RANGING ENERGY OPTIMIZATION FOR ROBUST SENSOR POSITIONING WITH COLLABORATIVE ANCHORS

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ABSTRACT

We propose a sensor positioning scheme for a wireless sensor network consisting of beacons as well as collaborative anchors (CA) to help sensors within a prescribed service area to locate themselves. We assume a robust performance is achieved in the sense that a prescribed location accuracy requirement is fulfilled within the service area. Under the assumption that the time-of-arrival and location estimators adopted achieve the associated Cramér-Rao bound, the performance of the location scheme is derived and analyzed. A ranging energy optimization problem is proposed, and a practical algorithm is presented. The effectiveness of this algorithm is illustrated by numerical experiments.

***Index Terms*—** Wireless sensor network, localization, Cramér-Rao bound.

1. INTRODUCTION

Sensor positioning is an important task of location-aware wireless sensor networks (WSNs). In practice, it is costly to equip each sensor with a global positioning system (GPS). Instead, a few beacons or anchors, which are fusion centers or sensors with known positions, are encompassed in most WSNs for locating sensors of interest. In general, a location scheme is carried out to first produce position-dependent measurements such as range, range-difference, and angle-of-arrival between a sensor and beacons, and then these measurements are conveyed somewhere for position estimation. To obtain these measurements, sensors and beacons usually need to emit ranging signals to each other.

The design of a location scheme highly depends on the feature of hardware available as well as application requirements. For instance, let's consider locating a sensor with unsynchronized beacons. When the beacons are connected to a central processing unit (CPU) and the application requires the CPU to know the sensor position, this CPU may initiate two-way ranging to obtain range measurements between the sensor and beacons, and then estimate the sensor position [1]. If the application requires the sensor to be aware of its position, the sensor may initiate two-way ranging to obtain these measurements and then estimate its own position.

Determining the ranging energy for sensors and beacons is another important issue. On the one hand, the ranging energy should be sufficiently high to produce reliable measurements. On the other hand, the ranging energy should be reduced to prolong system lifetime, especially for sensors and beacons supported by a battery. Ranging energy optimization has been addressed in [1] for a robust sensor positioning system based on two-way ranging between a sensor and beacons.

In this paper, we consider sensor positioning in a WSN consisting of beacons as well as collaborative anchors (CA) to help sensors within a prescribed service area to locate themselves. The sensor positioning features a robust performance in the sense that a prescribed accuracy requirement is fulfilled within the service area. In particular, this paper contains the following contributions:

- We propose a location scheme for the considered WSN. In particular, beacons and CAs are required to emit ranging signals, while sensors only need to estimate TOAs of these signals, and produce a set of range-difference measurements to estimate their own positions. Under the assumption that the time-of-arrival (TOA) and location estimators adopted achieve the associated Cramér-Rao bound (CRB), the performance of this location scheme is derived and analyzed.
- We propose a ranging energy optimization problem as well as a practical algorithm. Though we assume an optimistic scenario where the associated CRBs are achieved, the optimization results set lower bounds to the ranging energy required in more realistic scenarios, and give a sense of how much ranging energy has to be allocated at least.

The rest of this paper is organized as follows. In the next section, we describe the considered WSN. In Section III, we elaborate on the location scheme and study its performance. After that, we propose a ranging energy optimization problem, as well as a practical algorithm in Section IV. In Section V, numerical experiments are presented. Finally, some conclusions wrap up this paper.

2. SYSTEM DESCRIPTIONS

We consider a 2-dimensional WSN with M beacons, and beacon m is located at a known coordinate $\mathbf{p}_m = [x_{\mathrm{b},m}, y_{\mathrm{b},m}]^T$ ($m = 1, \cdots, M$). Note that M should be no smaller than 2, which will be explained in the next section. In addition, there are N CAs which help locating sensors unaware of their positions. Specifically, CA n, which is located at a known coordinate $\mathbf{q}_n = [x_{\mathrm{c},n}, y_{\mathrm{c},n}]^T$, is responsible for collaborating with the beacons such that sensors lying within a service area S_n can locate themselves. In practice, the CAs may be the cluster heads equipped with GPS or manually installed in clustered WSNs.

Let's say one of the sensors to be located within S_n has an unknown coordinate $\mathbf{u}_n = [x_{\mathrm{s},n}, y_{\mathrm{s},n}]^T$. For the simplicity of notation, we refer to this sensor as sensor n hereafter. We assume that the clocks of sensor n, CA n, and all beacons are unsynchronized but run at the same pace. We denote the distance between beacon m and sensor n by $D_{m,\mathbf{u}_n} = \|\mathbf{p}_m - \mathbf{u}_n\|_2$, the distance between CA n and sensor n by $d_{n,\mathbf{u}_n} = \|\mathbf{q}_n - \mathbf{u}_n\|_2$, and the distance between beacon m and CA n by $L_{m,n} = \|\mathbf{p}_m - \mathbf{q}_n\|_2$. We assume CA n knows its own coordinate as well as all beacons' coordinates, so that $\{L_{m,n}\}_{m=1}^{M}$ can be calculated by CA n itself. In addition, we assume all beacons' coordinates as well as CA n's coordinate, are known by sensor n.

