BLIND CALIBRATION OF SPARSE ARRAYS FOR DOA ESTIMATION WITH ANALOG AND ONE-BIT MEASUREMENTS

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ABSTRACT

In this paper, the focus is on the gain and phase calibration of sparse sensor arrays to localize more sources than the number of physical sensors. The proposed technique is a blind calibration method as it does not require any calibrator sources. Joint estimation of the gain errors, phase errors, and source directions is a complicated non-convex optimization problem, which is transformed into a convex optimization problem by exploiting the underlying algebraic structure. It is shown that the developed solver is suitable for analog as well as one-bit measurements. Numerical experiments based on sparse rulers are provided to illustrate the developed theory.

1. INTRODUCTION

To reduce the sensing and data processing costs, sparse sensing methods [1] are gaining attention. For direction-of-arrival (DOA) estimation in particular, by smartly placing sensor elements, one can resolve as many as $\mathcal{O}(K^2)$ sources using K sensors. Examples of such sensor placements include sparse rulers [2], minimum redundancy arrays (MRAs) [3], and co-prime arrays [4], to name a few.

In recent times, the use of one-bit quantized data has been gaining a lot attention in massive MIMO systems [5, 6]. Also in array processing, one-bit quantizers have been used in DOA estimation with conventional [7, 9] as well as sparse arrays [8].

In practice, each sensor in the array has a different gain and phase response. The gain and phase mismatch might be due to inherent uncertainties in the transducers (i.e., due to the manufacturing) as well as in the receiver electronics (i.e., anti-aliasing filters, amplifiers and analog-to-digital converters). With the gain and phase errors, the DOA estimates are significantly deteriorated and the array should be calibrated to correct such errors.

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Existing calibration methods for non-sparse arrays either exploit the Toeplitz structure of the covariance matrix related to the underlying linear array [10, 11], or iteratively find DOAs and calibrate, in an alternating manner for irregular arrays [12].

The main goal of this paper is to blindly calibrate the sparse arrays for DOA estimation with *analog* and *one-bit quantized* measurements. Blind calibration of sparse arrays for DOA estimation with analog measurements has been studied in [13], where the method proposed in [10] is used for estimating the gain errors. Instead of estimating the phase errors, they are modeled as a perturbation of the array manifold, and the DOAs are estimated based on the sparse total least squares (STLS), which is solved using an alternating minimization algorithm leading to a suboptimal solution. To alleviate these issues, we exploit the algebraic structure in the signal model and develop a *one-step* convex solver for the rank-constrained bilinear problem at hand. Unlike [13], for the infinite data records case (i.e., with no finite sample noise), we show via simulations that the proposed convex solver leads to the true solution.

2. PROBLEM FORMULATION

Consider a scenario with D uncorrelated far-field narrowband sources located at angles $\boldsymbol{\theta} = [\theta_1, \theta_2, \ldots, \theta_D]^T \in \mathbb{R}^D$. To compute these angles, we use an *irregularly-spaced sparse* linear array consisting of K sensors. The measurement data at time index t can be modeled as

$$\begin{aligned}\mathbf{x}(t) &= [x_1(t), x_2(t), \ldots, x_K(t)]^T \\ &= \operatorname{diag}\{\mathbf{g}\}\,[\mathbf{A}(\boldsymbol{\theta})\,\mathbf{s}(t) + \mathbf{n}(t)] \in \mathbb{C}^K, \qquad (1)\end{aligned}$$

where $\mathbf{g} = \boldsymbol{\psi} \odot \boldsymbol{\phi} = [\psi_1 e^{j\phi_1}, \psi_2 e^{j\phi_2}, \ldots, \psi_K e^{j\phi_K}]^T \in \mathbb{C}^K$ ($\odot$ is the element-wise product) is the vector that collects sensor uncertainties with ψ_i and ϕ_i being the gain and phase error of the ith sensor, respectively. The source signals are stacked in the length-D vector $\mathbf{s}(t)$ and the noise in the length-K vector $\mathbf{n}(t)$. The dth column of the array manifold, $\mathbf{A}(\boldsymbol{\theta}) =$

$[\mathbf{a}(\theta_1), \ldots, \mathbf{a}(\theta_D)] \in \mathbb{C}^{K \times D}$, denotes the response of the array towards the dth source and is given by

$$\mathbf{a}(\theta_d) = \left[e^{j\frac{2\pi r}{\lambda}\cos(\theta_d)p_1}, \ldots, e^{j\frac{2\pi r}{\lambda}\cos(\theta_d)p_K}\right]^T,$$

where $\{p_1 = 0, p_2, \cdots, p_K\}$ are the known sensor positions, λ is the wavelength of the source signals and r is the smallest inter-sensor spacing expressed in λ.

