Energy Detection of (Ultra-)Wideband PPM

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Abstract

In this paper, energy detectors are developed for wideband and ultra-wideband (UWB) pulse position modulation (PPM). Exact bit error probability (BEP) expressions are presented under different assumptions about the channel. More specifically, we present an expression for the instantaneous BEP for a specific channel realization as well as an expression for the average BEP for an i.i.d. zero-mean Gaussian channel. Simulation results corroborate the precision of the expressions.

1 Introduction

Pulse position modulation (PPM) is an M-ary orthogonal modulation scheme that has enjoyed great interest with the advent of ultra-wideband (UWB) communications [1, 2]. Different symbols are realized by shifting a pulse to distinct positions in time within the specified symbol duration. PPM is advantageous because of its simplicity and the ease of controlling delays [1] but the disadvantage is the relatively large bandwidth associated with it. This large bandwidth causes a large number of multipaths [3]. Thus channel estimation becomes a very important but complicated process. A number of solutions have been provided to circumvent this issue, e.g., as proposed in [4, 5, 6]. In order to reduce the overall system complexity and power consumption, we concentrate on noncoherent reception of PPM signals through energy detection [7, 8]. The resulting detection procedure is akin to a generalized maximum likelihood (GML) detector. The symbol decision is determined by the pulse position that contains more energy than the rest of the positions. We derive the optimal energy detectors for wideband and UWB PPM reception, and provide the corresponding bit error probability (BEP) expressions. We obtain expressions for different assumptions on the channel, and verify them through simulations. First of all, we consider a deterministic channel and present the instantaneous BEP for a specific channel realization. Next, we consider a stochastic channel and present the average BEP for an i.i.d. zero-mean Gaussian channel. We finally point out that the UWB PPM detector proposed in [7], whose performance degrades with an increasing spreading factor, is suboptimal, and we show that the performance of the optimal UWB PPM detector does not depend on the spreading factor.

2 Signal Model

Define the pulse position modulated transmitted signal $s_k(t)$ of length T for the kth information symbol $a_k \in \{0, 1, \ldots, M-1\}$ as $s_k(t) = g(t - kT - a_kT/M)$ where $g(t)$ is the unit-energy pulse waveform with support $[0, T_g]$. If $p(t)$ represents the impulse

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response of the physical communication channel, then the received signal corresponding to the kth information symbol is given by

$$x_k(t) = s_k(t) * p(t) + n_k(t) = h(t - kT - a_k T/M) + n_k(t)$$

where $n_k(t)$ is the additive noise corresponding to the kth information symbol, and $h(t) = g(t) * p(t)$ is the received pulse waveform with support $[0, T_h]$. Following Nyquist-rate sampling at rate N/T, the sampled received signal corresponding to the kth information symbol is given by

$$x_{k,i} = x_k(iT/N) = h_{i-kN-a_k N/M} + n_{k,i},$$

for $i = 0, 1, \ldots, N-1$, where $h_i = h(iT/N)$ and $n_{k,i} = n_k(iT/N)$, and where we have assumed that N/M is an integer. The support of h_i is given by $[0, L-1]$, where $L = \lceil NT_h/T \rceil$. Since we want to make the detection process separable in the different symbols, we do not want the symbols to overlap and we thus require $T_h \leq T/M$ or $L \leq N/M$.

3 Energy Detection of PPM

In order to reduce the overall system complexity and power consumption, we concentrate on noncoherent reception of PPM signals [7], which is akin to a GML detector. The symbol decision is based on finding the pulse position that contains the maximum energy. The symbol-by-symbol detection process does not require the estimation of the channel parameters. The energy of the multipath components is collected to increase the detection probability of the actual transmitted pulse. Let us focus on 2-PPM for simplicity ($M = 2$) and let us assume that we have knowledge of L (in practice this can be an overestimate of L). In that case, the detector can be built by incorporating only those samples in $x_{k,i}$ that contain symbol information, and we obtain

$$u_{k,1} = \sum_{i=0}^{L-1} x_{k,i}^2 \underset{1}{\overset{0}{\gtrless}} u_{k,2} = \sum_{i=N/2}^{N/2+L-1} x_{k,i}^2. \tag{1}$$

In the following, we present the theoretical performance of the above detector. We consider two different situations. First of all, we consider a deterministic channel h_i and present the instantaneous BEP given a specific channel realization. The average BEP for a certain channel distribution can then be estimated in simulations by averaging the BEP over different channel realizations drawn from the channel distribution. For one specific channel distribution, we can present the average BEP in closed form, namely when the channel h_i is i.i.d. zero-mean Gaussian distributed. This is discussed in the second part. Note that since all symbols are treated separately, we can simply consider $k = 0$ and drop the subscript k in the sequel of this section whenever it offers notational convenience.