We assume a robust performance is achieved for sensor positioning, in the sense that a prescribed location accuracy requirement is fulfilled for sensors within the service area associated with each CA. Specifically, we adopt the accuracy requirement that $\forall n \in \{1, \cdots, N\}$, $\forall \mathbf{u}_n \in S_n$, $\|\hat{\mathbf{u}}_n - \mathbf{u}_n\|_2$ should be smaller than R_e with probability higher than P_e, where $\hat{\mathbf{u}}_n$ denotes an estimate of $\mathbf{u}_n$, and the value of R_e and P_e is prescribed. Note that this form of requirement was first introduced by FCC to regulate the localization of mobile users.

We assume that the two-sided power spectral density of the additive white Gaussian noise (AWGN) at sensor n and CA n is equal to $\frac{N_\mathrm{s}}{2}$. We assume a LOS channel exists between any two nodes of the set consisting of sensor n, CA n, and beacon m. In addition, this LOS channel incurs a propagation delay $\frac{d}{c}$ and a path loss $\alpha d^{-\beta}$, where d is the propagation distance of this LOS channel, c stands for the signal propagation speed, α represents the path gain at 1 m, and β denotes the path loss exponent. We assume α and β are both known by system designers, and each channel remains unchanged during the localization process.

We assume either sensor n or CA n can estimate the TOAs of incoming ranging signals. In addition, we assume both beacon m and CA n are able to emit ranging signals. We assume beacons have a reliable power supply. To reduce the complexity of the system design, the ranging signals used by the beacons are all the same and equal to $s_\mathrm{b}(t)$ with energy E_b. We assume each CA has a weaker power supply than the beacons, and the ranging signal used by CA n is denoted by $s_{\mathrm{c},n}(t)$ with energy ϵ_n. In practice, these signals may be preamble parts of communication signals.

We assume a ranging signal can be received reliably by a sensor or a CA when its energy is above a threshold value E_t. We assume E_b is sufficiently high so that all sensors and CAs receive ranging signals from every beacon reliably. Each sensor can be located with the help of CA n if a ranging signal from CA n is received reliably. We assume S_n and ϵ_n are chosen so that sensors within S_n can be located with the help of CA n. In addition, sensors lying in the intersection of more service areas choose the CA from which stronger ranging signals are received. We assume $\{S_n\}_{n=1}^{N}$, $\{\epsilon_n\}_{n=1}^{N}$, and E_b are fixed and determined during the system design phase.

It has been shown that the CRB of TOA estimation depends on the root-mean-square (RMS) angular frequency of the adopted ranging signal. We assume all CAs' ranging signals have the same RMS angular frequency. To facilitate the following derivations, we define the RMS angular frequency of $s_\mathrm{b}(t)$ and $s_{\mathrm{c},n}(t)$ respectively as:

$$\omega_\mathrm{b} = \sqrt{\frac{\int_{-\infty}^{+\infty} |2\pi f S_\mathrm{b}(f)|^2 df}{\int_{-\infty}^{+\infty} |S_\mathrm{b}(f)|^2 df}} \quad (1)$$

$$\omega_\mathrm{c} = \sqrt{\frac{\int_{-\infty}^{+\infty} |2\pi f S_{\mathrm{c},n}(f)|^2 df}{\int_{-\infty}^{+\infty} |S_{\mathrm{c},n}(f)|^2 df}} \quad (2)$$

where $S_\mathrm{b}(f)$ and $S_{\mathrm{c},n}(f)$ represent the spectrum of $s_\mathrm{b}(t)$ and $s_{\mathrm{c},n}(t)$, respectively.

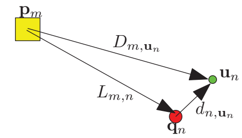

Fig. 1. An exemplary geometry related to beacon m, CA n, and sensor n.