Let us assume that $\mathbf{s}(t)$ and $\mathbf{n}(t)$ are mutually uncorrelated and have covariance matrices $\mathbb{E}\left\{\mathbf{s}(t)\mathbf{s}^H(t)\right\} = \text{diag}(\mathbf{r}_s)$ and $\mathbb{E}\left\{\mathbf{n}(t)\mathbf{n}^H(t)\right\} = \sigma_\text{n}\mathbf{I}$. We can then express the covariance matrix of $\mathbf{x}(t)$ as

$$\begin{aligned}\mathbf{R}_x &= \mathbb{E}\left\{\mathbf{x}(t)\mathbf{x}^H(t)\right\} \in \mathbb{C}^{K \times K}, \qquad (2)\\ &= \text{diag}\{\mathbf{g}\}\left[\mathbf{A}(\boldsymbol{\theta})\text{diag}(\mathbf{r}_s)\mathbf{A}^H(\boldsymbol{\theta}) + \sigma_\text{n}\mathbf{I}\right]\text{diag}\{\bar{\mathbf{g}}\}\end{aligned}$$

where $\bar{(\cdot)}$ denotes complex conjugation. By vectorizing $\mathbf{R}_x$, we get

$$\mathbf{r}_x = \text{diag}\{\bar{\mathbf{g}} \otimes \mathbf{g}\}\left[\mathbf{A}_\text{d}(\boldsymbol{\theta})\mathbf{r}_s + \sigma_\text{n}\mathbf{e}\right], \qquad (3)$$

where $\mathbf{A}_\text{d} = \bar{\mathbf{A}}(\boldsymbol{\theta}) \circ \mathbf{A}(\boldsymbol{\theta}) \in \mathbb{C}^{K^2 \times D}$ is the array manifold of the so-called *difference coarray* (hence the subscript "d"), $\circ$ is the Khatri-Rao product, and $\mathbf{e}$ is the vectorized identity matrix. In this work, we will restrict ourselves to irregularly-spaced sparse arrays such as MRAs [3], sparse rulers [2, 14], or coprime arrays [4] that allow identification of more sources than the number of physical sensors.

In practice, the sensor data are not analog as it is acquired through analog-to-digital converters, hence are quantized. One-bit quantizers are the most simple (in terms of implementation and power consumption) quantizers, which measure the sign of the real and imaginary parts using a comparator. For the analog measurements, $\mathbf{x}(t)$ described in (1), quantized one-bit measurements are denoted as [5, 6]

$$\begin{aligned}\mathbf{y}(t) &= [y_1(t), y_2(t), \ldots, y_K(t)]^T\\ &= [\mathcal{Q}\{x_1(t)\}, \mathcal{Q}\{x_2(t)\}, \ldots, \mathcal{Q}\{x_K(t)\}]^T, \qquad (4)\end{aligned}$$

where $\mathcal{Q}\{x_k(t)\} = \frac{1}{\sqrt{2}}\text{sgn}\{\text{Re}(x_k(t))\} + \frac{j}{\sqrt{2}}\text{sgn}\{\text{Im}\{x_k(t)\}\}$ with $\text{sgn}\{x\} = 1$ for $x \geq 0$ and $\text{sgn}\{x\} = -1$ otherwise. The covariance matrix of $\mathbf{y}(t)$ is denoted as $\mathbf{R}_y = \mathbb{E}\left\{\mathbf{y}(t)\mathbf{y}^H(t)\right\}$.

