in (3) with the optimal user allocation resulting from (2), the vector of optimal powers allocated to the users in the subset $S_J(n)$ in the n-th frequency bin, and the corresponding optimal user rates are given explicitly as :

$$\begin{pmatrix} P_{k_1,n}h_{k_1,n}^2 \\ \cdots \\ P_{k_j,n}h_{k_j,n}^2 \\ \cdots \\ P_{k_J,n}h_{k_J,n}^2 \end{pmatrix} = \begin{pmatrix} H_{k_1,n} - H_{k_1,k_2,n} \\ \cdots \\ H_{k_{j-1},k_j,n} - H_{k_j,k_{j+1},n} \\ \cdots \\ H_{k_{J-1},k_J,n} - \sigma_n^2 \end{pmatrix} \tag{4}$$

and

$$\begin{pmatrix} R_{k_1,n}^* \\ \cdots \\ R_{k_j,n}^* \\ \cdots \\ R_{k_J,n}^* \end{pmatrix} = b\log \begin{pmatrix} H_{k_1,n}/H_{k_1,k_2,n} \\ \cdots \\ H_{k_{j-1},k_j,n}/H_{k_j,k_{j+1},n} \\ \cdots \\ H_{k_{J-1},k_J,n}/\sigma_n^2 \end{pmatrix}. \tag{5}$$

with

$$H_{i,n} = \frac{\alpha_i}{\mu_i h_{in}^{-2}} \tag{6}$$

$$H_{i,j,n} = \frac{\alpha_j - \alpha_i}{\mu_j h_{jn}^{-2} - \mu_i h_{in}^{-2}}. \tag{7}$$

Equation (3) can be associated with a multiuser water-filling diagram as in [5]. Figure 1 is an example of a three-user water-filling diagram. The common water level is 1. The three containers (bold curves) are associated with the equivalent channels h_{kn}^2/μ_k, and are modified by $(1-\alpha_k)$ to reflect the different user relative priorities. The total area defined by the water level and the bottom of the composite container represents the equivalent power sum $\mu_1\overline{P}_1 + \mu_2\overline{P}_2 + \mu_3\overline{P}_3$. After the application of the power allocation algorithm in each equivalent subchannel, the water area can be subdivided in three parts (represented with three different patterns in Figure 1). The area of each part should be equivalent to the individual equivalent power $\mu\overline{P}_k$. There is a single set of coefficients $\{\mu_k\}$ satisfying the K power constraints simultaneously. These coefficients have to be computed iteratively.

3. MAXIMUM BALANCED RATES

The computation of maximum balanced rates is very complex as the optimal vectors $\boldsymbol{\alpha}$ and $\boldsymbol{\mu}$ have to be found, that satisfy simultaneously the K balanced rate equations and the K individual power constraints :

$$\boldsymbol{R}^*(\boldsymbol{\alpha},\boldsymbol{\mu}) = \sum_{n=1}^N \boldsymbol{R}_n^*(\boldsymbol{\alpha},\boldsymbol{\mu}) = g\boldsymbol{R}^1 \tag{8}$$

$$\boldsymbol{P}(\boldsymbol{\alpha},\boldsymbol{\mu}) = \sum_{n=1}^N \boldsymbol{P}_n(\boldsymbol{\alpha},\boldsymbol{\mu}) = \overline{\boldsymbol{P}} \tag{9}$$

where the non-zero elements of $\boldsymbol{P}_n$ and $\boldsymbol{R}_n^*$ are given by equations (4) and (5), $\boldsymbol{R}^1$ is the vector of single-user capacities (obtained by applying the single-user water-filling algorithm separately for each user), and $g < 1$ is a 'multiuser coefficient'. Actually, standard convex optimization methods could be used to get the exact solution $(\boldsymbol{\alpha}^*, \boldsymbol{\mu}^*)$. Taking into account the specific structure of the problem, we propose the following specialized algorithm, that updates alternately the α_k's (to obtain balanced rates) and the μ_k's (to obtain balanced powers) until both sets of constraints are satisfied.