3.1 Instantaneous BEP

We assume that the channel h_i is deterministic and that the noise n_i is i.i.d. zero-mean Gaussian distributed with variance σ^2, i.e., $n_i \sim \mathcal{N}(0, \sigma^2)$ for $i = 0, 1, \ldots, N-1$. Assuming a zero has been transmitted, this results into $x_i = h_i + n_i \sim \mathcal{N}(h_i, \sigma^2)$, $i = 0, 1, \ldots, L-1$. From (1), we can write the instantaneous BEP for the case a zero is transmitted as

$$P_e = P(u_1 < u_2),$$

where $u_1 = \sum_{i=0}^{L-1} x_i^2 = \sum_{i=0}^{L-1} (h_i + n_i)^2$ and $u_2 = \sum_{i=N/2}^{N/2+L-1} x_i^2 = \sum_{i=N/2}^{N/2+L-1} n_i^2$. Since $x_i = h_i + n_i \sim \mathcal{N}(h_i, \sigma^2)$, $i = 0, 1, \ldots, L-1$, u_1 is a non-central chi-square distributed random variable where the noncentrality parameter equals the instantaneous channel energy, $s^2 = E_h = \sum_{i=0}^{L-1} h_i^2$. The pdf of u_1 is given by [9]

$$p_{U_1}(u_1) = \frac{1}{2\sigma^2}\left(\frac{u_1}{s^2}\right)^{(L-2)/4} \exp\left[\frac{-(s^2+u_1)}{2\sigma^2}\right] I_{L/2-1}\left(\sqrt{u_1}\frac{s}{\sigma^2}\right), \; u_1 > 0,$$

where $I_\nu(z)$ is the modified Bessel function of the first kind [11, Eq. (8.445)]. Further, u_2 is a central chi-square distributed random variable since only the noise is involved. The pdf of u_2 is given by [9]

$$p_{U_2}(u_2) = \frac{1}{\sigma^L \, 2^{\frac{L}{2}} \, \Gamma\left(\frac{L}{2}\right)} u_2^{(L-2)/2} \exp\left[\frac{-u_2}{2\sigma^2}\right], \; u_2 > 0.$$

The instantaneous BEP is

$$P_e = 1 - \frac{1}{2\sigma^2\Gamma(\frac{L}{2})s^{(L-2)/2}} e^{\frac{-s^2}{2\sigma^2}} \int_0^\infty \gamma\left(\frac{L}{2}, \frac{u_1}{2\sigma^2}\right) u_1^{(L-2)/4} e^{\frac{-u_1}{2\sigma^2}} I_{L/2-1}\left(\sqrt{u_1}\frac{s}{\sigma^2}\right) du_1 \quad (2)$$

where $\gamma(.,.)$ is the lower incomplete gamma function given by $\gamma(n,u) = \int_0^u t^{n-1}e^{-t}dt$. Details regarding the derivation of (2) can be found in [10]. (2) only contains a single integral and can easily be computed numerically. Remark that P_e is the expression for the instantaneous BEP given a specific channel realization h_i. The average BEP $\bar{P}_e$ for a certain channel distribution can then be estimated in simulations by averaging the instantaneous BEP P_e over different channel realizations drawn from the channel distribution.