In this paper, we provide algorithms to jointly estimate K complex calibration parameters, $\mathbf{g}$, and D angles, $\boldsymbol{\theta}$, given (i) the covariance matrix related to the analog measurements $\mathbf{R}_x$ or (ii) the covariance matrix related to the one-bit quantized measurements $\mathbf{R}_y$. We will be particularly interested in the case where $D \gg K$.

3. JOINT CALIBRATION AND DOA ESTIMATION

3.1. Analog measurements

In this section, we will consider an approach where both the calibration errors (i.e.,gain and phase errors) and the source DOAs will be estimated jointly from (3).

Multiplying both sides of (3) with the diagonal calibration matrix $\text{diag}\{\bar{\mathbf{b}} \otimes \mathbf{b}\} = \text{diag}^{-1}\{\bar{\mathbf{g}} \otimes \mathbf{g}\}$, we have

$$\text{diag}\{\bar{\mathbf{b}} \otimes \mathbf{b}\}\mathbf{r}_x = \text{diag}\{\mathbf{r}_x\}(\bar{\mathbf{b}} \otimes \mathbf{b}) = \mathbf{A}_\text{d}(\boldsymbol{\theta})\mathbf{r}_s + \sigma_\text{n}\mathbf{e}, \quad (5)$$

where $\mathbf{b} = [b_1, b_2, \ldots, b_K]^T \in \mathbb{C}^K$ contains as its entries the element-wise inverse of $\mathbf{g}$.

Assuming that the directions are from a uniform grid of N points, with $N \gg D$, i.e., we assume that $\theta_d \in \left\{0, \frac{\pi}{N} \cdots, \frac{\pi(N-1)}{N}\right\}$, for $d = 1, 2, \ldots, D$, we can approximate (5) as

$$\text{diag}\{\mathbf{r}_x\}(\bar{\mathbf{b}} \otimes \mathbf{b}) = \mathbf{A}_\text{D}\,\boldsymbol{\sigma}_\text{s} + \sigma_\text{n}\mathbf{e}. \qquad (6)$$

Here, $\mathbf{A}_\text{D}$ is a $K^2 \times N$ dictionary matrix that consists of column vectors of the form $\bar{\mathbf{a}}(\bar{\theta}_n) \otimes \mathbf{a}(\bar{\theta}_n)$, with $\bar{\theta}_n$ being the nth point of the uniform direction grid, i.e., $\bar{\theta}_n = \frac{\pi n}{N}$, $n = 0, 1, \ldots, N-1$, and $\boldsymbol{\sigma}_\text{s}$ is a length-N vector containing the source powers of the corresponding discretized directions. It should be noted that finding the columns of $\mathbf{A}_\text{D}$ that correspond to the non-zero elements of $\boldsymbol{\sigma}_s$ corresponds to finding the DOAs. We can now write (6) equivalently as

$$\begin{bmatrix}\text{diag}\{\mathbf{r}_x\} & -\mathbf{A}_\text{D} & -\mathbf{e}\end{bmatrix}\begin{bmatrix}\bar{\mathbf{b}} \otimes \mathbf{b}\\ \boldsymbol{\sigma}_\text{s}\\ \sigma_\text{n}\end{bmatrix} = 0 \Leftrightarrow \mathbf{G}\boldsymbol{\alpha} = \mathbf{0}. \quad (7)$$

This is an under-determined system of equations with K^2 equations in $(K^2 + N + 1)$ unknowns in $\boldsymbol{\alpha}$.

We next exploit the structure in $\boldsymbol{\alpha}$ to solve the above system. Firstly, we have the rank-1 Kronecker structure $\bar{\mathbf{b}} \otimes \mathbf{b} = \text{vec}(\mathbf{B})$ with $\mathbf{B} = \mathbf{b}\mathbf{b}^H \in \mathbb{C}^{K \times K}$. Secondly, since we know that there are only D sources, we have $||\boldsymbol{\sigma}_\text{s}||_0 = D$, where $\|\cdot\|_0$ is the ℓ_0-norm that counts the number of non-zero entries of its argument. Finally, $\boldsymbol{\sigma}_\text{s}$ and σ_n are positive.