Algorithm 1 *Maximum balanced rates with individual power constraints (exact solution)*

- *Set* $i = 0$, $\boldsymbol{\mu}^{*(0)} = [1,\cdots,1]^T$, *and* $\boldsymbol{\alpha}^{*(0)} = \left[(R_1^1)^{-1},\cdots,(R_K^1)^{-1}\right]^T$.
- *While both sets of constraints (8) and (9) are not satisfied simultaneously :*
  - ***Rate balancing step****: update* $\boldsymbol{\alpha}^*$ *iteratively on the basis of (5), while keeping* $\boldsymbol{\mu}^*$ *constant, until the balanced rate constrains (8) are satisfied.*
  - ***Power balancing step****: update* $\boldsymbol{\mu}^*$ *iteratively on the basis of (4), while keeping* $\boldsymbol{\alpha}^*$ *constant, until the individual power constrains (9) are satisfied.*

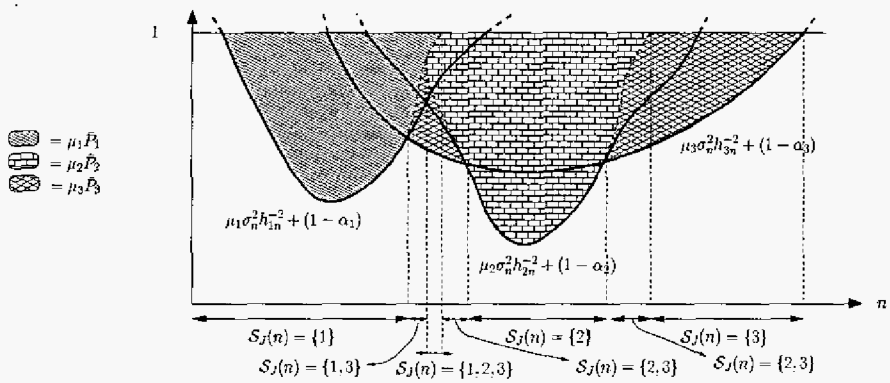

Fig. 1. Multiuser water-filling diagram

– *Set* $i = i + 1$

The vector updates can be computed from the Jacobian matrices $\frac{\partial \boldsymbol{P}}{\partial \boldsymbol{\mu}} = \sum_n \frac{\partial \boldsymbol{P}_n}{\partial \boldsymbol{\mu}}$ and $\frac{\partial \boldsymbol{R}}{\partial \boldsymbol{\alpha}} = \sum_n \frac{\partial \boldsymbol{R}_n}{\partial \boldsymbol{\alpha}}$, which are easily obtained from (4) and (5), respectively. The convergence of the proposed algorithm was found to be very slow. The main utility of this algorithm lies in the fact that it provides the *exact solution*, which is helpful in the evaluation of alternative algorithms providing suboptimal solutions.

4. SUBOPTIMAL SOLUTION

The alternative method we propose is based on the Iterative Multiuser Water-Filling algorithm (IMWF) [6]. The IMWF algorithm can be used to compute the maximum sum-rate (i.e. the same priority α_k is given to all users) of a frequency-selective multiple access channel, with individual power constraints. It is based on the observation that each power allocation $\boldsymbol{P}_k$ from the set of optimal power allocations $[\boldsymbol{P}_1, \cdots, \boldsymbol{P}_K]$ maximizing the rate-sum is obtained by single-user water-filling (SUWF) with an equivalent noise $\sigma_n'^2 = \sigma_n^2 + \sum_{k' \neq k} P_{k'n} h_{k'n}^2$. In other words, the set of K optimal power allocations should satisfy the *simultaneous water-filling* condition. The IMWF starts from arbitrary power allocations $\boldsymbol{P}_k$ and updates them successively by applying the SUWF algorithm with the $K - 1$ other allocations fixed. The obtained solution can be shown to converge quickly to the simultaneous WF solution.

