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Occupancy-based illumination control of LED lighting systems

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Light emitting diode (LED)-based systems are considered to be the future of lighting. We consider the problem of energy-efficient illumination control of such systems. Energy-efficient system design is based on two aspects: localised information on occupancy and optimisation of dimming levels of the LEDs. Specifically, we are interested in minimising the power consumption of an LED system, subject to providing uniform illumination at a pre-specified level around occupied zones, by determining the dimming levels of the LEDs. We show that this optimisation problem can be solved by linear programming and use the simplex algorithm to determine the dimming levels. The efficacy of our proposed system is evaluated in an office scenario by comparing it with a system that renders uniform illumination across the whole space.

List of symbols

A	Maximum illuminance per LED
A_0	Luminous flux
$\boldsymbol{b}$	Constraint vector of the linear program
C	Illuminance contrast
C_{th}	Maximum illuminance contrast in R_0
$\boldsymbol{d}$	Dimming vector
D	Number of dimming levels
E_i	Illuminance of the i-th LED
E_T	Total illuminance
$\boldsymbol{f}_u$	Vector of illuminance contribution per LED at u-th location in $\mathcal{U}$
$\boldsymbol{g}_v$	Vector of illuminance contribution per LED at v-th location in $\mathcal{V}$
h	Distance from ceiling to workspace plane
J	Number of occupants
l	Length of room

L	Illuminance level
$L_{\max}$	Target illuminance level in R_0
$L_{\min}$	Minimum target illuminance level outside R_0
m	Lambertian mode
$\boldsymbol{M}$	Coefficient matrix of the linear program
$\boldsymbol{n}$	Coefficient vector of the linear program
N_x	Number of LEDs along the length of the room
N_y	Number of LEDs across the width of the room
N_l	Number of evaluation points along the length of the room
N_w	Number of evaluation points across the width of the room
P_i	Power consumption of the i-th LED
P_{on}	Power consumption of LED while is on
P_{off}	Power consumption of LED while is off
P_T	Total power consumption of the lighting system
Q	Number of constraints
r_0	Radius of the region R_0

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R_0	Region surrounding the occupant
R_f	Feasible region for optimisation
s	Vector of slack variables of the linear program
$\mathcal{U}$	Discrete set of coordinate pairs in R_0 and in R_f
U	Number of coordinate pairs in $\mathcal{U}$
$\mathcal{V}$	Discrete set of coordinate pairs outside R_0 and in R_f
V	Number of coordinate pairs in $\mathcal{V}$
w	Width of room
x	x-coordinate
y	y-coordinate
Δl	Separation amongst evaluation points along the length of the room
Δw	Separation amongst evaluation points across the width of the room
Δ	Separation amongst LEDs on a uniformly spaced grid
Δx	Separation amongst LEDs along the length of the room
Δy	Separation amongst LEDs across the width of the room
Ω	Area of region R_0
$\Phi_{\frac{1}{2}}$	Semiangle of the light beam at half power

Notation: Given 2 coordinates (x_1, y_1) and (x_2, y_2), the distance between them, or the 2-norm of the difference between the coordinates is given by $||(x_2, y_2) - (x_1, y_1)||_2 = \sqrt{(x_2 - x_1)^2 + (y_2 - y_2)^2}$. For a real value x, its absolute value will be written as $|x|$ and its floor by $\lfloor x \rfloor$. For two positive numbers a and b, the remainder of division of a by b is given by the modulo operation, written as $a \bmod b$.

1. Introduction

Light emitting diodes (LEDs) are set to become the next generation source of energy efficient illumination. They offer longer life times, dynamic light effects and greater design flexibility. Flexibility in tuning LEDs in particular means that the design of LED-based

systems offers greater potential for energy savings.[1,2]

We consider the problem of illumination control of an LED-based lighting system. Illumination achieved by an LED system depends on the illumination radiation pattern and the dimming level of the individual LEDs. A Lambertian function[3,4] is commonly used to model the broad beam illumination pattern of an LED. The dimming level thus provides the degree of freedom to control illumination patterns realised by an LED system. The LED system is considered in a typical workplace setting of an office room with one or more occupants. We consider two instruments of energy-efficient system design in our framework. One aspect is that illumination is rendered at a higher level (at an illumination level required as per workspace norms) only in occupied regions. Occupancy may be determined by a presence detection sensor capable of determining localised occupancy information, that is the location of different occupants can be obtained. Alternatively, occupants may be equipped with a controller that conveys their location. The second aspect of energy-efficient design lies in the determination of dimming levels of the LEDs so as to minimise the total power consumption of the LED system. This optimisation is done under the constraint of achieving uniform illumination at a pre-specified level in occupied regions and a minimal illumination level elsewhere.

Different aspects of LED system design have been considered in past literature. The problem of illumination rendering has been treated by Yang *et al.*[5] and Moreno.[6] Linnartz *et al.*[7] presented the idea of modulating LED illumination pulses using code division multiplexing as a way to facilitate determining individual illumination contributions at a receiver. Solutions based on frequency division multiplexing as a means to determine (and control) individual LED dimming levels have been treated by Yang *et al.*[8,9]

A framework for the design of lighting systems based on daylight control and occupancy information was developed by Singhvi *et al*.[10] The approach used there considered the maximisation of occupant utility functions taking energy efficiency into consideration. Further, in solving for the illumination levels, the light sources were assumed to have narrow beams. As such, both the optimisation problem and the solution methodology considered there differ from our work.

The paper is structured as follows: in Section 2 we describe the LED lighting system and also present LED illumination models. The design of energy-efficient illumination control is formulated as an inequality constrained optimisation problem in Section 3. We analyse this optimisation problem as a linear program in Section 4. A simplex algorithm is employed to obtain the LED dimming levels. Under this illumination control algorithm, the performance of the LED system is evaluated in Section 5 using LUXEON LED models. Finally, conclusions are drawn in Section 6.

2. System description

We consider a lighting system with LEDs on a uniformly spaced grid in a room of length l and width w. Let N_x and N_y be the number of LEDs distributed along the length and width respectively of the room ceiling with separation

$$\Delta x = \frac{w}{N_x},$$
$$\Delta y = \frac{l}{N_y}.$$

For convenience, a coordinate system is assumed with the origin at the centre of the ceiling. The location of the i-th LED is given by the coordinate pair (x_i, y_i) where

$$x_i = \left(\alpha - \frac{N_x - 1}{2}\right)\Delta x,$$
$$\alpha = (i - 1) \bmod N_x$$
$$y_i = \left(\beta - \frac{N_y - 1}{2}\right)\Delta y,$$
$$\beta = \left\lfloor \frac{i - 1}{N_x} \right\rfloor$$
$$\text{for} \quad i = 1, 2, \ldots, N_x N_y.$$

All the measurements of illuminance are taken at a plane parallel to the ceiling located at a distance h, measured perpendicular from the LEDs' plane. This distance represents a normal height for a working place, for example a desk. As such, there are two planes (as depicted in Figure 1) – one in which the LEDs are placed and the other is the workspace plane. We will not introduce a z-coordinate to distinguish the two planes for clarity of exposition since the difference will be clear from the context.

