Introduction

Novel single-sensor seismic acquisition systems (possibly in combination with high-productivity source methods) allow for more complete sampling of the wavefield, more efficiently and affordably than traditional systems using field arrays. In this way, single-sensor systems enable the application of advanced wavefield-based processing and imaging methods.However, one persistent difficulty with land surveys – and single sensor surveys in particular – is that the acquisition grid is often irregular due to obstacles and surface conditions. As a result, in some applications, such as multiple elimination methods or imaging, regularization of data may be necessary. At the same time, filtering of steeply dipping events, for example, may be desirable. Another application of filtering irregular data is decimation. This is a data-reduction step often carried out before imaging, a process sometimes referred to as digital group forming (Özbek and Ferber, 2005). In both cases the filter is a low-pass (anti-alias) filter. The purpose of our work is to develop a robust filtering method that accounts for nonuniform sampling and that can output traces at regular intervals. We consider the method robust if different realizations of the receiver locations lead to small differences in the array output. The output should approximate the output one would get in the ideal case, i.e. if no receiver misplacements had occurred. In order to accomplish these goals, the proposed method takes advantage of the fact that new acquisition systems are equipped with GPS/RTK, and we therefore know the location of each receiver with high (but still limited) accuracy. One approach to filtering non-uniformly sampled data is to first reconstruct the data on the uniform (nominal) grid and perform filtering afterwards. Filtering can then be carried out as a simple discrete convolution. The reconstruction problem in general has been extensively studied in many engineering applications. For example in exploration geophysics, Duijndam et al. (1999) propose a method that can reconstruct spatially band-limited data using parametric inversion. Another approach is proposed by Özbek and Ferber (2005) as part of their digital group forming algorithm. They interpolate a predesigned filter to the actual receiver locations and each resulting filter coefficient is reweighted based on the sampling density around its location. Our proposed method has common elements with both of these two approaches. Similar to the method of Özbek and Ferber (2005), our method designs a different FIR filter per output location, based on a prototype filter to be approximated. Unlike it, however, the filter coefficients are calculated as the result of a least-squares problem. In common with band-limited reconstruction (Duijndam et al. (1999)), our method also assumes that the data is band-limited, but our aim is to design filters having limited number of taps that combine regularization and filtering. The limited number of taps means that each designed filter is applied only to a relatively small part of the data, covered by the physical length of the filter, in this way minimizing the contribution of data from potentially different (sub) surface conditions in the output trace. Although we only consider the one-dimensional low-pass filtering problem here, our method can be extended to allow for arbitrary multi-dimensional filters. We discuss both the implementation in the space domain and in the wave number domain.

Theory

For simplicity we only consider filtering along one spatial dimension, x. The data is represented by the function $d(t,x)$, which is sampled in time at instants $t_i = i\Delta t$ and in space at the N locations x_j, $j = 0,\dots,N-1$. The nominal locations of the receivers are defined on the grid $\bar{x}_n = n\Delta x$. The actual locations of the receiver are assumed to lie on the denser grid $\bar{\bar{x}}_m = m\delta x = m(\Delta x/M)$ with $M \geq 1$ and integer. This is not overly restrictive, since M can be large and the precise receiver locations are known with high, but nonetheless limited, accuracy. We also define the indicator function $s(\bar{\bar{x}}_m)$ takes the value $s(\bar{\bar{x}}_m) = 1$ when a receiver is present at $\bar{\bar{x}}_m$ and the value zero otherwise. Let $h(\bar{x}_i)$, $i = 0,1,\dots,L_f - 1$ be an FIR filter of length L_f. All the filter taps lie in the range $\mathscr{A} = [0,\ L_f\Delta x)$. The main idea is to design a new FIR filter $g_l(x)$ for each output location $\check{x}_l$, such that

$$\sum_{n=0}^{N-1} h(\check{x}_l - \bar{x}_n)d(t,\bar{x}_n) = \sum_{m=0}^{NM-1} g_l(\check{x}_l - \bar{\bar{x}}_m)s(\bar{\bar{x}}_m)d(t,\bar{\bar{x}}_m) \tag{1}$$

holds. In other words, for each output location, the output of $g_l(x)$ applied to the actual gathered data should ideally be identical to that of $h(x)$ applied to the data that would have been gathered if no misplacements had occurred. We require that $g_l(x) = 0$ for $x \notin \mathscr{A}$, so that $g_l(x)$ has the same spatial support as $h(x)$. Since $d(t,x)$ is assumed to be band-limited, $d(t,\bar{\bar{x}}_m)$ can be approximated with the aid of a discrete version of the sinc-interpolation kernel,

$$d(t,\bar{\bar{x}}_m) \approx \sum_{q=0}^{N-1} \mathrm{sincd}(N;\bar{\bar{x}}_m,\bar{x}_q) d(t,\bar{x}_q). \tag{2}$$