Figure 2 gives a two-user capacity region with individual power constraints (curve ABCDE). Line d is the locus of balanced rate pairs. The IMWF algorithm can be used to obtain the power allocation corresponding to point C ($\alpha_1 = \alpha_2 = \frac{1}{2}$). The rates obtained with this solution are unbalanced : a higher priority should be given to user 2 ($\alpha_2 > \alpha_1$), which involves that the decoding order would be $(1, 2)$. When the highest priority is given to user 2 ($\alpha_1 = 0, \alpha_2 = 1$), the solution consists of *successive water-filling* : $\boldsymbol{P}_2$ is first computed with the SUWF algorithm ignoring the presence of user 1, $\boldsymbol{P}_1$ is then computed with the same algorithm and an equivalent noise including the effect of user 2 signal. Obviously, the maximum balanced rate pair ($BC \cap d$) results from a compromise between the simultaneous WF solution (C) and the successive WF solution (B). A lower bound for the BC curve can be found by considering the allocation $\boldsymbol{P}_2(x) = x\boldsymbol{P}_2^B + (1 - x)\boldsymbol{P}_2^C$ for user 2, and the SUWF allocation $\boldsymbol{P}_1(x)$ for user 1, with noise including the effect of $\boldsymbol{P}_2(x)$. The rate pairs obtained with this allocation policy are given by the dotted line BC. For some x in the range $[0, 1]$, balanced rates are obtained that are very close to the maximum balanced rates. A comparison with the rates obtained by algorithm 1 confirmed that the proposed solution is a tight lower bound for the optimal solution.

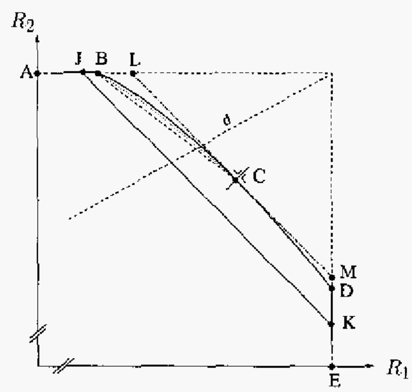

Fig. 2. Two-user capacity region and IMWF algorithm

Simple lower and upper bounds $g_1 < g < g_2$ on the multiuser coefficient g in (8) can be obtained as follows. The ALCME pentagon is obviously an upper bound for the convex capacity region. The corresponding balanced rate pair ($LC \cap d$) is $(g_2 R_1^1, g_2 R_2^1)$. The AJKE pentagon is the capacity region obtained when the SUWF algorithm is applied separately for every user. This pentagon appears to be a lower bound for the true capacity region. The corresponding balanced rate pair ($JK \cap d$) is $(g_1 R_1^1, g_1 R_2^1)$.

Generalizing these ideas to multiuser channels with an arbitrary number of users K is not easy as the optimal decoding order ($K!$ different combinations) is a priori unknown. The next algorithm is based on the idea that a higher priority should be given to the users to whom the IMWF algorithm is unfair.

Algorithm 2 *Maximum balanced rates with individual power constraints (approximate solution)*

- *Apply the SUWF and IMWF algorithms on the set* $\mathcal{S}_K = \{1, \cdots, K\}$. *Compute the corresponding power allocations, denoted by* $\{\boldsymbol{P}_k^1\}$ *and* $\left\{\boldsymbol{P}_k^{\mathcal{S}_K}\right\}$, *respectively. Compute the associated user rates, denoted by* $\{R_k^1\}$ *and* $\left\{R_k^{\mathcal{S}_K}\right\}$, *respectively.*
- *Compute initial bounds* g_1 *and* g_2 *as follows :*

$$g_1 = \frac{\sum_n \mathcal{C}\left(\sum_k P_{kn}^1 h_{kn}^2 / \sigma_n^2\right)}{\sum_k R_k^1} \quad (10)$$

$$g_2 = \frac{\sum_k R_k^{\mathcal{S}_K}}{\sum_k R_k^1} \quad (11)$$

with $\mathcal{C}(x) \triangleq b\log(1+x)$.

- *Set* $k_K = \arg\min_k R_k^{\mathcal{S}_K}/R_k^1$ *: the highest priority is given to the user whose normalized rate would be the lowest if the same priority was given to all users.*
- *While the balanced rate constraints are not satisfied :*
  - *Set* $i = K$ *and* $g^* = \frac{1}{2}(g_1 + g_2)$.
  - *Set* $\boldsymbol{P}_{k_K}(x_K) = x_K \boldsymbol{P}_{k_K}^1 + (1 - x_K)\boldsymbol{P}_{k_K}^{\mathcal{S}_K}$. *Compute* x_K *such that :*