We assume that the location of an occupant is determined either by a presence detection sensor or by a user-equipped controller. It is desired to achieve uniform illumination at a prescribed level surrounding
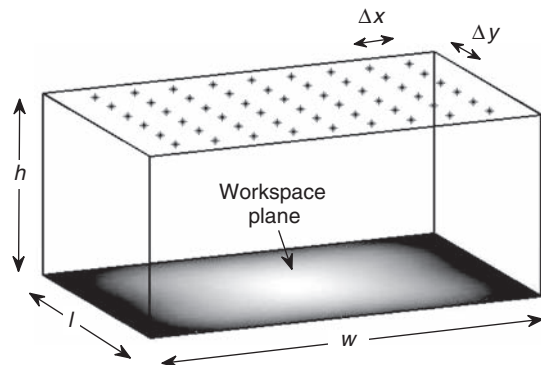

Figure 1 Illumination of an LED lighting system

the occupant locations. In unoccupied areas, it is desired to have a minimal illumination level. Both levels are chosen so as to meet required illumination norms. In practice, uniform illumination means that variations in the illumination level must be below a certain threshold. The distortion in illumination pattern at location (x, y) with respect to a target illuminance level L is characterised by the illuminance contrast, as given by Weber's law,[11]

$$C(E(x, y; h), L) = \frac{E(x, y; h) - L}{L} \quad (1)$$

where $E(x, y; h)$ is the illuminance at point (x, y) and distance h.

3. Problem formulation

We now mathematically formalise the illumination control problem.

Denote by d, the N_x $N_y \times 1$ dimming vector, given by

$$d = [d_1, \ldots, d_{N_x N_y}],$$

where $0 \leq d_i \leq 1$ is the dimming level of the i-th LED. $d_i = 0$ means that the LED is dimmed off while $d_i = 1$ represents that the LED is at its maximum illumination.

Given J known locations (x_j, y_j) of occupants, it is desired to have a uniform illumination level, $L_{\max}$, in regions around the occupant locations. Denote this whole region by R_o:

$$R_o = \{(x, y) : ||(x, y) - (x_j, y_j)||_2 \leq r_0,$$
$$j = 1, \ldots, J\} \quad (2)$$

and its area by Ω. The constant r_0 may be chosen as per workspace norms and occupant comfort. Thus at any point in R_o, we have the contrast between the total illumination and

$L_{\max}$ to be lower than a prescribed contrast C_{th}. Furthermore, the mean illumination level over R_0 is desired to be $L_{\max}$. Outside region R_o, it is desired that the illumination level be at least $L_{\min}$.

We seek to minimise the total power consumed by the lighting system under the illumination constraints in the occupied and unoccupied regions. Formally, we want to determine the optimum dimming vector $d^\star$ to solve

$$d^\star = \arg\min_{d} \sum_{i=1}^{N_x N_y} P_i(d_i) \quad \text{s.t.}$$

$$\begin{cases} |C(E_T(x, y, d; h), L_{\max})| \leq C_{th}, \\ \quad \forall (x, y) \in R_o \\ E_T(x, y, d; h) \geq L_{\min}, \\ \quad \forall (x, y) \notin R_o \\ \frac{1}{\Omega} \int_{(x,y) \in R_o} E_T(x, y, d; h) \partial x \partial y = L_{\max} \\ 0 \leq d_i \leq 1, \quad i = 1, \ldots, N_x N_y. \end{cases} \quad (3)$$

Here, $P_i(d_i)$ is the average power consumption of the i-th LED at dimming level d_i. $E_T(x, y, d; h)$ is the total illuminance at point (x, y) and distance h resulting when using dimming vector d.

Some comments are in order regarding the optimisation problem in (3). Note that in the region outside R_o, we only require an illumination level of at least $L_{\min}$, which is different from the requirement of uniform illumination of $L_{\max}$ inside R_o. This is due to the practical reason that it is not possible to achieve uniform illumination in this region owing to edge effects (e.g. on the boundaries outside R_o and near the walls). Further, we shall assume a feasible solution exists for problem (3). That is, the LED system is designed in the first place such that illumination control can be done as per (3).

3.1 Power consumption

The illumination intensity of an LED is typically controlled using pulse width modulation (PWM).[12] The dimming level d_i is in fact the duty cycle of the PWM waveform. Hence, the average power consumed by the i-th LED over one waveform cycle is

$$P_i(d_i) = d_i P_{\text{on}} + (1 - d_i)P_{\text{off}} \qquad (4)$$

where P_{on} and P_{off} are the power consumptions while the LED is on and off, respectively. In practice, $P_{\text{off}} = 0$. Hence

$$P_i(d_i) = d_i P_{\text{on}}.$$

Then, the total power, P_T, consumed by the lighting system is the summation of the average power of each LED:

$$P_T = \sum_{i=1}^{N_x N_y} d_i P_{\text{on}}. \qquad (5)$$

Thus, minimising the total power consumption is equivalent to minimising the sum of the dimming levels of the LEDs,

$$\arg\min_{\boldsymbol{d}} \sum_{i=1}^{N_x N_y} P_i(d_i) \equiv \arg\min_{\boldsymbol{d}} \sum_{i=1}^{N_x N_y} d_i. \qquad (6)$$

3.2 Illumination pattern model

A widely used model for the illumination pattern of an LED is the generalised Lambertian function.[3,4] The illuminance, in the workspace plane, at location (x, y) and a distance h for a single LED located at (x_i, y_i) is

$$E_i(x, y; h)$$
$$= A\left[1 + \frac{||(x, y) - (x_i, y_i)||_2^2}{h^2}\right]^{-\frac{m+3}{2}} \qquad (7)$$

with

$$A = \frac{(m + 1)A_0}{2\pi h^2}$$

where A_0 is the luminous flux of the light and m is the Lambertian mode ($m > 0$). This mode is related to the semiangle of the light beam at half power, $\Phi_{\frac{1}{2}}$, determined by

$$m = -\frac{\ln(2)}{\ln\left(\cos\left(\Phi_{\frac{1}{2}}\right)\right)}.$$

The overall illumination at position (x, y) is then equal to the combined contribution of every LED. Thus, with the i-th LED at dimming level d_i and the illumination pattern in (7), the total illuminance is written as

$$E_T(x, y, \boldsymbol{d}; h)$$

$$= \sum_{i=1}^{N_x N_y} d_i E_i(x, y; h)$$

$$= A \sum_{i=1}^{N_x N_y} d_i\left[1 + \frac{||(x, y) - (x_i, y_i)||_2^2}{h^2}\right]^{-\frac{m+3}{2}}. \quad (8)$$

Note that the above model neglects reflections of light occurring in the room. In practice, these contributions need to be accounted for in the overall illumination. This can be done by incorporating a reflection model for a given room[13] or by actually measuring light intensities using appropriate sensors.

