Overview of Spectrum Sensing for Cognitive Radio

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***Abstract*—We present a survey of state-of-the-art algorithms for spectrum sensing in cognitive radio. The algorithms discussed range from energy detection to sophisticated feature detectors. The feature detectors that we present all have in common that they exploit some known structure of the transmitted signal. In particular we treat detectors that exploit cyclostationarity properties of the signal, and detectors that exploit a known eigenvalue structure of the signal covariance matrix. We also consider cooperative detection. Specifically we present data fusion rules for soft and hard combining, and discuss the energy efficiency of several different sensing, sleeping and censoring schemes in detail.**

***Index Terms*—spectrum sensing, cognitive radio, signal detection**

I. Introduction

Spectrum is a scarce resource, and licensed spectrum is intended to be used only by the spectrum owners. Cognitive radio is a new concept of reusing licensed spectrum in an unlicensed manner [1], [2]. The motivation for cognitive radio is various measurements of spectrum utilization, that show unused resources in frequency, time and space [3], [4]. The introduction of cognitive radios will inevitably create increased interference and thus degrade the quality of service of the primary system. The impact on the primary system, for example in terms of increased interference, must be kept at a minimal level. To keep the impact at an acceptable level, secondary users must sense the spectrum to detect whether it is available or not. Secondary users must be able to detect very weak primary user signals [5], [6], [7]. Spectrum sensing is one of the most essential components of cognitive radio.

In the following we will present some topics in spectrum sensing for cognitive radio, that have been of great interest in recent research. We will highlight some fundamental problems and present techniques for signal detection.

II. Model

As a preliminary, we set up the model for signal detection. Assume that $\mathbf{y}$ is a received vector of length N, that consists of a signal plus noise. That is

$$\mathbf{y} = \mathbf{x} + \mathbf{w},$$

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where $\mathbf{x}$ is a signal vector, and $\mathbf{w}$ is a noise vector. The noise $\mathbf{w}$ is assumed to be i.i.d. zero-mean complex Gaussian with variance σ^2. That is, $\mathbf{w} \sim \mathcal{C}N(\mathbf{0}, \sigma^2\mathbf{I})$.

We wish to detect whether there is a signal present or not. That is, we want to discriminate between the following two hypotheses:

$$\begin{aligned} H_0 &: \mathbf{y} = \mathbf{w}, \\ H_1 &: \mathbf{y} = \mathbf{x} + \mathbf{w}. \end{aligned} \tag{1}$$

The optimal (uniformly most powerful) Neyman-Pearson test is to compare the log-likelihood ratio to a threshold. That is

$$\Lambda \triangleq \log\left(\frac{P(\mathbf{y}|H_1)}{P(\mathbf{y}|H_0)}\right) \underset{H_0}{\overset{H_1}{\gtrless}} \eta.$$

Clearly, the log-likelihood ratio depends on the distribution of the signal to be detected.

III. Energy Detection

Initially we will present one of the simplest signal models, for which the optimal detector is the energy detector [8]. We assume that the signal to be detected does not have any known structure that could be used for detection. Thus, we assume that the signal is also zero-mean circularly symmetric complex Gaussian $\mathbf{x} \sim \mathcal{C}N(\mathbf{0}, \gamma^2\mathbf{I})$. Then, under H_0, $\mathbf{y}|H_0 \sim \mathcal{C}N(\mathbf{0}, \sigma^2\mathbf{I})$ and under H_1, $\mathbf{y}|H_0 \sim \mathcal{C}N(\mathbf{0}, (\sigma^2 + \gamma^2)\mathbf{I})$. The log-likelihood ratio is

$$\log\left(\frac{P(\mathbf{y}|H_1)}{P(\mathbf{y}|H_0)}\right) = \log\left(\frac{\frac{1}{\pi^N(\sigma^2+\gamma^2)^N}\exp(-\frac{\|\mathbf{y}\|^2}{\sigma^2+\gamma^2})}{\frac{1}{\pi^N\sigma^{2N}}\exp(-\frac{\|\mathbf{y}\|^2}{\sigma^2})}\right).$$

By removing all constants that are independent of the received vector $\mathbf{y}$, we obtain the optimal Neyman-Pearson test

$$\Lambda_{\text{e}} \triangleq \|\mathbf{y}\|^2 = \sum_{i=0}^{N-1} |y_i|^2 \underset{H_0}{\overset{H_1}{\gtrless}} \eta_{\text{e}}. \tag{2}$$

Hence, the optimal detector, in the Neyman-Pearson sense, is in this case the energy detector also known as radiometer [8]. In essence the energy detector measures the received energy during a finite time interval, and compares it to a predetermined threshold. The performance of the energy detector is well known, cf. [9], and can be written in closed form. The probability of false alarm P_{FA} is given by

$$P_{\text{FA}} \triangleq \Pr(\Lambda_{\text{e}} > \eta_{\text{e}}|H_0) = 1 - F_{\chi^2_{2N}}\left(\frac{2\eta_{\text{e}}}{\sigma^2}\right), \tag{3}$$

where $F_{\chi^2_{2N}}(\cdot)$ denotes the cumulative distribution function of a χ^2-distributed random variable with $2N$ degrees of freedom.