From (3), it can be seen that $\bar{\mathbf{g}} \otimes \mathbf{g}$ and $\mathbf{r}_\text{s}$ share a common scalar factor and due to the Kronecker structure of $\bar{\mathbf{g}} \otimes \mathbf{g}$ there is a phase ambiguity. To resolve these ambiguities, we require two reference sensors. This observation is consistent with the discussion in [10]. Without loss of generality, we choose $g_i = b_i = 1$ for $i = 1, 2$.

Taking into account all the aforementioned constraints, we can now formally pose the joint calibration and DOA estimator as the solution to

$$\begin{aligned}\underset{\boldsymbol{\alpha}, \mathbf{b}, \mathbf{B}}{\text{minimize}} \quad & \|\mathbf{G}\boldsymbol{\alpha}\|_2^2\\ \text{subject to} \quad & \boldsymbol{\alpha} = [\text{vec}(\mathbf{B})^T, \boldsymbol{\sigma}_\text{s}^T, \sigma_\text{n}]^T\\ & ||\boldsymbol{\sigma}_\text{s}||_0 = D, \boldsymbol{\sigma}_\text{s} \succeq \mathbf{0}, \sigma_\text{n} \geq 0\\ & \mathbf{B} = \mathbf{b}\mathbf{b}^H, b_1 = b_2 = 1.\end{aligned} \qquad (8)$$

This is a non-convex optimization problem due to the ℓ_0-norm cardinality constraint and rank-one quadratic equality constraint on $\mathbf{B}$. By replacing the ℓ_0-norm with its convex approximation $\mathbf{1}^T\boldsymbol{\sigma}_\text{s}$ (recall that $\boldsymbol{\sigma}_\text{s}$ is positive) and replacing the quadratic equality constraint with the inequality constraint $\mathbf{B} \succeq \mathbf{b}\mathbf{b}^H$ whose Schur complement is $\begin{bmatrix} \mathbf{B} & \mathbf{b} \\ \mathbf{b}^H & 1 \end{bmatrix} \succeq 0$, we get the following convex optimization problem

$$\begin{aligned} \underset{\boldsymbol{\alpha},\mathbf{b},\mathbf{B}}{\text{minimize}} \quad & \|\mathbf{G}\boldsymbol{\alpha}\|_2^2 \\ \text{subject to} \quad & \boldsymbol{\alpha} = [\text{vec}(\mathbf{B})^T, \boldsymbol{\sigma}_\text{s}^T, \sigma_\text{n}]^T \\ & \mathbf{1}^T\boldsymbol{\sigma}_\text{s} \leq D, \boldsymbol{\sigma}_\text{s} \succeq \mathbf{0}, \sigma_\text{n} \geq 0 \\ & \begin{bmatrix} \mathbf{B} & \mathbf{b} \\ \mathbf{b}^H & 1 \end{bmatrix} \succeq 0, b_1 = b_2 = 1. \end{aligned} \tag{9}$$

This is a semidefinite programming problem that can be solved with any of the off-the-shelf solvers. It should be noted that the resolution of DOA estimates from the above solution is restricted by the chosen grid and may suffer from grid mismatch issues if the true directions are not in the predefined grid. To avoid such issues, grid-free DOA estimation methods such as spatial smoothing MUSIC (SS MUSIC) [15, 16] can be used after applying the calibration estimates of $\mathbf{b}$ obtained from (9).

3.2. One-bit measurements

The *arcsine rule* [17,18] for the complex Gaussian vectors relates the covariance matrices of $\mathbf{x}(t)$ and its one-bit quantized version $\mathbf{y}(t)$ as [19]:

$$\mathbf{R}_y = \frac{2}{\pi}\text{arcsin}\left(\mathbf{Q}^{-1/2}\mathbf{R}_x\mathbf{Q}^{-1/2}\right) \tag{10}$$

where $\mathbf{Q} = P\,\text{diag}(\boldsymbol{\psi})^2$ is a diagonal matrix with $[\mathbf{Q}]_{i,i} = [\mathbf{R}_x]_{i,i} = \psi_i^2\left(\sum_{d=1}^{D}[\mathbf{r}_s]_d + \sigma_\text{n}\right) = \psi_i^2 P$. We evaluate $\text{arcsin}(\cdot)$ of a matrix element-wise and use the notation $\text{arcsin}(a) = \text{arcsin}(\text{Re}\{a\}) + j\text{arcsin}(\text{Im}\{a\})$. The above relationship enables DOA estimation with one-bit quantized measurements [7, 19–21].