3.2 Average BEP for an i.i.d. zero-mean Gaussian Channel

In this subsection, we consider a specific channel distribution for which we can find an expression of the average BEP $\bar{P}_e$ in closed form. More specifically, we assume that the channel h_i is i.i.d. zero-mean Gaussian distributed with variance 1, i.e., $h_i \sim \mathcal{N}(0,1)$ for $i = 0, 1, \ldots, L-1$, and that the noise n_i is i.i.d. zero-mean Gaussian distributed with variance σ^2, i.e., $n_i \sim \mathcal{N}(0,\sigma^2)$ for $i = 0, 1, \ldots, N-1$. Assuming a zero has been transmitted, this results into $x_i = h_i + n_i \sim \mathcal{N}(0, 1+\sigma^2)$, $i = 0, 1, \ldots, L-1$. Although this might not be the most realistic channel model, it provides us the opportunity to study the influence of certain channel and noise parameters on the average BEP $\bar{P}_e$. From (1), we can write the average BEP for the case a zero is transmitted as

$$\bar{P}_e = P(u_1 < u_2),$$

where $u_1 = \sum_{i=0}^{L-1} x_i^2 = \sum_{i=0}^{L-1} (h_i + n_i)^2$ and $u_2 = \sum_{i=N/2}^{N/2+L-1} x_i^2 = \sum_{i=N/2}^{N/2+L-1} n_i^2$. Since $x_i = h_i + n_i \sim \mathcal{N}(0, 1+\sigma^2)$, $i = 0, 1, \ldots, L-1$, u_1 now is a central chi-square distributed random variable instead of a non-central chi-square distributed random variable. The pdf of u_1 is given by [9]

$$p_{U_1}(u_1) = \frac{u_1^{\frac{L}{2}-1}}{\sigma_1^L 2^{\frac{L}{2}}\Gamma\left(\frac{L}{2}\right)} e^{\frac{-u_1}{2\sigma_1^2}}, \; \sigma_1^2 = 1 + \sigma^2.$$

Further, u_2 is again a central chi-square distributed random variable. The pdf of u_2 is given by [9]

$$p_{U_2}(u_2) = \frac{u_2^{\frac{L}{2}-1}}{\sigma_2^L 2^{\frac{L}{2}} \Gamma\left(\frac{L}{2}\right)} e^{\frac{-u_2}{2\sigma_2^2}}, \ \sigma_2^2 = \sigma^2.$$

The average BEP is

$$\bar{P}_e = 1 - \frac{2\Gamma(L)}{L[\Gamma(\frac{L}{2})]^2} \left[\frac{\sigma_1\sigma_2}{\sigma_1^2+\sigma_2^2}\right]^L {}_2F_1\left(1, L; \frac{L}{2}+1; \frac{\sigma_1^2}{\sigma_1^2+\sigma_2^2}\right) \quad (3)$$

where ${}_2F_1(.,.;.;.)$ is the Gaussian hypergeometric function that is defined by [11, Eq. (9.14.2)]. Details regarding the derivation of (3) can be found in [10]. Hence, we have obtained a closed form expression for the average BEP for an i.i.d. zero-mean Gaussian channel h_i.

3.3 Simulation Results

In this subsection, we will illustrate the above BEP expressions by means of some simulation examples. We consider a 2-PPM system with samples taken at Nyquist rate and a channel of length $L = 3$. Let us first focus on the instantaneous BEP and make the same assumptions as in Section 3.1. The instantaneous BEP will be plotted against the instantaneous SNR. Defining the instantaneous channel energy as $E_h = \sum_{i=0}^{L-1} h_i^2$, and the instantaneous SNR can be written as

$$\eta = \frac{E_h}{\sigma^2}.$$

Note that the instantaneous BEP P_e only depends on this instantaneous SNR η, and not on the distribution of the energy over the different channel taps. We compare our exact expression (2) with the approximate expression derived in [7]

$$P_e \approx Q\left(\left[2\frac{1}{\eta} + L\left(\frac{1}{\eta^2}\right)^2\right]^{-1/2}\right), \quad (4)$$

where $Q(.)$ is the Q-function. Note that we have adapted the expression of [7] to our context and notation. We will come back to this equation later on when we discuss energy detection for UWB PPM signals. The results are plotted in Figure 1. We clearly observe that the Gaussian approximation of the channel energy made in [7] does not hold for this example since L is too small. That is why the approximate instantaneous BEP of [7] severely underestimates the exact instantaneous BEP of (2). Let us next focus on the average BEP for an i.i.d. zero-mean Gaussian channel and make the same assumptions as in Section 3.2. The average BEP will be plotted against the average SNR. Defining the average channel energy as $\bar{E}_h = E\{\sum_{i=0}^{L-1} h_i^2\} = L$ and the average SNR is defined as