Now, using (6) and (8), our original problem (3) can be rewritten as

$$\mathbf{d}^{\star} = \arg\min_{\mathbf{d}} \sum_{i=1}^{N_x N_y} d_i \quad \text{s.t.}$$

$$\begin{cases} |\sum_{i=1}^{N_x N_y} d_i E_i(x, y; h) - L_{\max}| \\ \quad \leq L_{\max} C_{th}; \quad \forall (x, y) \in R_o \\ \sum_{i=1}^{N_x N_y} d_i E_i(x, y; h) \geq L_{\min}, \\ \quad \forall (x, y) \notin R_o \\ \sum_{i=1}^{N_x N_y} d_i \left[\frac{1}{\Omega} \int_{(x,y) \in R_o} E_i(x, y; h) \partial x \partial y \right] \\ \quad = L_{\max} \\ 0 \leq d_i \leq 1, \quad i = 1, \dots, N_x N_y. \end{cases} \quad (9)$$

3.3 Illumination uniformity

The feasibility of obtaining a uniform illumination pattern depends on the beamwidth of the LEDs and the amount of overlap of their patterns, that is the separation amongst LEDs.

There is a trade-off between these two parameters. When the LEDs are on a uniform grid, with an illumination pattern as defined by (7), the maximum separation between two consecutive LEDs $(\Delta = \Delta x = \Delta y)$ that ensures a uniform illumination is given by the approximation[14]

$$\Delta = h \sqrt{\frac{1.2125}{m - 3.349}} \quad (10)$$

for $N_x > 4$ and $N_y > 4$ and $m > 30$. This gives us an upper threshold for the maximum separation amongst LEDs to ensure that a uniform illumination is feasible.

4. Algorithm for illumination control

Note that the objective function as well as the constraints of the optimisation problem in (9) are linear in $\{d_i\}$.

To write (9) in the standard form of a linear optimisation problem,[15] we first discretise the constraints. To do this, we divide the workspace plane into a uniform spaced grid with N_l and N_w number of points along the length and width of the room, respectively. The separation between points is given by

$$\Delta w = \frac{w}{N_w},$$

$$\Delta l = \frac{l}{N_l}.$$

The k-th location is given by the coordinate pair (x_k, y_k) where

$$x_k = \left(\gamma - \frac{N_w - 1}{2} \right) \Delta w,$$

$$\gamma = (k - 1) \bmod N_w$$

$$y_k = \left(\zeta - \frac{N_l - 1}{2} \right) \Delta l,$$

$$\zeta = \left\lfloor \frac{k - 1}{N_w} \right\rfloor$$

$$\text{for} \quad k = 1, 2, \dots, N_w N_l.$$

Let R_f be the region within which the target illumination levels are feasible. Outside this region (in practice, this corresponds to points near the walls of the office) those levels are not achievable owing to edge effects and thus the constraints at these points are not considered.

Furthermore, let us define two discrete sets of coordinate pairs $\mathcal{U}$ and $\mathcal{V}$ given by

$$
\begin{aligned}
\mathcal{U} = &\{(x_k, y_k) : (x_k, y_k) \in R_o \text{ and} \\
&(x_k, y_k) \in R_f; \quad k = 1, \ldots, N_w N_l\}, \\
\mathcal{V} = &\{(x_k, y_k) : (x_k, y_k) \notin R_o \text{ and} \\
&(x_k, y_k) \in R_f; \quad k = 1, \ldots, N_w N_l\}.
\end{aligned} \tag{11}
$$

Let U and V be the number of coordinate pairs in the sets $\mathcal{U}$ and $\mathcal{V}$, respectively. The u-th or v-th coordinate pair (x, y) in $\mathcal{U}$ or $\mathcal{V}$ is denoted by $\mathcal{U}_u$ or $\mathcal{V}_v$, respectively.

Rewriting the constraints in (9) and evaluating the constraints in their respective sets, we obtain

$$
\sum_{i=1}^{N_x N_y} d_i E_i(\mathcal{U}_u; h) \leq L_{\max} C_{th} + L_{\max},
$$
$$
u = 1, \ldots, U;
$$
$$
-\sum_{i=1}^{N_x N_y} d_i E_i(\mathcal{U}_u; h) \leq L_{\max} C_{th} - L_{\max},
$$
$$
u = 1, \ldots, U;
$$
$$
-\sum_{i=1}^{N_x N_y} d_i E_i(\mathcal{V}_v; h) \leq -L_{\min},
$$
$$
v = 1, \ldots, V;
$$
$$
\sum_{i=1}^{N_x N_y} d_i \left[\frac{1}{U} \sum_{u=1}^{U} E_i(\mathcal{U}_u; h) \right] = L_{\max}. \tag{12}
$$

Using (12), the optimisation problem of (9) can be written in matrix form as

$$
\boldsymbol{d}^\star = \arg\min_{\boldsymbol{d}} \mathbf{1}_{N_x N_y \times 1}^T \boldsymbol{d} \quad \text{s.t.}
$$
$$
\begin{cases}
\boldsymbol{M}\boldsymbol{d} \leq \boldsymbol{b} \\
\boldsymbol{n}^T \boldsymbol{d} = L_{\max} \\
0 \leq d_i \leq 1, \quad i = 1, \ldots, N_x N_y
\end{cases} \tag{13}
$$

where $\mathbf{1}_{N \times 1}$ is the vector $[1, 1, \ldots, 1]^T$ of size $N \times 1$ and

$$
\boldsymbol{M} = \begin{bmatrix} \boldsymbol{M}_1 \\ \boldsymbol{M}_2 \\ \boldsymbol{M}_3 \end{bmatrix}, \quad \boldsymbol{b} = \begin{bmatrix} \boldsymbol{b}_1 \\ \boldsymbol{b}_2 \\ \boldsymbol{b}_3 \end{bmatrix}, \quad \boldsymbol{n} = \frac{1}{U} \sum_{u=1}^{U} \boldsymbol{f}_u
$$

with

$$
\boldsymbol{M}_1 = \begin{bmatrix} \boldsymbol{f}_1^T \\ \vdots \\ \boldsymbol{f}_U^T \end{bmatrix}, \quad \boldsymbol{M}_2 = -\boldsymbol{M}_1 \ \boldsymbol{M}_3 = \begin{bmatrix} -\boldsymbol{g}_1^T \\ \vdots \\ -\boldsymbol{g}_V^T \end{bmatrix}
$$

and

$$
\begin{aligned}
\boldsymbol{f}_u &= [E_1(\mathcal{U}_u; h), \ldots, E_{N_x N_y}(\mathcal{U}_u; h)]^T, \\
&u = 1, \ldots, U ; \\
\boldsymbol{g}_v &= [E_1(\mathcal{V}_v; h), \ldots, E_{N_x N_y}(\mathcal{V}_v; h)]^T, \\
&v = 1, \ldots, V ; \\
\boldsymbol{b}_1 &= (L_{\max} C_{th} + L_{\max}) \mathbf{1}_{U \times 1}; \\
\boldsymbol{b}_2 &= (L_{\max} C_{th} - L_{\max}) \mathbf{1}_{U \times 1}; \\
\boldsymbol{b}_3 &= -L_{\min} \mathbf{1}_{V \times 1}.
\end{aligned}
$$