The matrix $\mathbf{Q}^{-1/2}\mathbf{R}_x\mathbf{Q}^{-1/2}$ in (10) simplifies to

$$P^{-1}\text{diag}\{\boldsymbol{\phi}\}\left(\mathbf{A}(\boldsymbol{\theta})\,\text{diag}\{\mathbf{r}_\text{s}\}\mathbf{A}^H(\boldsymbol{\theta}) + \sigma_\text{n}\mathbf{I}\right)\text{diag}\{\boldsymbol{\phi}^H\},$$

where $\boldsymbol{\phi} = [e^{j\phi_1}, \ldots, e^{j\phi_K}]^T$ contains the phase errors. This essentially means that with one-bit quantization, the gain errors drop out and we are required to estimate only the phase errors and the source directions. Using the above simplification and vectorizing (10), we arrive at

$$\tilde{\mathbf{r}}_y = \sin\left(\frac{\pi}{2}\mathbf{r}_y\right) = P^{-1}\,\text{diag}\{\bar{\boldsymbol{\phi}} \otimes \boldsymbol{\phi}\}\left[\mathbf{A}_\text{d}(\boldsymbol{\theta})\mathbf{r}_s + \sigma_\text{n}\mathbf{e}\right],$$

which can be further approximated using the dictionary matrix $\mathbf{A}_\text{D}$ as in (6) to

$$\text{diag}\{\tilde{\mathbf{r}}_y\}(\boldsymbol{\phi} \otimes \bar{\boldsymbol{\phi}}) = \mathbf{A}_\text{D}\,\tilde{\boldsymbol{\sigma}}_\text{s} + \tilde{\sigma}_\text{n}\mathbf{e},$$

or equivalently to

$$\begin{bmatrix} \text{diag}(\tilde{\mathbf{r}}_y) & -\mathbf{A}_\text{D} & -\mathbf{e} \end{bmatrix} \begin{bmatrix} \text{vec}(\boldsymbol{\Phi}) \\ \tilde{\boldsymbol{\sigma}}_\text{s} \\ \tilde{\sigma}_\text{n} \end{bmatrix} = 0 \Leftrightarrow \tilde{\mathbf{G}}\tilde{\boldsymbol{\alpha}} = \mathbf{0}. \tag{11}$$

Here, $\tilde{\boldsymbol{\sigma}}_\text{s} = P^{-1}\boldsymbol{\sigma}_\text{s}$, $\tilde{\sigma}_\text{n} = P^{-1}\sigma_\text{n}$, and $\text{vec}(\boldsymbol{\Phi}) = \boldsymbol{\phi} \otimes \bar{\boldsymbol{\phi}}$ with $\boldsymbol{\Phi} = \bar{\boldsymbol{\phi}}\boldsymbol{\phi}^T$. Letting $\boldsymbol{\sigma} = [\tilde{\boldsymbol{\sigma}}_\text{s}^T, \tilde{\sigma}_\text{n}]^T$, it is easy to see that $\mathbf{1}^T\boldsymbol{\sigma} = 1$. Based on (11), the convex optimization problem (9) for one-bit measurement simplifies to

$$\begin{aligned} \underset{\tilde{\boldsymbol{\alpha}},\boldsymbol{\phi},\boldsymbol{\Phi}}{\text{minimize}} \quad & \|\tilde{\mathbf{G}}\tilde{\boldsymbol{\alpha}}\|_2^2 \\ \text{subject to} \quad & \tilde{\boldsymbol{\alpha}} = [\text{vec}(\boldsymbol{\Phi})^T, \tilde{\boldsymbol{\sigma}}^T]^T \\ & \mathbf{1}^T\boldsymbol{\sigma} \leq 1, \boldsymbol{\sigma} \succeq \mathbf{0} \\ & \begin{bmatrix} \boldsymbol{\Phi} & \bar{\boldsymbol{\phi}} \\ \boldsymbol{\phi}^T & 1 \end{bmatrix} \succeq 0, [\boldsymbol{\phi}]_1 = [\boldsymbol{\phi}]_2 = 1, \end{aligned} \tag{12}$$