$$R_{k_K} = \sum_n \mathcal{C}\left(\frac{P_{k_K n}(x_K) h_{k_K n}^2}{\sigma_n^2}\right) = g^* R_{k_K}^1 \quad (12)$$

  x_K *can be found by a simple binary search in the interval* $[0, 1]$.
  - *For* $i = K - 1$ *down to* 2 *:*
    * *Set* $\mathcal{S}_i = \mathcal{S}_{i+1} \setminus \{k_{i+1}\}$
    * *Compute the equivalent noise* $\boldsymbol{I}_i$ *including the additive noise* $\boldsymbol{\sigma}^2$ *and the interference from higher-priority users :* $\boldsymbol{I}_i = \boldsymbol{\sigma}^2 + \sum_{j=i+1}^{K} \boldsymbol{P}_{k_j}^T \boldsymbol{h}_{k_j}^2$.
    * *Apply the IMWF algorithm on the set of remaining users* $\mathcal{S}_i$, *with an equivalent noise power* $\boldsymbol{I}_i$. *The obtained power allocations and the associated user rates are denoted* $\left\{\boldsymbol{P}_k^{\mathcal{S}_i}(\boldsymbol{I}_i)\right\}_{k\in\mathcal{S}_i}$ *and* $\left\{R_k^{\mathcal{S}_i}(\boldsymbol{I}_i)\right\}_{k\in\mathcal{S}_i}$.
    * *Set* $k_i = \arg\min_{k\in\mathcal{S}_i} R_k^{\mathcal{S}_i}(\boldsymbol{I}_i)/R_k^1$ *: the next higher priority is given to the user in* $\mathcal{S}_i$ *whose normalized rate would be the lowest if the same priority was given to all users in* $\mathcal{S}_i$.
    * *Apply the SUWF algorithm on user* k_i, *with an equivalent noise power* $\boldsymbol{I}_i$. *The obtained power allocation is* $\boldsymbol{P}_{k_i}^1(\boldsymbol{I}_i)$ *and the associated rate is* $R_{k_i}^1(\boldsymbol{I}_i)$.
    * *If* $R_{k_i}^{\mathcal{S}_i}(\boldsymbol{I}_i) > g^* R_{k_i}^1$, *set* $g_1 = g^*$, *return to* $i = K$.
    * *Else, if* $R_{k_i}^1(\boldsymbol{I}_i) < g^* R_{k_i}^1$, *set* $g_2 = g^*$, *return to* $i = K$.
    * *Else, set* $\boldsymbol{P}_{k_i}(x_i) = x_i \boldsymbol{P}_{k_i}^1(\boldsymbol{I}_i) + (1 - x_i)\boldsymbol{P}_{k_i}^{\mathcal{S}_i}(\boldsymbol{I}_i)$. *Compute* x_i *such that :*

$$R_{k_i} = \sum_n \mathcal{C}\left(\frac{P_{k_i n}(x_i) h_{k_i n}^2}{I_{in}}\right) = g^* R_{k_i}^1 \quad (13)$$

    x_i *can be found by a simple binary search in the interval* $[0, 1]$.
  - *Set* $k_1 = \mathcal{S}_2 \setminus \{k_2\}$.
  - *Compute the equivalent noise* $\boldsymbol{I}_1 = \boldsymbol{\sigma}^2 + \sum_{j=2}^{K} \boldsymbol{P}_{k_j}^T \boldsymbol{h}_{k_j}^2$
  - *Compute* $\boldsymbol{P}_{k_1}$ *by applying the SUWF algorithm on user* k_1, *with an equivalent noise power* $\boldsymbol{I}_1$. *The associated rate is* R_{k_1}.
  - *If* $R_{k_1} > g^* R_{k_1}^1$ *: set* $g_1 = g^*$*; else set* $g_2 = g^*$

Table 1. Maximum balanced rates (in Mbits/s)

Scenario	User 5	User 10	User 15	User 20	g
SU	127.41	59.85	16.04	7.06	1
MU	77.94	36.61	9.81	4.32	0.61
TDMA	31.85	14.96	4.01	1.77	0.25