We now use slack variables[15] to transform the inequality constraints $\boldsymbol{M}\boldsymbol{d} \leq \boldsymbol{b}$ into equality constraints. Let the $(2U + V) \times 1$ vector of slack variables $\boldsymbol{s}$ be written as

$$
\boldsymbol{s} = [s_1, s_2, \ldots, s_{2U+V}]^T
$$
$$
s_q \geq 0, \quad q = 1, \ldots, 2U + V.
$$

Hence, (13) can be posed as

$$
\boldsymbol{d}^\star = \arg\min_{\boldsymbol{d}} \mathbf{1}_{N_x N_y \times 1}^T \boldsymbol{d} \quad \text{s.t.}
$$
$$
\begin{cases}
\boldsymbol{M}\boldsymbol{d} + \boldsymbol{s} = \boldsymbol{b} \\
\boldsymbol{n}^T \boldsymbol{d} = L_{\max} \\
0 \leq d_i \leq 1, \quad i = 1, \ldots, N_x N_y \\
s_q \geq 0, \quad q = 1, \ldots, 2U + V.
\end{cases} \tag{14}
$$

Now our problem is in the standard form of a linear optimisation problem with an additional upper bound for the variables $\{d_i\}$. For such problems, there are known efficient methods[16] such as the simplex algorithm for obtaining an exact solution.

The solution of (14) obtained from the simplex algorithm results in continuous values for d_i lying between 0 and 1. As a final step, we discretise the resulting vector $d^\star$. Assuming D levels for dimming an LED, we proceed to map each element of $d^\star$ to the nearest dimming level (multiple of $\frac{1}{D}$). It is clear that this final step introduces an error in the solution which is inversely proportional to the number of levels D (see also the following section). For a high resolution level D, this error is negligible.

4.1 Computational complexity

In practice, the simplex method converges in less than $3Q$ iterations,[17] where Q is the number of constraints (here, $Q = N_x N_y + 2U + V + 1$). In comparison, a full search method with a resolution of D levels for dimming the LEDs requires $D^{N_x N_y}$ iterations. Hence, a full search algorithm is not feasible to use even when the number of LEDs is moderately large.

5. Numerical example

We consider a typical indoor office scenario, with the parameters shown in Table 1. The illumination lighting parameters comply with the recommendations of the European Committee for Standardization.[18]

The parameters from the Luxeon Rebel,[3] which produces a Lambertian radiation pattern with $\Phi_{\frac{1}{2}} = 60°$, are chosen for testing. These values are listed in Table 2. A single LED provides approximately 14.3 lx in the axis direction. Hence, the radiation pattern of the i-th LED over the workspace plane is

Table 1 Indoor lighting parameters

| Parameter	Value
l (m)	4
w (m)	6
h (m)	2
L_{max} (lx)	500
L_{min} (lx)	300
r_0 (m)	1

Table 2 LED parameters

Parameter	Value
$\Phi_{\frac{1}{2}}$ (degrees)	60
Lambertian mode (m)	1
Luminous flux (A_0) (lm)	180
Maximum illuminance (A) per LED (lx)	14.3
Power consumed (P_{on}) per LED (W)	2.24

given by

$$E_i^{(60)}(x, y; h)$$
$$= 14.3\left[1 + \frac{||(x,y)-(x_i,y_i)||_2^2}{4}\right]^{-2}. \quad (15)$$

Additionally, a second narrower beamwidth of $10.5°$ is tested. Most commercial lenses offer different beamwidths, from around $5°$ to $40°$, with different gains in illuminance.[19] Very narrow beams are excluded due to the fact that in those cases the best choice of dimming levels is the trivial solution of turning on the LEDs in the surrounding of the occupant to achieve uniform illumination and dimming off the others to maintain the minimum level L_{min}. The maximum illuminance per LED using a lens with angle $\Phi_{\frac{1}{2}} = 10.5°$ is 180 lx (lambertian mode, $m = 41$). The illumination pattern with $\Phi_{\frac{1}{2}} = 10.5°$ is given by

$$E_i^{(10.5)}(x, y; h)$$
$$= 180\left[1 + \frac{||(x,y)-(x_i,y_i)||_2^2}{4}\right]^{-22}.$$
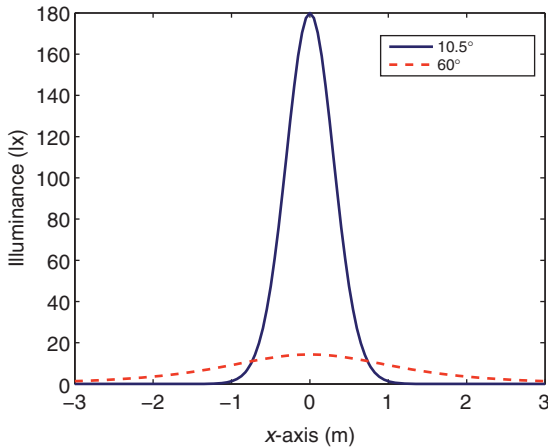
Figure 2 Illuminance pattern for different beamwidths

Table 3 Error in contrast for different bit resolutions

Bit resolution	Error (%)
4	4.254
5	1.241
6	0.361
7	0.343
8	0.099
9	0.098
10	0.017
11	0.017
12	0.008

The corresponding patterns for $\Phi_{\frac{1}{2}}$ of 10.5 and $60°$ are shown in Figure 2.

For dimming the LED, a resolution of 8 bits is chosen. This allows $D = 256$ different levels of illumination per LED. Furthermore, the error introduced in the calculated contrast is low. The error in the resulting contrast within R_0 for different bit resolutions is shown in Table 3. This error is calculated with an occupant located at the centre of the room and $\Phi_{\frac{1}{2}} = 60°$.

The separation of the LEDs is chosen in a way that makes it possible to render a uniform illumination distribution across the plane. Thus, using (10), the maximum separation $(\Delta = \Delta x = \Delta y)$ for each case is

Table 4 Additional parameters of LED lighting system

Parameter	Value
$\Delta x = \Delta w$ (m)	0.3
$\Delta y = \Delta l$ (m)	0.3
$N_x = N_w$	20
$N_y = N_l$	13

calculated. For values for $\Phi_{\frac{1}{2}}$ of $10.5°$ and $60°$, we obtain a maximum separation Δ of 0.3598 m and 1.287 m, respectively.

Finally, the separation is chosen as $\Delta = 0.3$ m. That means

$$N_x = \frac{6}{0.3} \approx 20,$$

$$N_y = \frac{4}{0.3} \approx 13$$

which in total represents 260 LEDs distributed over a uniform grid on the ceiling. The constraints are evaluated at the same coordinate pairs (x, y) of the LEDs. These parameters are summarised in Table 4.

5.1 Performance comparison

We compare our proposed system and method (labelled, SM-2) with a system that renders uniform illumination at $L_{\max}$ across the whole room (labelled, SM-1[1]). The metric for comparison is the power consumed calculated from (5), while additionally considering the contrasts achieved.