3. LOCATION SCHEME AND PERFORMANCE ANALYSIS

3.1. Procedure of the location scheme

The location scheme proceeds as follows. First, beacons broadcast their ranging signals one after another. Let's say beacon m broadcasts $s_\mathrm{b}(t)$ of energy E_b. At sensor n, the TOA of this signal is denoted by $t_{\mathbf{p}_m\to\mathbf{u}_n}$, but estimated by sensor n as $\hat{t}_{\mathbf{p}_m\to\mathbf{u}_n} = t_{\mathbf{p}_m\to\mathbf{u}_n} + e_{\mathbf{p}_m\to\mathbf{u}_n}$. At CA n, the TOA of this signal is denoted by $t_{\mathbf{p}_m\to\mathbf{q}_n}$, but estimated by CA n as $\hat{t}_{\mathbf{p}_m\to\mathbf{q}_n} = t_{\mathbf{p}_m\to\mathbf{q}_n} + e_{\mathbf{p}_m\to\mathbf{q}_n}$. Here, $e_{\mathbf{p}_m\to\mathbf{u}_n}$ and $e_{\mathbf{p}_m\to\mathbf{q}_n}$ denote the TOA estimation error at sensor n and CA n, respectively.

After estimating the TOAs of all the beacons' ranging signals, CA n emits $s_{\mathrm{c},n}(t)$ of energy ϵ_n at time $t_{\mathbf{q}_n}$ to sensor n. We assume a perfect calibration is achieved so that $t_{\mathbf{q}_n}$ is precisely known by CA n. At sensor n, the TOA of this signal is denoted by $t_{\mathbf{q}_n\to\mathbf{u}_n}$, but estimated by sensor n as $\hat{t}_{\mathbf{q}_n\to\mathbf{u}_n} = t_{\mathbf{q}_n\to\mathbf{u}_n} + e_{\mathbf{q}_n\to\mathbf{u}_n}$, where $e_{\mathbf{q}_n\to\mathbf{u}_n}$ denotes the TOA estimation error at sensor n.

Note that $\{t_{\mathbf{q}_n\to\mathbf{u}_n}, \hat{t}_{\mathbf{q}_n\to\mathbf{u}_n}\}$ and $\{t_{\mathbf{p}_m\to\mathbf{u}_n}, \hat{t}_{\mathbf{p}_m\to\mathbf{u}_n}\}_{m=1}^{M}$ are recorded with the internal clock of sensor n, while $t_{\mathbf{q}_n}$ and $\{t_{\mathbf{p}_m\to\mathbf{q}_n}, \hat{t}_{\mathbf{p}_m\to\mathbf{q}_n}\}_{m=1}^{M}$ are recorded with the internal clock of CA n. As a result, we can see that, $t_{\mathbf{q}_n\to\mathbf{u}_n} - t_{\mathbf{p}_m\to\mathbf{u}_n}$ is equal to the difference between the signal propagation time from beacon m to sensor n via CA n and that from beacon m to sensor n directly. As illustrated in Figure 1, this difference is equal to $\frac{L_{m,n}+d_{n,\mathbf{u}_n}}{c} + (t_{\mathbf{q}_n} - t_{\mathbf{p}_m\to\mathbf{q}_n}) - \frac{D_{m,\mathbf{u}_n}}{c}$. Note that the processing delay $t_{\mathbf{q}_n} - t_{\mathbf{p}_m\to\mathbf{q}_n}$ at CA n is also taken into account. This means that $c(t_{\mathbf{q}_n\to\mathbf{u}_n} - t_{\mathbf{p}_m\to\mathbf{u}_n}) - [c(t_{\mathbf{q}_n} - t_{\mathbf{p}_m\to\mathbf{q}_n}) + L_{m,n}]$ is equal to $d_{n,\mathbf{u}_n} - D_{m,\mathbf{u}_n}$. Though $t_{\mathbf{q}_n\to\mathbf{u}_n}$, $t_{\mathbf{p}_m\to\mathbf{u}_n}$, and $t_{\mathbf{p}_m\to\mathbf{q}_n}$ are unknown, their estimates can be used to produce a measurement denoted by r_m:

$$r_m = c(\hat{t}_{\mathbf{q}_n\to\mathbf{u}_n} - \hat{t}_{\mathbf{p}_m\to\mathbf{u}_n}) - [c(t_{\mathbf{q}_n} - \hat{t}_{\mathbf{p}_m\to\mathbf{q}_n}) + L_{m,n}] \quad (3)$$

It is easy to find that $r_m = d_{n,\mathbf{u}_n} - D_{m,\mathbf{u}_n} + c(e_{\mathbf{q}_n\to\mathbf{u}_n} - e_{\mathbf{p}_m\to\mathbf{u}_n} + e_{\mathbf{p}_m\to\mathbf{q}_n})$, which is a measurement of the range difference $d_{n,\mathbf{u}_n} - D_{m,\mathbf{u}_n}$. Motivated by the above analysis, the final step of the location scheme is that, CA n evaluates a set of data $\{c(t_{\mathbf{q}_n} - \hat{t}_{\mathbf{p}_m\to\mathbf{q}_n}) + L_{m,n}\}_{m=1}^{M}$ and then transmits them to sensor n through data packets. After receiving these data, sensor n produces a set of range-difference measurements $\{r_m\}_{m=1}^{M}$, based on which sensor n estimates its own position with $\mathbf{q}_n$ and $\{\mathbf{p}_m\}_{m=1}^{M}$. Note that the location estimator using range-difference measurements needs at least two measurements, and therefore M should be no smaller than 2.