$$\bar{\eta} = \frac{\bar{E}_h}{\sigma^2} = \frac{L}{\sigma^2}.$$

In Fig. 2, we compare our exact expression (3) with two other curves: the simulated average BEP and the average BEP obtained by averaging (4) over different channel realizations. We can clearly see that our exact expression matches the simulations, whereas the result based on [7] again underestimates the exact average BEP.

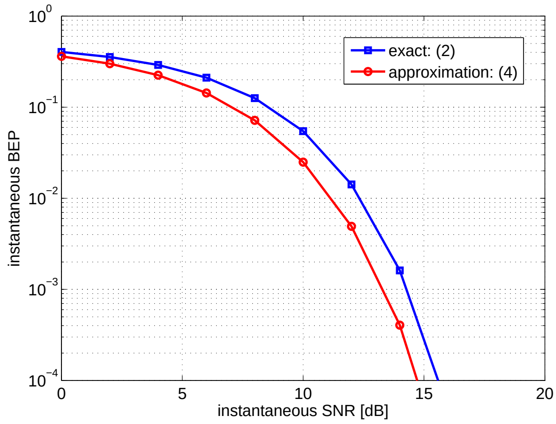

Figure 1: Instantaneous BEP of 2-PPM with $L = 3$ taps.

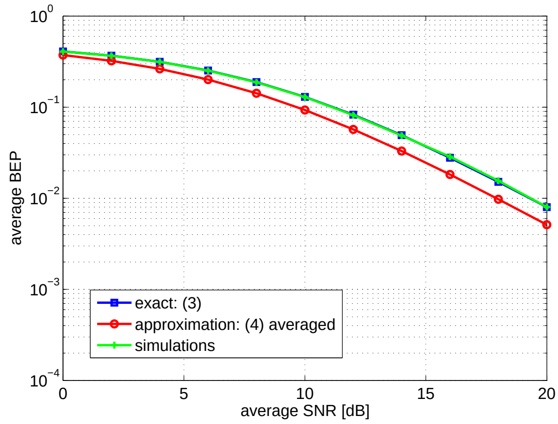

Figure 2: Average BER of 2-PPM with $L = 3$ taps.

4 Energy Detection of UWB PPM Signals

In this section, we extend the analysis to single-user UWB systems employing PPM. We follow the model presented in [7] and improve upon it. Every symbol now consists of N_f frames, each with frame time T_f, so that the symbol time is given by $T = N_f T_f$. The motivation for a multiple frame transmission has been attributed to the FCC limits on the signal power spectral density. Repeating a pulse N_f times, reduces the energy of an individual pulse for a constant symbol energy. The transmitted and received signal for the kth information symbol can now respectively be written as

$$s_k(t) = \sum_{j=0}^{N_f-1} g(t - (j + kN_f)T_f - a_k T_f/M)$$

and

$$x_k(t) = s_k(t) * p(t) + n_k(t) = \sum_{j=0}^{N_f-1} h(t - jT_f - kT - a_k T_f/M) + n_k(t).$$

Following Nyquist-rate sampling at rate N/T_f, the sampled received signal corresponding to the kth information symbol is now given by

$$x_{k,i} = x_k(iT_f/N) = \sum_{j=0}^{N_f-1} h_{i-jN-kNN_f-a_k N/M} + n_{k,i}, \tag{5}$$

for $i = 0, 1, \ldots, N_f N - 1$, using the same definitions and assumptions as before. Separating the detection process in the different symbols now also means that the different frames may not overlap, and thus we require $T_h \leq T_f/M$ or $L \leq N/M$. Stacking the NN_f received samples related to the kth symbol, $\mathbf{x}_k = [x_{k,0}, x_{k,1}, \ldots, x_{k,NN_f}]^T$, we can write (5) as