We shall consider two values for the contrast threshold, C_{th}. One value is a tighter choice of $C_{th} = 0.05$ so as to provide a higher uniformity in illumination. The other is $C_{th} = 0.3$ which is as per the recommended limit.[18]

We first consider a single occupant located at the centre of the office room, with

[1]The dimming levels are optimised under SM-1 for minimum power consumption subject to a maximum contrast C_{th} and mean illumination level $L_{\max}$ over R_f.

$C_{th} = 0.05$. Performance results comparing SM-1 and SM-2 for this scenario are shown in Table 5. In Figure 3, we depict the feasible region R_f for different values of $\Phi_{\frac{1}{2}}$ under SM-2. The obtained illumination patterns for each $\Phi_{\frac{1}{2}}$ are plotted in Figures 4 and 5, respectively. As indicated in Table 5, in both cases, the contrast is kept below the threshold of 0.05. With SM-2, we observe power savings higher than 33% in comparison to SM-1.

Additionally, with $\Phi_{\frac{1}{2}} = 60°$, the dimming levels for an occupant located at the centre of the office room for SM-2 and SM-1 are

Table 5 Performance comparison of SM-1 and SM-2 (C_{th}=0.05)

$\Phi_{\frac{1}{2}}$ (degrees)	10.5	60
Maximum contrast (SM-1)	0.05	0.05
Maximum contrast (SM-2)	0.05	0.05
Power (SM-1) (W)	277.63	331.86
Power (SM-2) (W)	184.80	205.52
Reduction in Power (%)	33.43	38.07

plotted in Figures 6 and 7, respectively. We observe close to the borders a larger number of LEDs at a high dimming level with SM-1 than with SM-2.

Next, with $\Phi_{\frac{1}{2}} = 60°$, we consider the performance of SM-2 with varying locations of the occupant. Due to symmetry, results are shown corresponding to locations where $x \geq 0$ and $y \geq 0$.

The illumination uniformity obtained within R_0 and power savings are shown in Figures 8 and 9, respectively for different locations of an occupant. As can be seen, the uniformity is maintained below 0.05 and the total power saving is more than 30%. The lowest power saving is obtained close to the corners of the room because fewer LEDs are contributing to the illumination at those locations. Thus, more LEDs need to be at higher illumination levels to illuminate the occupant workspace.

Finally, in Table 6, a performance comparison of SM-1 and SM-2 is shown for different $\Phi_{\frac{1}{2}}$ with $C_{th} = 0.3$. The illumination
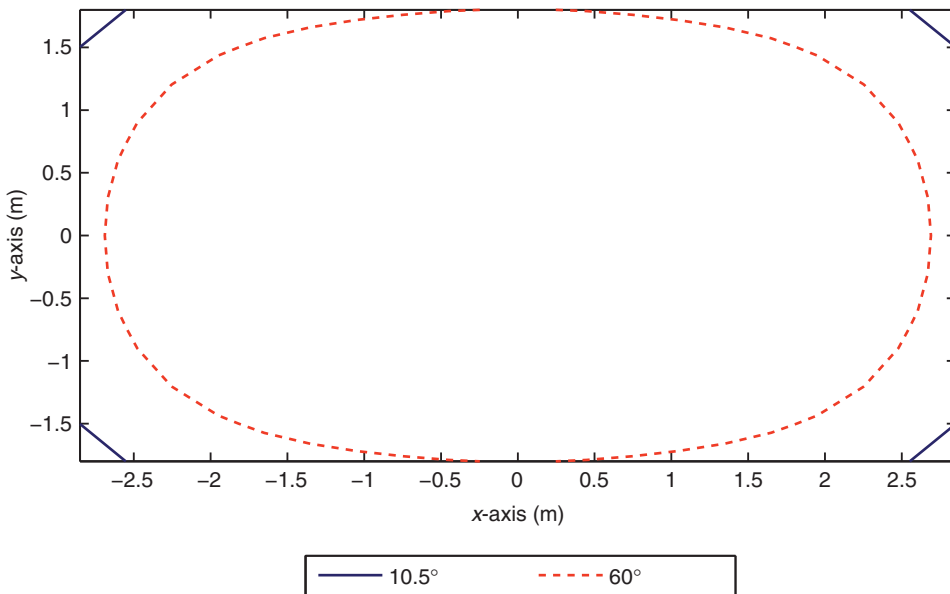

Figure 3 Region R_f for different angles $\Phi_{\frac{1}{2}}$ and $C_{th} = 0.05$

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patterns, dimming levels and power savings are depicted in Figures 10–15. A looser value of 0.3 for C_{th} (i.e. lower uniformity) implies that some locations will have illumination levels as low as 0.7 L_{max}. For SM-1, those locations are principally at the borders of R_f, whereas for SM-2 these are at the borders of R_0 (see Figures 10 and 11, and cf. Figures 4
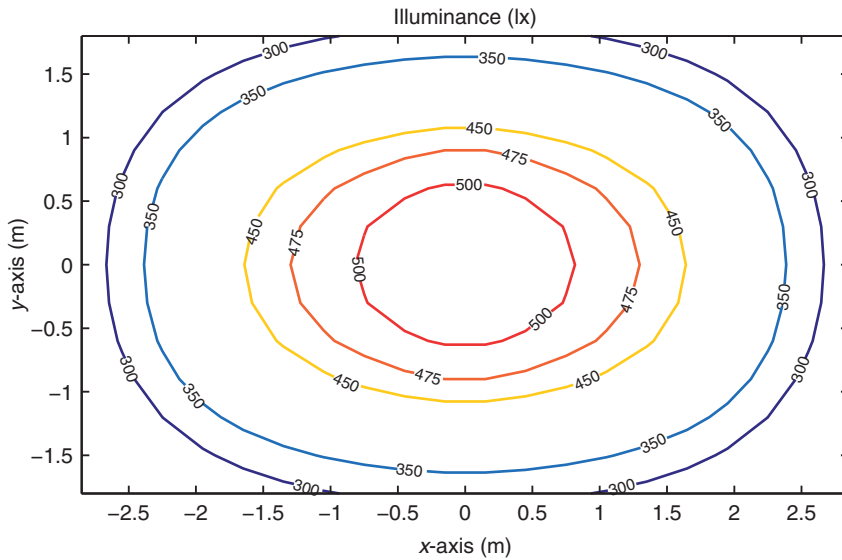
Figure 4 Illuminance pattern under SM-2 ($\Phi_{\frac{1}{2}} = 60°$, $C_{th} = 0.05$)
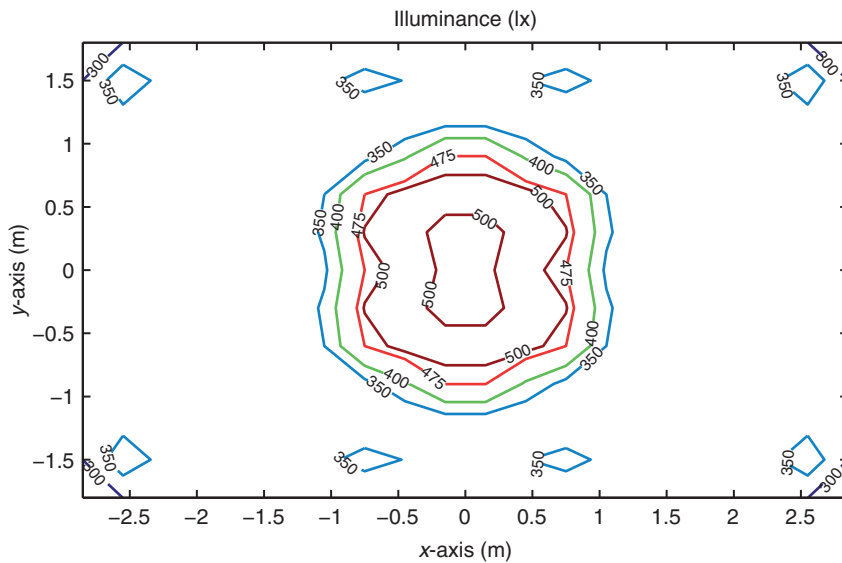
Figure 5 Illuminance pattern under SM-2 ($\Phi_{\frac{1}{2}} = 10.5°$, $C_{th} = 0.05$)