The definition of $\mathrm{sincd}(\cdot)$ can be found in Kontakis (2013). Substituting (2) in (1) we get

$$\sum_{n=0}^{N-1} \underbrace{h(\check{x}_l - \bar{x}_n)}_{h_{l\,n,1}} \underbrace{d(t,\bar{x}_n)}_{d_{n,1}} \approx \sum_{m=0}^{NM-1} \underbrace{g_l(\check{x}_l - \bar{\bar{x}}_m)}_{g_{l\,m,1}} \underbrace{s(\bar{\bar{x}}_m)}_{S_{m,m}} \sum_{q=0}^{N-1} \underbrace{\mathrm{sincd}(N;\bar{\bar{x}}_m,\bar{x}_q)}_{Q_{m,q}} d(t,\bar{x}_q)$$

In matrix-vector notation, this can be written as

$$\mathbf{h}_l^T \mathbf{d} \approx \mathbf{g}_l^T \mathbf{SQd} \tag{3}$$

where the elements of $\mathbf{h}_l$, $\mathbf{d}$, $\mathbf{g}_l$, the diagonal of $\mathbf{S}$ and $\mathbf{Q}$ are given by $h_{l\,n,1}$, $d_{n,1}$, $g_{l\,m,1}$, $S_{m,m}$ and $Q_{m,q}$ respectively. We would like (3) to hold irrespective of the (unknown) data $\mathbf{d}$. A sufficient condition for this to hold is

$$\mathbf{h}_l^T \approx \mathbf{g}_l^T \mathbf{SQ}$$

It is possible to find a suitable $\mathbf{g}_l$ by solving the least-squares problem

$$\min_{\mathbf{g}_l} \left\{ ||\mathbf{h}_l^T - \mathbf{g}_l^T \mathbf{SQ}||_2^2 \right\} \equiv \min_{\tilde{\mathbf{g}}_l} \left\{ ||\mathbf{h}_l^T - \tilde{\mathbf{g}}_l^T \tilde{\mathbf{Q}}||_2^2 \right\},$$

where $\tilde{\mathbf{g}}_l^T$ is formed by removing its elements corresponding to the zeros of $\mathbf{g}_l^T \mathbf{S}$. Similarly, $\tilde{\mathbf{Q}}$ is constructed after removing the rows of $\mathbf{Q}$ corresponding to the rows that would have been set to zero by the product $\mathbf{g}_L^T \mathbf{SQ}$. The problem has a closed-form solution given by $\tilde{\mathbf{g}}_l^T = \mathbf{h}_l^T \tilde{\mathbf{Q}}^T \left(\tilde{\mathbf{Q}}\tilde{\mathbf{Q}}^T\right)^{-1}$. If $\tilde{\mathbf{d}}$ is the irregularly sampled data, then our filtering method works as explained in Figure 1.

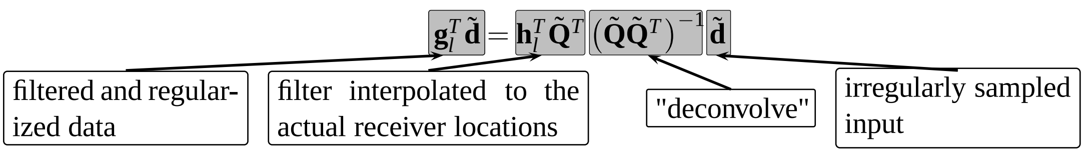

Figure 1 *Explanation of the different terms in our formulation*

A different approach is to approximate the ideal lowpass filter which is defined in the wavenumber domain. In this respect, the method to be outlined next can be viewed as a least-squares filter design method for designing a linear space-*varying* filter $\mathbf{G}'$, which is similar to traditional least-squares FIR filter design for space- or time- invariant filters. Each row of $\mathbf{G}'$ represents a filter with at most L_f taps. Let $\mathbf{F}$ be an N-point discrete Fourier transform matrix. Then the objective is to find $\mathbf{G}'$ such that