$$\mathbf{x}_k = \mathbf{u}(a_k, \mathbf{h}) + \mathbf{n}_k$$

where $\mathbf{h} = [h_0, h_1, \ldots, h_{L-1}]^T$ and $\mathbf{n}_k = [n_{k,0}, n_{k,1}, \ldots, n_{k,NN_f}]^T$. $\mathbf{u}(a_k, \mathbf{h})$ is the useful signal part. Assuming $n_{k,i}$ is i.i.d. zero-mean Gaussian distributed with variance σ^2, the pdf of the received signal $\mathbf{x}_k$ can be written as

$$p(\mathbf{x}_k|a_k,\mathbf{h}) = C\exp\left\{-\frac{1}{2\sigma^2}\|\mathbf{x}_k - \mathbf{u}(a_k,\mathbf{h})\|_2^2\right\} \quad (6)$$

where C is some positive constant. Using the generalized maximum likelihood criterion, it is clear that in order to maximize (6), we need to minimize the squared 2-norm, which can be written as

$$\Lambda(a_k,\mathbf{h}) = \|\mathbf{x}_k - \mathbf{u}(a_k,\mathbf{h})\|_2^2 = \sum_{j=0}^{N_f-1}\sum_{l=0}^{L-1}(h_l^2 - 2h_l x_{k,P_{j,l}}) \quad (7)$$

where $P_{j,l} = jN+a_kN/M+l$ for notational simplicity. Now taking the partial derivative with respect to h_l while keeping a_k fixed, we obtain

$$\frac{\partial\Lambda(a_k,\mathbf{h})}{\partial h_l} = 2N_f h_l - 2\sum_{j=0}^{N_f-1} x_{k,P_{j,l}}.$$

Now minimizing the cost function with respect to $\mathbf{h}$ would mean setting every gradient with respect to h_l to zero, which yields the following optimal estimate for h_l:

$$\hat{h}_l = \frac{1}{N_f}\sum_{j=0}^{N_f-1} x_{k,P_{j,l}}. \quad (8)$$

Defining $\hat{\mathbf{h}} = [\hat{h}_0, \hat{h}_1, \ldots, \hat{h}_{L-1}]^T$ and substituting (8) in (7), we finally obtain $\Lambda(a_k,\hat{\mathbf{h}}) = -N_f\sum_{l=0}^{L-1}\hat{h}_l^2$. As a result, the symbol a_k can be found by solving the following problem

$$\min_{a_k}\ \Lambda(a_k,\hat{\mathbf{h}}) = \max_{a_k}\ \sum_{l=0}^{L-1}\hat{h}_l^2. \quad (9)$$

Let us at this point define the instantaneous SNR as

$$\eta = \frac{N_f E_h}{\sigma^2}.$$

From (9) and (8), it can then be observed that the decision result will be independent of the number of frames N_f for the same instantaneous SNR η. We can explain this as follows. The estimate of h_l in (8) is obtained by averaging samples over different frames, which on one hand decreases the noise energy by a factor of N_f but on the other hand also decreases the signal energy by a factor of N_f due to the fact that the instantaneous SNR η is kept constant. Hence, the performance of the estimate of h_l does not change with N_f and thus also the solution to (9) does not change with N_f since it only involves the estimate of h_l. Replacing $\hat{h}_l$ in (9) by the value obtained from (8), we can write

$$\min_{a_k}\Lambda(a_k,\hat{\mathbf{h}}) = \max_{a_k}\sum_{l=0}^{L-1}\left[\frac{1}{N_f}\sum_{j=0}^{N_f-1} x_{k,P_{j,l}}\right]^2 = \max_{a_k}\sum_{l=0}^{L-1}\left[\frac{1}{N_f}\sum_{j=0}^{N_f-1} x_{k,jN+a_kN/M+l}\right]^2. \quad (10)$$

So we can see that the optimal procedure consists of first averaging and then squaring, and the related performance is independent of the number of frames N_f if the instantaneous SNR η is kept constant. The instantaneous BEP P_e can thus be computed using (2). This is in contrast to the procedure proposed in [7], consisting of first squaring and then averaging:

$$\hat{a}_k = \arg\max_{a_k} \frac{1}{N_f} \sum_{j=0}^{N_f-1} \sum_{l=0}^{L-1} x^2_{k,jN+a_kN/M+l}. \tag{11}$$