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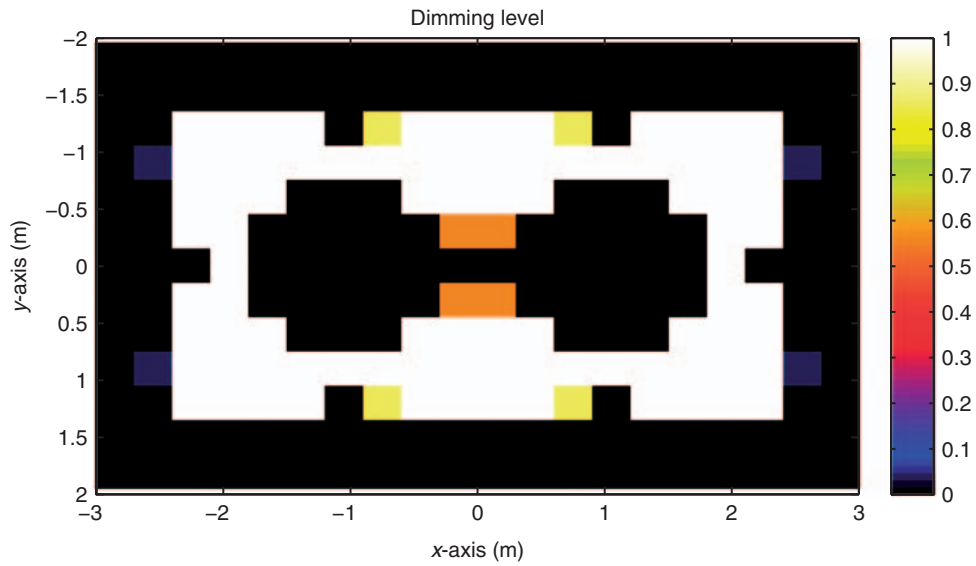
Figure 6 Dimming levels under SM-2 ($\Phi_{\frac{1}{2}} = 60°$, $C_{th} = 0.05$)
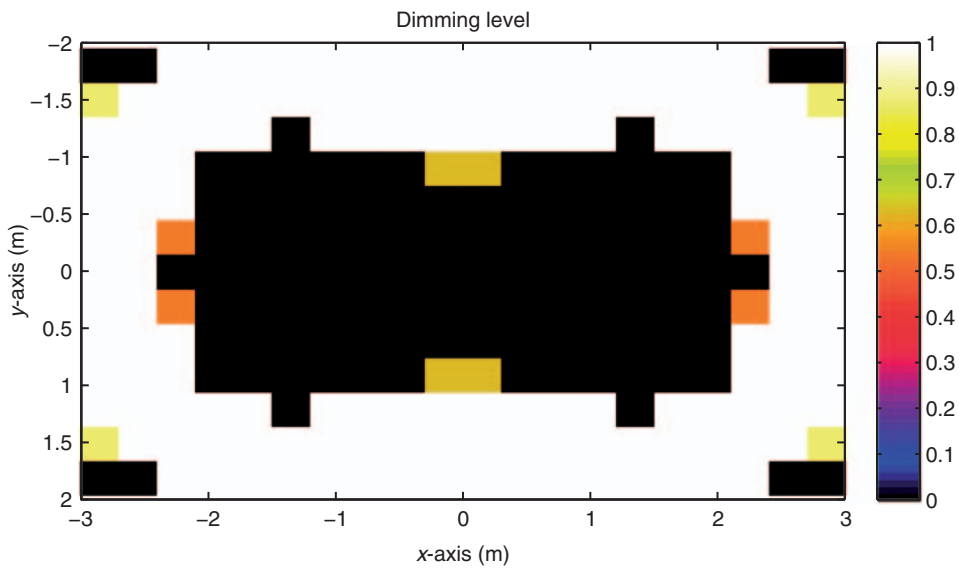
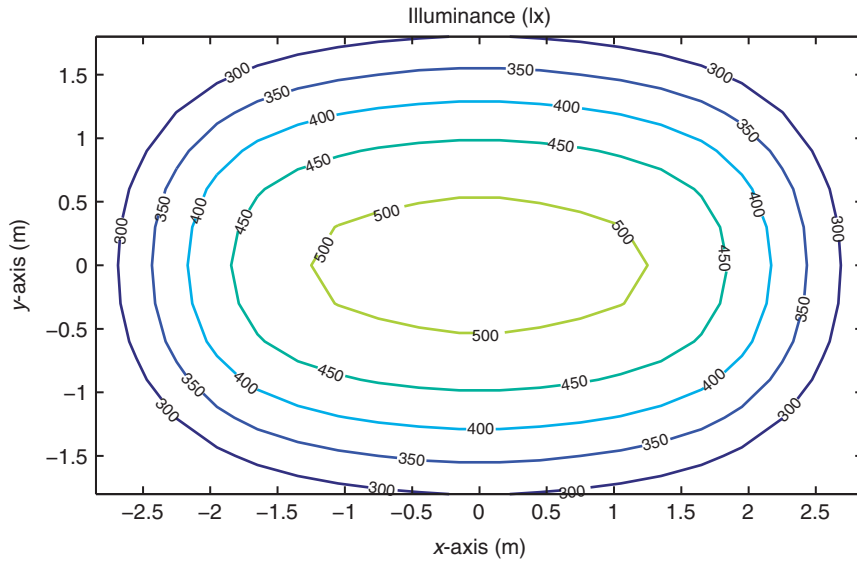**Figure 7** Dimming levels under SM-1 ($\Phi_{\frac{1}{2}} = 60°$, $C_{th} = 0.05$)
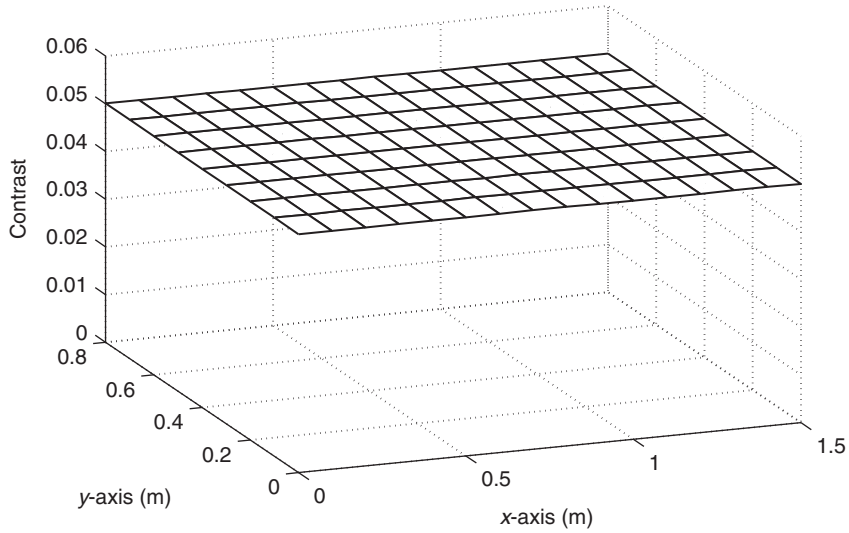
Figure 8 Maximum contrast for different locations of an occupant under SM-2 ($\Phi_{\frac{1}{2}} = 60°$, $C_{th} = 0.05$)
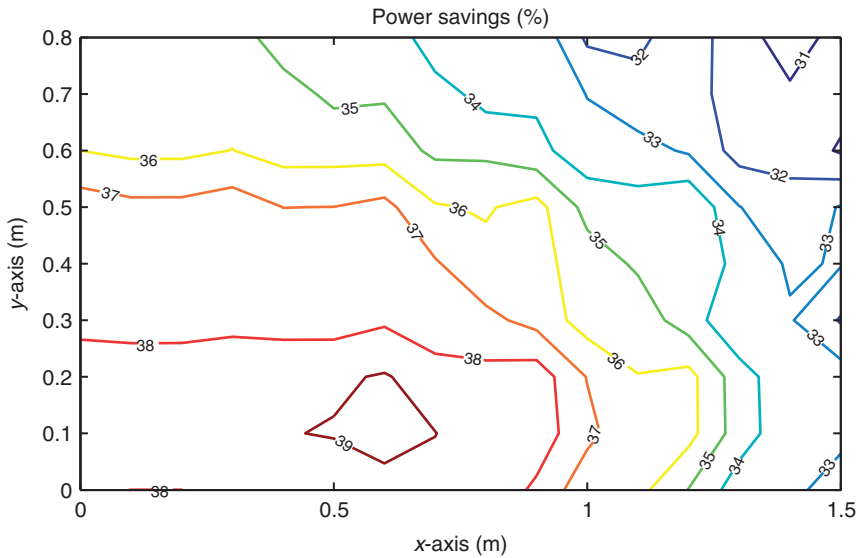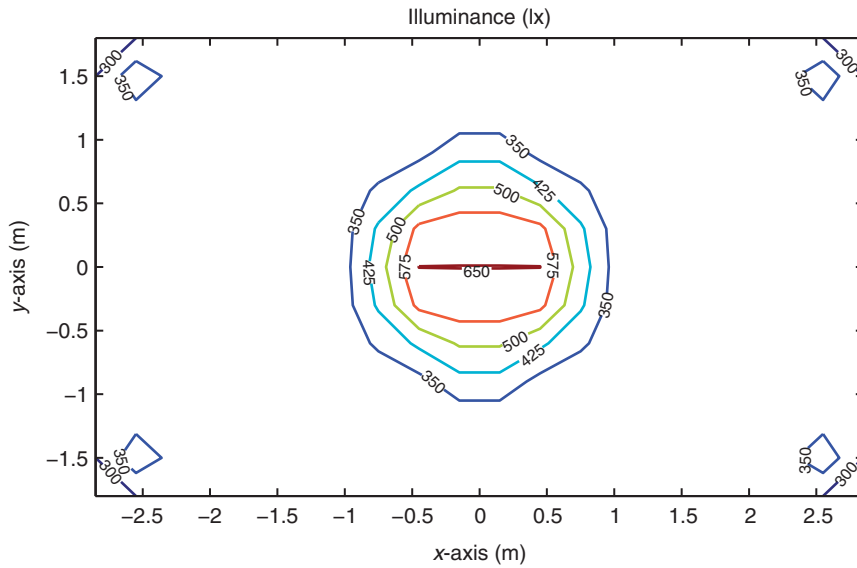