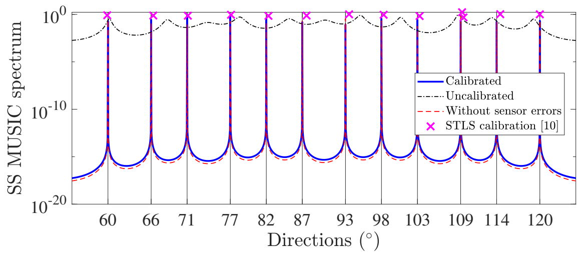
where the rank-one relaxation for $\boldsymbol{\Phi} = \bar{\boldsymbol{\phi}}\boldsymbol{\phi}^T$ is derived similar to (9).

4. NUMERICAL EXPERIMENTS

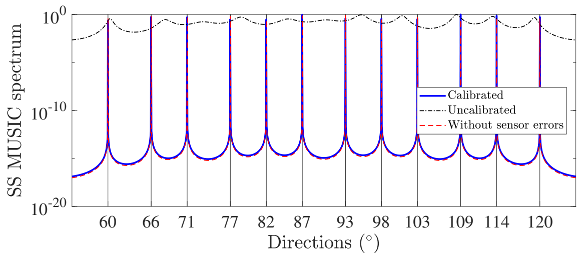
In this section, we present numerical simulations to illustrate the performance of the source DOA estimation based on the proposed solver in (9) for the analog data and in (12) for the one-bit quantized data. We consider a scenario with eight sensors, i.e., $K = 8$ arranged in a linear array configuration with $r = 0.5\lambda$ and $\{0, 1, 2, 3, 4, 10, 15, 20\}$ being the sensor positions. This forms a length-21 sparse ruler and has a hole-free-uniform difference co-array. Twelve unit power sources, i.e., $D = 12 > K$, whose DOAs are chosen uniformly in the cosine space within the sector between $60°$ and $120°$ are considered with signal-to-noise (SNR) being 10 dB for analog and one-bit quantized data. We calibrate with respect to the first two reference sensors in the array. The nominal gain and phase for the reference sensors are considered as 1 and $0°$, respectively. The gain errors, $\boldsymbol{\psi}$, and phase errors, $\boldsymbol{\phi}$, are picked from a realization of a uniform distribution over the interval of $[-2,\ 2]$ dB and $[-40°,\ 40°]$, respectively.

Once the calibration errors and grid-based DOA estimates are obtained by solving (9) or (12), the array is calibrated and the continuous (off-the-grid) DOAs may be obtained using SS MUSIC. In Fig. 1, the SS MUSIC spectra based on the analog and one-bit quantized data are presented. It is evident that the DOA estimates of an uncalibrated array are not useful with many unresolved sources. However, after calibrating the analog and one-bit quantized data, we see that all the sources are perfectly resolved with improved angular resolution and comparable to the ideal scenario without any sensor errors. In particular, for the case with infinite data records, we see that in Fig. 1(a) and Fig. 1(b) we in fact obtain the true solution for both the analog and one bit measurements suggesting the exactness of the convex approximation in (9).

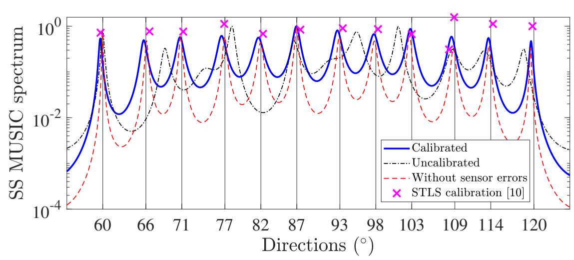
On the other hand, following the STLS calibration approach [13] for analog measurements leads to a sub optimal solution, where source DOAs are not perfectly recovered as well as more number of sources are identified, even for infinite data records as seen in Fig. 1(a). Further in Fig. 1(c), for finite data records with 1000 snapshots, the performance



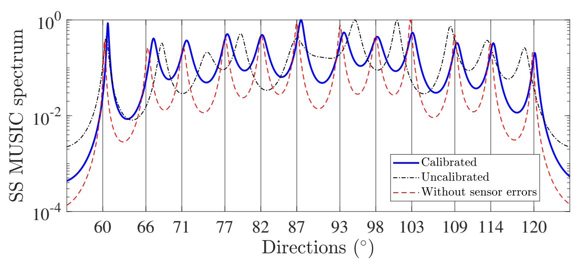
(a) Analog measurements with infinite snapshots.