The related instantaneous BEP can be approximated by [7]

$$P_e \approx Q\left(\left[2\left(\frac{1}{\eta}\right) + N_f L \left(\frac{1}{\eta}\right)^2\right]^{-1/2}\right), \tag{12}$$

which increases significantly with N_f. Hence, we can conclude that the approach in [7] is clearly suboptimal. Note that for $N_f = 1$ both approaches are equivalent and that is why we could use (12) with $N_f = 1$ as a performance benchmark for the instantaneous BEP in Section 3.3.

4.1 Simulation Results

Let us consider the pulse waveform $g(t)$ given by the second derivative of a Gaussian pulse with unit energy and a duration of 1 nsec. Further, let us generate a channel $p(t)$ using the IEEE 802.15.3a CM1 channel model [3], which is a line-of-sight channel model. We focus on a bandwidth of 1 GHz, corresponding to a sample rate of $N/T =$ 0.5 ns. Since the CM1 channel model has a delay spread of about 20 ns, we take $L = 40$. Figure 3 shows the simulated instantaneous BEP for the proposed method (10) and compares this with (12) which is an approximation of the instantaneous BEP for the method of [7]. We observe that the performance of the proposed method does not change with N_f, whereas the method of [7] is severely influenced by N_f. Fig. 4 finally compares the simulated instantaneous BEP of the proposed method (10) with the exact expression of (2) and the approximated expression of (4) for $L = 20$ and $L = 40$. Clearly, the exact expression corresponds to the simulated results, whereas the approximated expression slightly deviates (the deviation decreases as L increases). In this case, the difference between (2) and (4) is smaller as in Fig. 1 because we are dealing with larger values of L here. Further, we notice that decreasing L below the delay spread (from $L = 40$ to $L = 20$) can have a positive effect on the performance, since we are capturing less noise energy in the considered intervals.

5 Conclusion

We have presented PPM signal models for wideband and UWB signals along with their theoretical expressions for the BEP. We have looked at the instantaneous BEP for a specific channel realization as well as the average BEP for an i.i.d. zero-mean Gaussian channel. Our theoretical analysis portrays the exact behavior of the signal models. We have also presented an UWB PPM detector along with its theoretical BEP expression which outperforms an existing detector.

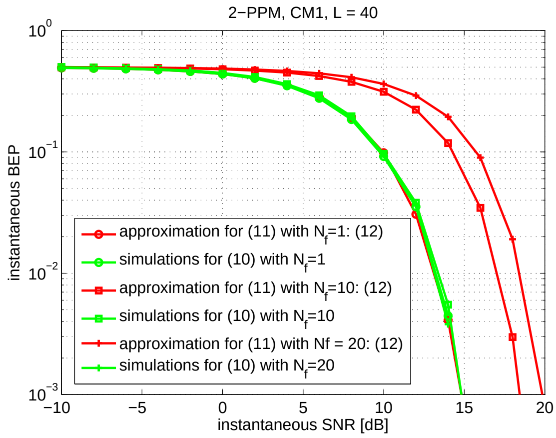

Figure 3: Instantaneous BEP comparison between (10) and (11) for 2-PPM UWB.

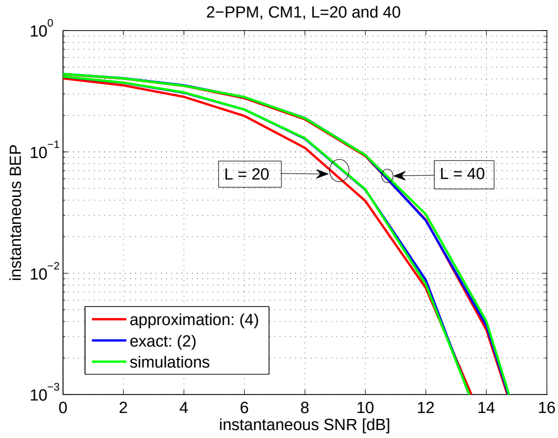

Figure 4: Instantaneous BEP of proposed method (10) for 2-PPM UWB.

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