Figure 9 Power savings of SM-2 over SM-1 for different locations of an occupant under SM-2 ($\Phi_{\frac{1}{2}} = 60°$, $C_{th} = 0.05$)

Table 6 Performance comparison of SM-1 and SM-2 ($C_{th} = 0.3$)

$\Phi_{\frac{1}{2}}$ (degrees)	10.5	60
Maximum contrast (SM-1)	0.3	0.3
Maximum contrast (SM-2)	0.3	0.08
Power (SM-1) (W)	254.71	237.98
Power (SM-2) (W)	182.85	202.21
Reduction in Power (%)	28.21	15.03

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Figure 10 Illuminance pattern under SM-2 ($\Phi_{\frac{1}{2}} = 60°$, $C_{th} = 0.3$)

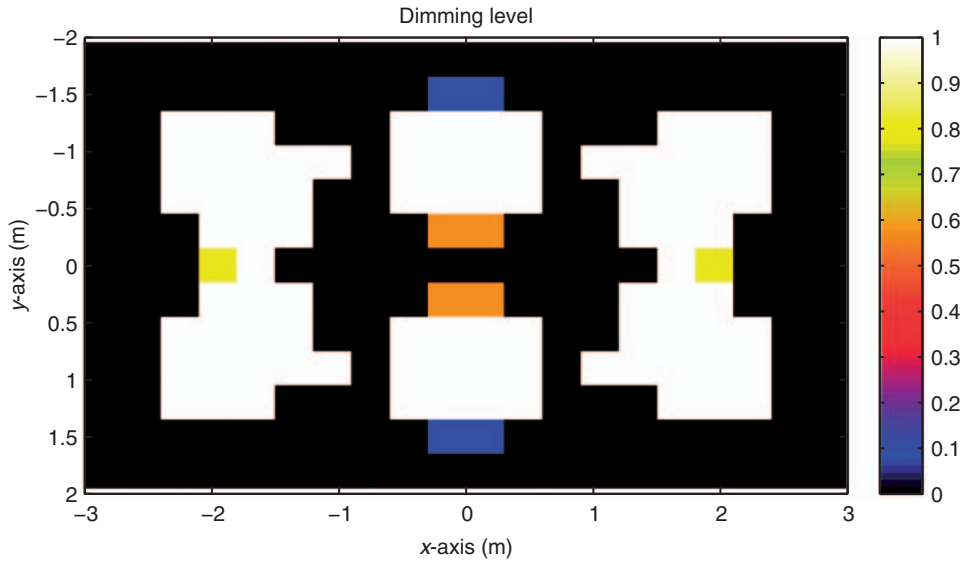**Figure 11** Illuminance pattern under SM-2 ($\Phi_{\frac{1}{2}} = 10.5°$, $C_{th} = 0.3$)