3.2. Performance derivation

We assume the TOA estimates at sensor n and CA n are independent of each other, and they achieve the associated CRBs with unbiased Gaussian distributions. In fact, this can be asymptotically accomplished by a maximum-likelihood estimator. The CRB of

TOA estimation has been derived for similar scenarios in [1]. Using the formulas given there, we can show that the CRBs of estimating $t_{\mathbf{q}_n\to\mathbf{u}_n}$, $t_{\mathbf{p}_m\to\mathbf{u}_n}$, and $t_{\mathbf{p}_m\to\mathbf{q}_n}$ are equal to $\kappa\gamma d_{n,\mathbf{u}_n}^{\beta}\epsilon_n^{-1}$, $\kappa D_{m,\mathbf{u}_n}^{\beta}E_{\mathrm{b}}^{-1}$, and $\kappa L_{m,n}^{\beta}E_{\mathrm{b}}^{-1}$, respectively, where $\kappa = \frac{N_{\mathrm{s}}}{2\alpha\omega_{\mathrm{b}}^2}$, and $\gamma = \frac{\omega_{\mathrm{b}}^2}{\omega_{\mathrm{c}}^2}$. Therefore, $e_{\mathbf{q}_n\to\mathbf{u}_n}$, $e_{\mathbf{p}_m\to\mathbf{u}_n}$, and $e_{\mathbf{p}_m\to\mathbf{q}_n}$ are Gaussian distributed as $\mathcal{N}(0,\kappa\gamma d_{n,\mathbf{u}_n}^{\beta}\epsilon_n^{-1})$, $\mathcal{N}(0,\kappa D_{m,\mathbf{u}_n}^{\beta}E_{\mathrm{b}}^{-1})$, and $\mathcal{N}(0,\kappa L_{m,n}^{\beta}E_{\mathrm{b}}^{-1})$, respectively.

Let's stack all M measurements into a vector $\mathbf{r} = [r_1,\cdots,r_M]^T$. According to our assumptions, $\mathbf{r}$ is Gaussian distributed. Let's denote its mean and covariance by $\mathbf{v}_{\mathbf{u}_n}$ and $\mathbf{\Sigma}_{\mathbf{u}_n}$, respectively. It is easy to show that the m-th entry of $\mathbf{v}_{\mathbf{u}_n}$ is equal to $d_{n,\mathbf{u}_n} - D_{m,\mathbf{u}_n}$. After simple manipulations, $\mathbf{\Sigma}_{\mathbf{u}_n}$ can be expressed as:

$$\mathbf{\Sigma}_{\mathbf{u}_n} = c^2\kappa(E_{\mathrm{b}}^{-1}\mathbf{\Lambda}_{\mathbf{u}_n} + \gamma d_{n,\mathbf{u}_n}^{\beta}\epsilon_n^{-1}\mathbf{1}_M\mathbf{1}_M^T) \quad (4)$$

where $\mathbf{\Lambda}_{\mathbf{u}_n}$ is an $M\times M$ diagonal matrix with the m-th diagonal entry equal to $D_{m,\mathbf{u}_n}^{\beta} + L_{m,n}^{\beta}$, and $\mathbf{1}_M$ is an $M\times 1$ column vector with all entries equal to 1.

We assume the location estimator adopted by sensor n is unbiased and achieves the associated CRB, i.e., the mean and covariance of the produced location estimate are equal to $\mathbf{u}_n$ and $\mathbf{F}_{\mathbf{u}_n}^{-1}$, respectively, where $\mathbf{F}_{\mathbf{u}_n}$ denotes the Fisher information matrix. Note that this can be achieved by methods based on semidefinite programming as shown in [2].

In practice, the performance of a location scheme is usually evaluated by the mean square error (MSE) of the location estimator. Therefore, the performance of the location estimator at sensor n can be evaluated by $\mathrm{Tr}(\mathbf{F}_{\mathbf{u}_n}^{-1})$, namely the trace of $\mathbf{F}_{\mathbf{u}_n}^{-1}$. In particular, $\mathbf{F}_{\mathbf{u}_n}$ can be evaluated by:

$$\mathbf{F}_{\mathbf{u}_n} = \mathbb{E}_{\mathbf{r}}\left[(\nabla_{\mathbf{u}_n}L_{\mathbf{u}_n})(\nabla_{\mathbf{u}_n}L_{\mathbf{u}_n})^T\right] \quad (5)$$

where $\mathbb{E}_{\mathbf{r}}(\cdot)$ denotes the ensemble average with respect to $\mathbf{r}$, and $L_{\mathbf{u}_n}$ is the log-likelihood function of $\mathbf{r}$ with respect to $\mathbf{u}_n$:

$$L_{\mathbf{u}_n} = -\frac{\ln\left((2\pi)^M\det(\mathbf{\Sigma}_{\mathbf{u}_n})\right) + (\mathbf{r}-\mathbf{v}_{\mathbf{u}_n})^T\mathbf{\Sigma}_{\mathbf{u}_n}^{-1}(\mathbf{r}-\mathbf{v}_{\mathbf{u}_n})}{2} \quad (6)$$