$$\mathbf{H}_w \mathbf{Fd} = (\mathbf{FG}'\mathbf{SQF}^H)\mathbf{Fd}, \tag{4}$$

where $\mathbf{H}_w$ is a diagonal matrix whose diagonal elements are given by sampling of the wavenumber response $H_w(k) = 1, k \in [-k_{\text{pass}}, +k_{\text{pass}}]$ and $H_w(k) = 0$ otherwise. k_{pass} is the largest normalized wavenumber in the passband of the filter. Similar to the space domain method, a sufficient condition for (4) to hold regardless of the spectrum is

$$\mathbf{H}_w \approx (\mathbf{FG}'\mathbf{SQF}^H),$$

which can be again stated as a least-squares problem:

$$\min_{\mathbf{G}'}\{||\mathbf{W}\odot(\mathbf{H}_w-(\mathbf{F}\mathbf{G}'\mathbf{S}\mathbf{Q}\mathbf{F}^H)||_F^2\}, \tag{5}$$

where $\odot$ denotes the Hadamard (elementwise) product and $||\cdot||_F$ the Frobenius norm. $\mathbf{W}$ can be used to assign different weights on the optimization of the passband and stopband region of the filter. A detailed solution of (5) is not provided due to space restrictions.

Results

In order to test the performance of our algorithms, synthetic data were created using a finite difference method. The nonuniform sampling is simulated by oversampling (at .5 m) and then using only a subset of the traces. A simple velocity model was used with three layers. The wavelet has a bandwidth of approximately 70Hz and was generated by a source at x=+500m. The receiver perturbations are chosen from a uniform distribution in the range $[0m,\ 4m]$. The rest of the parameters as well as the velocity model and the wavenumber response of the prototype filter are shown in Figure 2. Without perturbations, applying the filter in Figure 2 to the data gives the output in Figure 3. The output locations are $\check{x}_l = l\Delta x,\ l = 0,\dots,N$. In order to test the performance of the proposed method, we applied our method to 50 realizations of the perturbed receiver locations. We also implemented a one-spatial dimension version of the algorithm outlined in Özbek and Ferber (2005), that uses the trapezoidal rule instead of Delaunay triangulation. We will further refer to this method as geometry compensated filtering (GCF). The results are presented in Figure 4. The average outputs of the algorithms show that the location perturbations have a more detrimental effect at higher frequencies. The reason for this is that the spatial bandwidth is also larger at those frequencies, which leads to more energy leakage resulting from nonuniform sampling (Beutler and Leneman, 1968). Our methods display better accuracy than our implementation of GCF, which can be verified by the plots in the third row. This can be attributed to the fact that we attempt a deconvolution of the effects of nonuniform sampling, much in the same way as described in Duijndam et al. (1999).

Parameter	Value
Δx	5m
δx	0.5m
M	10
N	250
N_t	1501
Δt	1ms
L_f	7

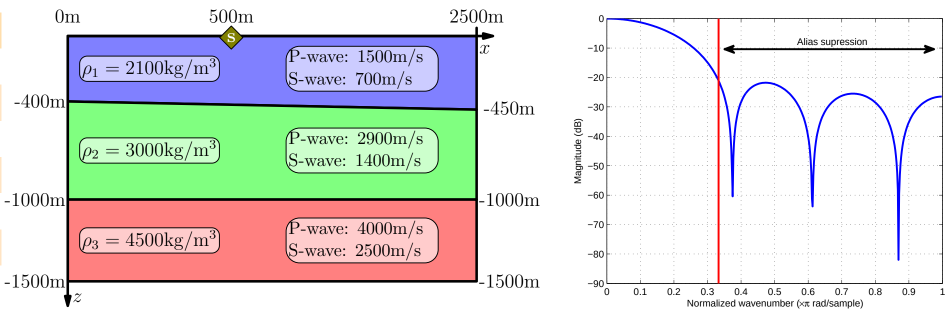

Figure 2 *From left to right: the simulation parameters, the velocity model and the prototype FIR filter.*