(b) One-bit quantized measurements with infinite snapshots.



(c) Analog measurements with 1000 snapshots.

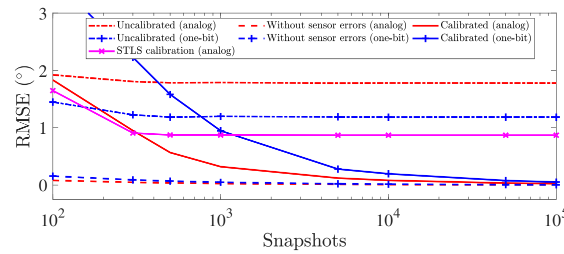


(d) One-bit quantized measurements with 1000 snapshots.

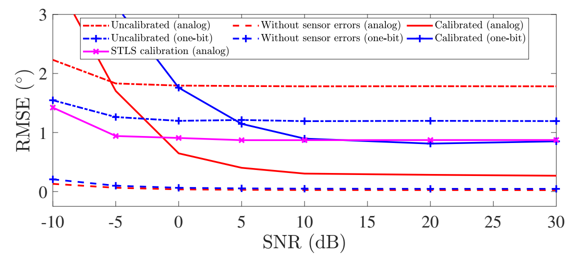
Fig. 1. SS MUSIC spectra for $K = 8$, $D = 12$ and SNR = 10 dB. The black grid lines denote the true source directions.

of our proposed method is better than the STLS calibration approach [13].

In Fig. 2, the root mean squared error (RMSE) of the DOA estimates obtained using SS MUSIC for different SNRs and for different number of data snapshots are shown. Here, we use $K = 8$ with sensors placed as before and $D = 3$ with $\boldsymbol{\theta} = [78°, 90°, 102°]$. The RMSE is computed for the source at 90° using 1000 independent Monte-Carlo trials, but with fixed gain and phase errors that were chosen as mentioned before. From Fig. 2(a), we can observe that as the number of snapshots increases for both analog and one-bit data, the RMSE of the DOA estimate after calibration approaches the



(a) SNR = 10 dB.



(b) Snapshots = 1000.

Fig. 2. RMSE of the DOA estimates for the source at 90° obtained from SS MUSIC with $K = 8$, $D = 3$ at $\boldsymbol{\theta} = [78°, 90°, 102°]$.

ideal scenario without any sensor errors.

Finally, Fig. 2(b) shows the RMSE for different SNRs. For analog data, similar to Fig. 2 (a), the RMSE of the DOA estimate after calibration decreases and approaches the scenario without any sensor errors. However for SNR above 15 dB, we see that the RMSE saturates as expected for the considered sparse arrays [22]. On the other hand for the one-bit quantized data, the reduction in the RMSE after calibration is not significant suggesting that the finite sample errors are dominant and require more snapshots for improved behavior. Furthermore, the RMSE for the STLS calibration saturates both with increase in the number of snapshots as well as increase in the SNR, as it converges to a sub-optimal solution.

5. CONCLUSIONS

In this paper, we proposed a blind calibration technique for sparse arrays based on analog as well as one-bit data. Based on the proposed approach, we showed that it is indeed possible to jointly estimate calibration errors and source directions using a *one-step* approach by exploiting the underlying algebraic structure and convex optimization techniques. It is shown that for both analog as well as one-bit data with infinite data records, we in fact obtain the optimal solution suggesting the exactness of the convex relaxations. Furthermore, through simulation, we show that even for finite data records we are able to recover all the source DOAs.

In the future, it is of considerable interest to improve the performance of the proposed approach especially for the finite data record scenario with a relatively low number of snapshots.

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