By some mathematical manipulations, we can show that:

$$\mathbf{F}_{\mathbf{u}_n} = \mathbf{\Phi}_{\mathbf{u}_n}\mathbf{\Sigma}_{\mathbf{u}_n}^{-1}\mathbf{\Phi}_{\mathbf{u}_n}^T \quad (7)$$

where $\mathbf{\Phi}_{\mathbf{u}_n} = \nabla_{\mathbf{u}_n}(\mathbf{v}_{\mathbf{u}_n}^T)$ is a $2\times M$ matrix with the m-th column equal to $\frac{\mathbf{u}_n-\mathbf{q}_n}{d_{n,\mathbf{u}_n}} - \frac{\mathbf{u}_n-\mathbf{p}_m}{D_{m,\mathbf{u}_n}}$. Using the matrix inversion formula $(\mathbf{J}+\mathbf{x}\mathbf{y}^T)^{-1} = \mathbf{J}^{-1} - \frac{\mathbf{J}^{-1}\mathbf{x}\mathbf{y}^T\mathbf{J}^{-1}}{1+\mathbf{y}^T\mathbf{J}^{-1}\mathbf{x}}$, we can further reduce (7) to:

$$\mathbf{F}_{\mathbf{u}_n} = \rho E_{\mathrm{b}}(\mathbf{A}_{\mathbf{u}_n} - \eta_{\mathbf{u}_n}\mathbf{B}_{\mathbf{u}_n}) \quad (8)$$

where $\rho = \kappa^{-1}c^{-2}$ and

$$\mathbf{A}_{\mathbf{u}_n} = \mathbf{\Phi}_{\mathbf{u}_n}\mathbf{\Lambda}_{\mathbf{u}_n}^{-1}\mathbf{\Phi}_{\mathbf{u}_n}^T \quad (9)$$

$$\mathbf{B}_{\mathbf{u}_n} = (\mathbf{\Phi}_{\mathbf{u}_n}\mathbf{g}_{\mathbf{u}_n})(\mathbf{\Phi}_{\mathbf{u}_n}\mathbf{g}_{\mathbf{u}_n})^T. \quad (10)$$

Here $\mathbf{g}_{\mathbf{u}_n} = \mathbf{\Lambda}_{\mathbf{u}_n}^{-1}\mathbf{1}_M$, and $\eta_{\mathbf{u}_n}$ is expressed by:

$$\eta_{\mathbf{u}_n} = \frac{1}{(\epsilon_n/E_{\mathrm{b}})\gamma^{-1}d_{n,\mathbf{u}_n}^{-\beta} + \sum_{m=1}^{M}(L_{m,n}^{\beta}+D_{m,\mathbf{u}_n}^{\beta})^{-1}} \quad (11)$$

3.3. Impact of ranging energy on performance

The MSE performance of the location estimator at sensor n can be evaluated by: $\mathrm{Tr}(\mathbf{F}_{\mathbf{u}_n}^{-1}) = \lambda_{\mathbf{u}_n,1}^{-1} + \lambda_{\mathbf{u}_n,2}^{-1}$, where $\lambda_{\mathbf{u}_n,1}$ and $\lambda_{\mathbf{u}_n,2}$ denote the two eigenvalues of $\mathbf{F}_{\mathbf{u}_n}$ ($\lambda_{\mathbf{u}_n,1} \le \lambda_{\mathbf{u}_n,2}$). We have the following theorems about the impact of E_{b} and ϵ_n on $\lambda_{\mathbf{u}_n,1}$ and $\lambda_{\mathbf{u}_n,2}$:

Theorem 1 *Both $\lambda_{\mathbf{u}_n,1}$ and $\lambda_{\mathbf{u}_n,2}$ are non-decreasing when ϵ_n increases.*