Figure 12 Dimming levels under SM-2 ($\Phi_{\frac{1}{2}} = 60°$, $C_{th} = 0.3$)
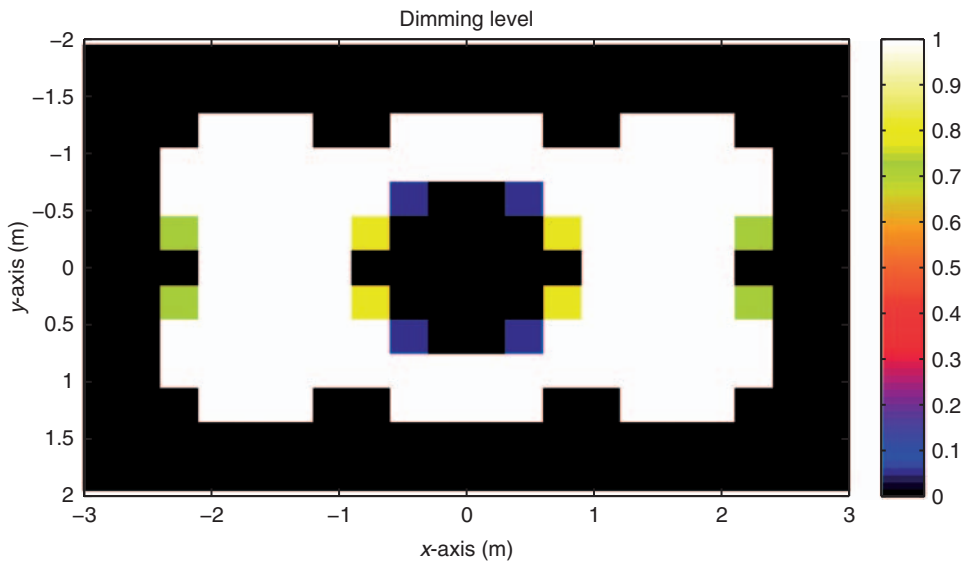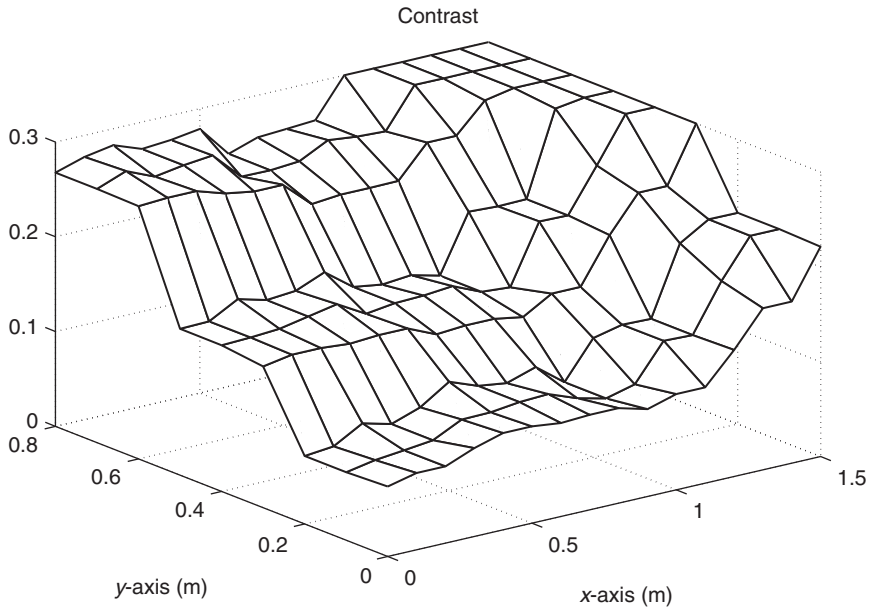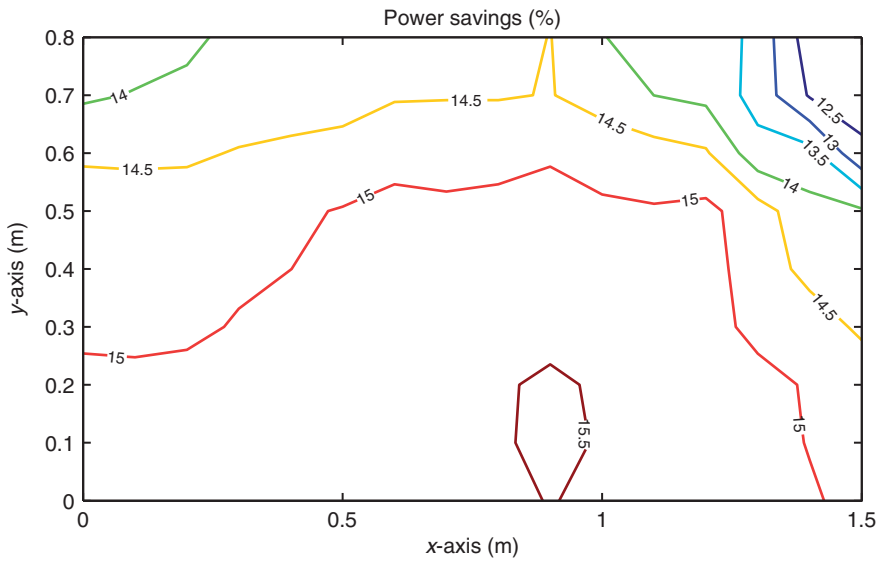

Figure 13 Dimming levels under SM-1 ($\Phi_{\frac{1}{2}} = 60°$, $C_{th} = 0.3$)

Contrast

Figure 14 Maximum contrast for different locations of an occupant under SM-2 ($\Phi_{\frac{1}{2}}=60°$, $C_{th}=0.3$)

Power savings (%)

Figure 15 Power savings of SM-2 over SM-1 for different locations of an occupant under SM-2 ($\Phi_{\frac{1}{2}}=60°$, $C_{th}=0.3$)

and 5). It is noteworthy to mention that the power saving with $C_{th} = 0.3$ is less than that with $C_{th} = 0.05$ (Figures 9 and 15). This can be understood by looking at the optimised dimming levels under SM-1 and SM-2 for $C_{th} = 0.05$ and $C_{th} = 0.3$ (refer to Figures 6, 7, 12 and 13). Under SM-1 with $C_{th} = 0.05$, a larger number of LEDs are at maximum power when compared with that of $C_{th} = 0.3$ (as shown in Figures 7 and 13, respectively). However, under SM-2 the dimming levels for $C_{th} = 0.05$ and $C_{th} = 0.3$ are quite comparable (as shown in Figures 6 and 12).

6. Conclusions

We formulated the design of occupancy-based uniform illumination control of LED lighting systems as a constrained optimisation problem. This problem can be solved using linear programming methods, and a simplex algorithm was used to obtain an optimal solution. We then compared the energy efficiency of such an LED system (SM-2) with an LED system that renders uniform illumination across the entire space (SM-1). For single occupancy configurations, we showed that substantial savings are achieved with the proposed design.

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