5. RESULTS

The case of a regular-pattern wireline access network with matched terminations is considered. The basic network structure is given on the top of Figure 3. The example considered here includes $K = 20$ derivations and identical cable segments of length $d_k = d'_k = 15$ m. The bottom of Figure 3 gives the channel frequency responses $|H_k(\omega)|^2$ in the bandwidth $[0, B]$ with $B = 10$ MHz. Through the combined effect of cable losses and multiple reflections on the cable derivations, the channel gains for the remote users can go below -100 dB at some frequencies. The noise level is chosen as -120 dBm/Hz. The power budget for every user is $\overline{P}_k = 10$ mW. For simplicity, the results presented in this

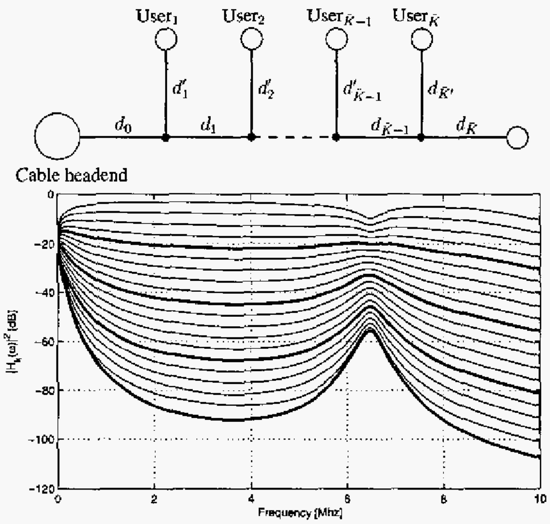

Fig. 3. Wireline multiaccess network and associated channel responses

section are restricted to the $K = 4$-user case, with users 5, 10, 15 and 20. The corresponding channel responses are given by the bold curves in Figure 3. Table 1 gives the single-user rates (first row), the maximum balanced rates (second row), and the TDMA balanced rates (last row, obtained by allocating time-slots of equal duration to every user) obtained in that scenario. By an appropriate power allocation, every user can achieve 61 % of its single user rate in a multiuser scenario, which is a significant improvement with respect to the trivial TDMA solution (providing 25 % of the single user rates). The top of Figure 4 gives the SUWF allocations, normalized with respect to a flat power allocation of 1 mW/MHz. The lower part of Figure 4 gives the optimal power allocation associated with the maximum balanced rates. As the remote users tend to concentrate their power $\overline{P}_k$ in a small fraction of the available bandwidth, a logarithmic scale was chosen for the vertical axis. This power allocation was obtained by algorithm 1. The suboptimal power allocation obtained by algorithm 2 is given in the third graph. It appears as a compromise between the SUWF power allocation (first graph) and the IMWF allocation (second graph). The rate loss associated with the suboptimal power allocation is negligible (less than 0.1 %).

6. CONCLUSIONS

This paper extended previous results on the balanced capacity of wireline multiaccess channels by introducing individual power constraints in the uplink. The computation of the balanced capacity was shown to require the resolution of two sets of nonlinear equations. Finally, an alternative algorithm was shown to provide a close-to-optimum solution.

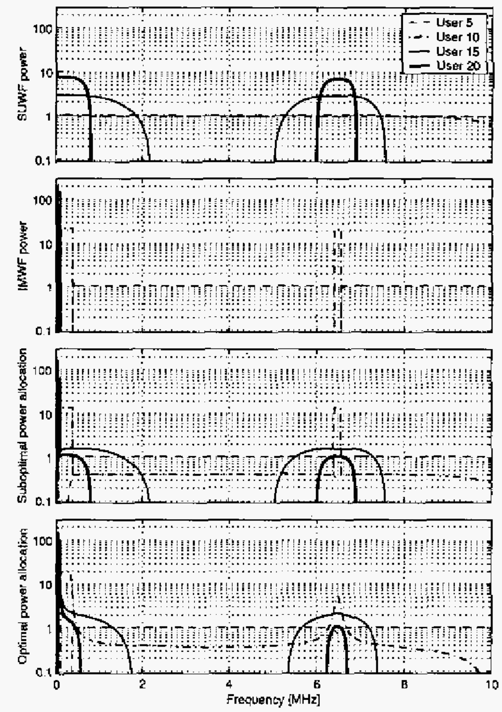

Fig. 4. Normalized power allocations

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