Proof: We can see that $\eta_{\mathbf{u}_n}$ reduces as ϵ_n increases. Suppose ϵ_n is increased by a small amount. As a consequence, $\eta_{\mathbf{u}_n}$ becomes $\eta'_{\mathbf{u}_n} = \eta_{\mathbf{u}_n} - \delta$ where δ is a positive value, while $\mathbf{F}_{\mathbf{u}_n}$, $\lambda_{\mathbf{u}_n,1}$, and $\lambda_{\mathbf{u}_n,2}$ become $\mathbf{F}'_{\mathbf{u}_n} = \mathbf{F}_{\mathbf{u}_n} + \delta\rho E_{\mathrm{b}}\mathbf{B}_{\mathbf{u}_n}$, $\lambda'_{\mathbf{u}_n,1}$, and $\lambda'_{\mathbf{u}_n,2}$ ($\lambda'_{\mathbf{u}_n,1} \le \lambda'_{\mathbf{u}_n,2}$), respectively. Since $\mathbf{B}_{\mathbf{u}_n}$ is positive semidefinite, $\lambda'_{\mathbf{u}_n,1} \ge \lambda_{\mathbf{u}_n,1}$ and $\lambda'_{\mathbf{u}_n,2} \ge \lambda_{\mathbf{u}_n,2}$ hold according to Corollary 4.3.3 in [3]. Therefore, both $\lambda_{\mathbf{u}_n,1}$ and $\lambda_{\mathbf{u}_n,2}$ are non-decreasing when ϵ_n increases. ■

Theorem 2 *Both $\lambda_{\mathbf{u}_n,1}$ and $\lambda_{\mathbf{u}_n,2}$ are non-decreasing when E_{b} increases.*

Proof: Suppose E_{b} is reduced by a small amount to E'_{b}. As a consequence, $\mathbf{\Sigma}_{\mathbf{u}_n}$, $\mathbf{F}_{\mathbf{u}_n}$, $\lambda_{\mathbf{u}_n,1}$, and $\lambda_{\mathbf{u}_n,2}$ become $\mathbf{\Sigma}'_{\mathbf{u}_n}$, $\mathbf{F}'_{\mathbf{u}_n}$, $\lambda'_{\mathbf{u}_n,1}$, and $\lambda'_{\mathbf{u}_n,2}$ ($\lambda'_{\mathbf{u}_n,1} \le \lambda'_{\mathbf{u}_n,2}$), respectively.

Let's define a series of matrices $\{\mathbf{C}_m\}_{m=0}^{M}$, where $\mathbf{C}_0 = \mathbf{\Sigma}_{\mathbf{u}_n}$, and $\mathbf{C}_m = \mathbf{C}_{m-1} + \epsilon_m\mathbf{e}_m\mathbf{e}_m^T$ when $1 \le m \le M$. Here, $\epsilon_m = \rho^{-1}(E_{\mathrm{b}}'^{-1} - E_{\mathrm{b}}^{-1})(D_{m,\mathbf{u}_n}^{\beta} + L_{m,n}^{\beta})$ is positive, and $\mathbf{e}_m$ is an $M\times 1$ all-zero column vector except that the m-th entry is equal to 1. Using the matrix inversion formula, we can show that $(1 \le m \le M)$:

$$\mathbf{C}_m^{-1} = \mathbf{C}_{m-1}^{-1} - \mu_m\mathbf{h}_m\mathbf{h}_m^T \quad (12)$$

where $\mathbf{h}_m = \mathbf{C}_{m-1}^{-1}\mathbf{e}_m$ and $\mu_m = \frac{1}{\epsilon_m^{-1} + \mathbf{e}_m^T\mathbf{C}_{m-1}^{-1}\mathbf{e}_m}$.

We can see that $\mathbf{C}_M = \mathbf{\Sigma}'_{\mathbf{u}_n}$. Using (12) successively, we can show that:

$$\mathbf{\Sigma}_{\mathbf{u}_n}'^{-1} = \mathbf{\Sigma}_{\mathbf{u}_n}^{-1} - \sum_{m=1}^{M}\mu_m\mathbf{h}_m\mathbf{h}_m^T \quad (13)$$

Inserting (13) into (7), we can show that

$$\mathbf{F}'_{\mathbf{u}_n} = \mathbf{F}_{\mathbf{u}_n} - \mathbf{D}_{\mathbf{u}_n} \quad (14)$$

where $\mathbf{D}_{\mathbf{u}_n} = \sum_{m=1}^{M}\mu_m(\mathbf{\Phi}_{\mathbf{u}_n}\mathbf{h}_m)(\mathbf{\Phi}_{\mathbf{u}_n}\mathbf{h}_m)^T$.