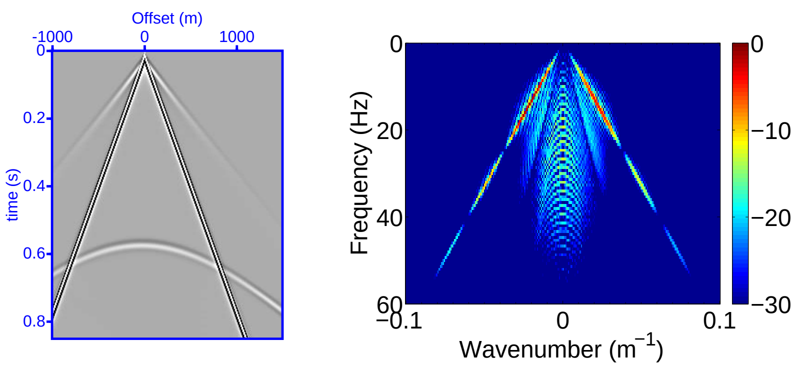

Figure 3 *a) Regularly sampled input data and b) output in the ideal case of uniformly sampled data.*

Additionally, we can see that our method gives a result that is more robust especially at higher frequencies, i.e. it has a lower standard deviation than GCF. Note that areas that correspond to the zeros of the prototype filter are not attenuated as much in our method. This is because the individually designed filters do not always have zeros at the exact same locations as the prototype filter. Since each output trace is the result of filtering data with a different filter the composite f-k spectrum does not necessarily inherit the regular zero spacing from the prototype filter. In our experience, the wavenumber domain method yields slightly worse results than the space domain method. This might be attributed to the weighting scheme we used. Optimizing the weights remains to be studied in future work.

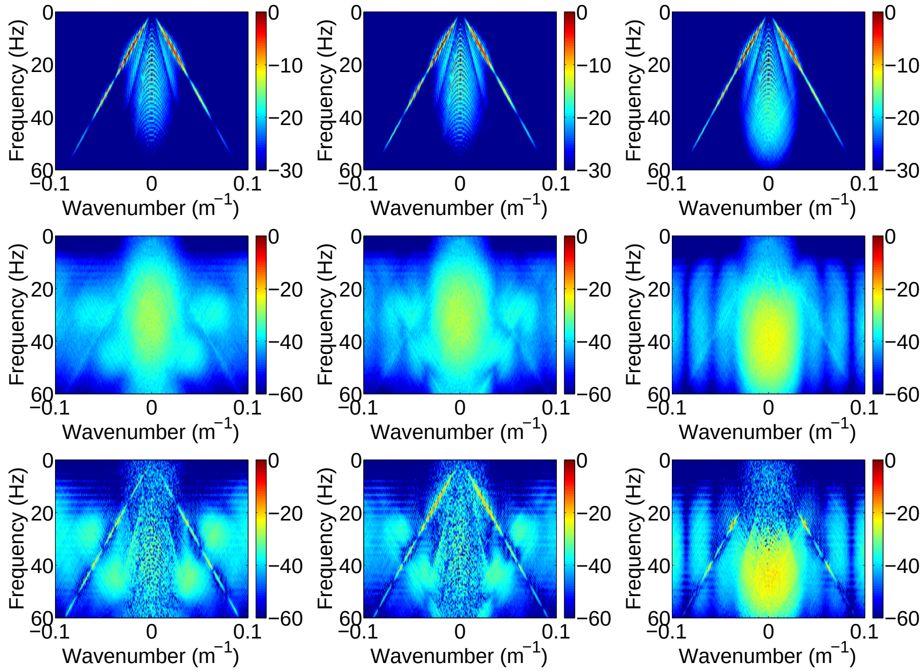

Figure 4 *From left to right column: space domain method, wavenumber domain method and the nonuniform filtering method proposed in Özbek and Ferber (2005). From top to bottom row: average, standard deviation and average difference from the ideal filtered output seen in Figure 3. These results are over 50 different realizations of the receiver locations.*

Conclusions

We formulated a method for jointly filtering and regularizing nonuniformly sampled seismic data as an optimization problem in the space and in the wavenumber domain. Our results show good filtering and regularization accuracy and a high degree of robustness when compared to another similar method. The increase in robustness, however, comes at the price of a higher computational cost. The wavenumber domain formulation of our method does not seem to yield significantly better results, but this may be improved by choosing different relative weights for the stop and pass band in the optimization scheme.

References

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