Apparently μ_m is positive, and thus $\mathbf{D}_{\mathbf{u}_n}$ is positive semidefinite. Therefore, $\lambda'_{\mathbf{u}_n,1} \le \lambda_{\mathbf{u}_n,1}$ and $\lambda'_{\mathbf{u}_n,2} \le \lambda_{\mathbf{u}_n,2}$ according to Corollary 4.3.3 in [3]. This means that both $\lambda_{\mathbf{u}_n,1}$ and $\lambda_{\mathbf{u}_n,2}$ remain unchanged or decrease as E_{b} reduces. Equivalently, both $\lambda_{\mathbf{u}_n,1}$ and $\lambda_{\mathbf{u}_n,2}$ remain unchanged or increase as E_{b} increases. Therefore, both $\lambda_{\mathbf{u}_n,1}$ and $\lambda_{\mathbf{u}_n,2}$ are non-decreasing when E_{b} increases. ■

Based on the above two theorems, $\mathrm{Tr}(\mathbf{F}_{\mathbf{u}_n}^{-1})$ is non-increasing as E_{b} or ϵ_n increases, which means that the MSE performance at $\mathbf{u}_n$ does not degrade as E_{b} or ϵ_n increases.

4. RANGING ENERGY OPTIMIZATION

To guarantee the robust performance for sensor positioning, i.e., $\forall n \in \{1,\cdots,N\}$, $\forall\mathbf{u}_n \in S_n$, $\|\hat{\mathbf{u}}_n - \mathbf{u}_n\|_2$ should be smaller than R_{e} with probability higher than P_{e}, we use the constraint

$$\lambda_{\min} \ge \lambda_{\mathrm{th}} \quad (15)$$

where $\lambda_{\min} = \min_{1\leq n\leq N} \min_{\mathbf{u}_n\in S_n} \lambda_{\mathbf{F}_{\mathbf{u}_n}}$, with $\lambda_{\mathbf{F}_{\mathbf{u}_n}}$ and λ_{th} denoting the minimal eigenvalue of $\mathbf{F}_{\mathbf{u}_n}$ and a threshold eigenvalue, respectively. Specifically, $\lambda_{\mathrm{th}} = -2R_{\mathrm{e}}^{-2}\ln(1-P_{\mathrm{e}})$ if $\hat{\mathbf{u}}_n$ is Gaussian distributed, and $\lambda_{\mathrm{th}} = 2R_{\mathrm{e}}^{-2}(1-P_{\mathrm{e}})^{-1}$ if $\hat{\mathbf{u}}_n$ has an unknown distribution. Note that this constraint was first derived in [1].

We consider the scenario where $\{\epsilon_n\}_{n=1}^N$ and $\{S_n\}_{n=1}^N$ are prescribed. According to our assumptions, S_n should be a subset of $\{\mathbf{u}_n : \alpha d_{n,\mathbf{u}_n}^{-\beta}\epsilon_n \geq E_{\mathrm{t}}\} = \{\mathbf{u}_n : d_{n,\mathbf{u}_n} \leq (\frac{\alpha\epsilon_n}{E_{\mathrm{t}}})^{1/\beta}\}$ so that sensors in S_n can receive the ranging signal of CA n reliably. Note that to ensure the beacons' ranging signals are reliably received by the CAs and the sensors, E_{b} should satisfy $\alpha D_{m,\mathbf{u}_n}^{-\beta} E_{\mathrm{b}} \geq E_{\mathrm{t}}, \forall S_n, \forall n, \forall m$, or equivalently $E_{\mathrm{b}} \geq E_{\min}$, where

$$E_{\min} = \max_{1\leq m\leq M} \max_{1\leq n\leq N} \max_{\mathbf{u}_n\in S_n} \{\alpha^{-1} D_{m,\mathbf{u}_n}^{\beta} E_{\mathrm{t}}\} \qquad (16)$$

For the considered scenario, we have the following theorem regarding the impact of E_{b} on $\lambda_{\min}$:

Theorem 3 *$\lambda_{\min}$ is non-decreasing when E_{b} increases.*

Proof: Suppose E_{b} is increased to be E'_{b}, and $\lambda_{\mathbf{F}_{\mathbf{u}_n}}$ and $\lambda_{\min}$ become $\lambda'_{\mathbf{F}_{\mathbf{u}_n}}$ and $\lambda'_{\min}$, respectively. Let's say $\lambda'_{\min} = \lambda'_{\mathbf{F}_{\mathbf{z}_j}}$, where $\mathbf{z}_j$ denotes a position within the service area S_j $(1 \leq j \leq N)$. According to **Theorem 2**, $\lambda'_{\mathbf{F}_{\mathbf{z}_j}} \geq \lambda_{\mathbf{F}_{\mathbf{z}_j}}$. Apparently, $\lambda_{\mathbf{F}_{\mathbf{z}_j}} \geq \lambda_{\min}$. Therefore, $\lambda'_{\min} \geq \lambda_{\min}$ holds, which means that $\lambda_{\min}$ is non-decreasing when E_{b} increases. ■

According to the above theorem, (15) can be fulfilled by increasing E_{b}. We consider the problem of finding the minimal E_{b} fulfilling (15) and (16). A difficulty encountered is that each service area is continuous. To address this problem, we uniformly sample a set of grid points $G_n = \{g_{n,k}\}_{k=1}^{K_n}$ in S_n, and replace S_n with G_n, $\forall n$, whenever computing $\lambda_{\min}$ and $E_{\min}$. If (15) is satisfied when $E_{\mathrm{b}} = E_{\min}$, $E_{\min}$ is the output for the optimal E_{b}. Otherwise, we find a sufficiently high value E_{H} for E_{b} so that (15) is fulfilled. Then, we use the following algorithm to find the optimal E_{b}:

$E_{\mathrm{U}} = E_{\mathrm{H}}$; $E_{\mathrm{L}} = E_{\min}$;
while $|E_{\mathrm{U}} - E_{\mathrm{L}}|$ is smaller than a tolerance value **do**
  $E_{\mathrm{M}} = \frac{E_{\mathrm{U}}+E_{\mathrm{L}}}{2}$;
  if $\lambda_{\min}$ when $E_{\mathrm{b}} = E_{\mathrm{M}}$ is smaller than λ_{th} **then**
    $E_{\mathrm{L}} = E_{\mathrm{M}}$;
  else
    $E_{\mathrm{U}} = E_{\mathrm{M}}$;
  end if
end while
Output E_{M} as the optimal E_{b};

Note that this optimal E_{b} computed over $\{G_n\}_{n=1}^N$ is smaller than the true optimal one over $\{S_n\}_{n=1}^N$, since the feasible set of E_{b} enabling the robust positioning performance over $\{S_n\}_{n=1}^N$ is a subset of that over $\{G_n\}_{n=1}^N$. However, it is a reasonable approximation as the sampling distance is sufficiently small, since it approaches the true optimal one as the sampling spacing reduces.

5. NUMERICAL EXPERIMENTS

We consider a WSN with 3 beacons located at $\mathbf{p}_1 = [-3,-3]^T$, $\mathbf{p}_2 = [3,-3]^T$, and $\mathbf{p}_3 = [0,3]^T$. There are two CAs located at $\mathbf{q}_1 = [-1,0]^T$ and $\mathbf{q}_2 = [1,0]^T$. The parameters are set as: $\alpha = 0.5$, $\beta = 2$, $E_{\mathrm{t}} = 0.5$ J, $\frac{\omega_{\mathrm{b}}}{2\pi} = \frac{\omega_{\mathrm{c}}}{2\pi} = 1$ GHz, $N_{\mathrm{s}} = 1$ W/Hz, $c = 3\times 10^8$ m/s, $R_{\mathrm{e}} = 5$ cm, and $P_{\mathrm{e}} = 0.8$.

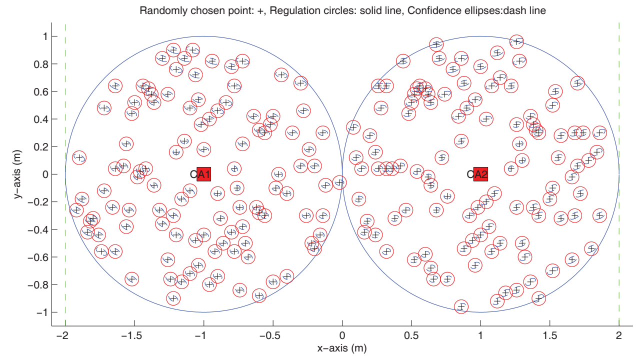

Fig. 2. Confidence ellipses and regulation circles for randomly chosen positions.

We assume S_1 and S_2 are two circular areas of radius 1 m and centered at $\mathbf{q}_1$ and $\mathbf{q}_2$, respectively. We set $\epsilon_1 = \epsilon_2 = 1$ J so that sensors in those areas receive ranging signals of associated CAs reliably. We sample S_1 and S_2 with uniform spacing equal to 2 cm to produce G_1 and G_2, respectively. We use G_1 and G_2 to replace S_1 and S_2 respectively in numerical experiments.

The computed $E_{\min}$ is equal to 50 J. Assuming the location estimate of each sensor is Gaussian distributed, the minimal E_{b} fulfilling the robust positioning performance is computed to be 221.3 J. We randomly chose a set of positions, and for each position we plot the regulation circle of radius R_{e} as well as the confidence ellipse in which the location estimate falls with probability P_{e}. It is shown that each confidence ellipse is contained in the corresponding regulation circle, which means that the accuracy requirement is fulfilled at each position chosen.

6. CONCLUSIONS

We have proposed a sensor positioning scheme for a WSN consisting of beacons as well as CAs to help sensors within a prescribed service area to locate themselves. We assume a robust performance is achieved in the sense that a prescribed location accuracy requirement is fulfilled within the service area. Assuming the associated CRBs can be achieved, we have analyzed the performance of the location scheme. A ranging energy optimization problem as well as a practical algorithm have been proposed. The effectiveness of this algorithm has been illustrated by numerical experiments.

7. REFERENCES

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