Thus, given a false alarm probability, we can derive the threshold η from

$$\eta_{\mathrm{e}} = F^{-1}_{\chi^2_{2N}}\left(1 - P_{\mathrm{FA}}\right)\frac{\sigma^2}{2}. \quad (4)$$

The probability of detection is given by

$$P_{\mathrm{D}} \triangleq \Pr\left(\Lambda_{\mathrm{e}} > \eta_{\mathrm{e}} | H_1\right) = 1 - F_{\chi^2_{2N}}\left(\frac{2\eta_{\mathrm{e}}}{\sigma^2 + \gamma^2}\right). \quad (5)$$

The energy detector is universal in the sense that it can detect any type of signal, and does not require any knowledge about the signal to be detected. On the other hand, for the same reason it does not exploit any potentially available knowledge about the signal. Moreover, the noise power needs to be known to set the decision threshold (4).

IV. Fundamental Limits on Detection

Cognitive radios must be able to detect very weak primary user signals [5]. However, there are some fundamental limits for detection in low SNR. For example, to set the decision threshold of the energy detector (4), the noise variance σ^2 must be known. If the knowledge of the noise variance is imperfect, clearly the threshold will be erroneous. It is well known that the performance of the energy detector quickly deteriorates if the noise variance is imperfectly known (cf. [6], [10]). Due to uncertainties in the model assumptions, robust detection is impossible below a certain SNR level, known as the SNR wall [10], [11]. It was shown in [10] that errors in the noise power assumption introduces SNR walls to any moment-based detector. This was further extended in [11] to any model uncertainties, such as assuming perfect white and stationary noise, flat fading, ideal filters and infinite precision A/D converters. These results hold for detectors with imperfect assumptions. However, it is possible to circumvent, or at least mitigate the problem of SNR walls by taking the imperfections into account. For example, it was shown in [11] that noise calibration improves the detector robustness. Exploiting some known features of the signal to be detected can also improve the detector performance and robustness.

V. Feature Detection

If the signal to be detected is perfectly known, the optimal detector is a matched filter (cf. [9]). In practice the signal is never perfectly known, but there is some knowledge about the signal. It is usually known what kind of primary users that are to be detected, and the transmitted signals are to some extent determined by standards and regulations. Thus, some features of the signal to be detected are usually known. In the following, we will describe some detectors exploiting known features of the signal, both to improve performance and to circumvent the problem of model uncertainties, for example imperfectly known noise variance.

A. Cyclostationarity

Most man-made signals show periodic patterns related to symbol rate, chip rate, channel code or cyclic prefix, that can be appropriately modeled as a cyclostationary random process [12]. A discrete-time zero-mean stochastic process y_t is said to be second-order cyclostationary if its time-varying covariance function $R(t,T) \triangleq E\left[y_t y^*_{t+T}\right]$ is periodic in t (cf. [13], [12]). Hence, $R(t,T)$ can be expressed by a Fourier series

$$R(t,T) = \sum_{\alpha} R^{\alpha}_y(T)e^{j\alpha t}, \quad (6)$$

where the sum is over integer multiples of fundamental frequencies and their sums and differences. The Fourier coefficients depend on the time lag T and are given by

$$R^{\alpha}_y(T) = \frac{1}{T_0}\sum_{t=0}^{T_0-1} R(t,T)e^{-j\alpha t}. \quad (7)$$

The Fourier coefficients $R^{\alpha}_y(T)$ are also known as the *cyclic covariance* with cyclic frequency α. The process y_t is said to be cyclostationary if there exists an α such that $R^{\alpha}_y(T) > 0$. The *cyclic spectrum* of the signal $\mathbf{y}$ is the Fourier coefficient

$$S_y(\alpha,\omega) = \sum_{T} R^{\alpha}_y(T)e^{-j\omega T}.$$

The cyclic spectrum is the density of correlation for cyclic frequency α. Knowing some of these cyclic characteristics of a signal, one can construct detectors that exploit the cyclostationarity of the signal [13], [14], [15], [16], [17] and benefit from the spectral correlation.

There has been a huge interest in detection of OFDM signals recently. One reason is that many of the current and future technologies for wireless communication, such as WiFi, WiMAX, LTE and DVB-T, use OFDM signalling. Therefore it is reasonable to assume that cognitive radios must be able to detect OFDM signals. Another reason is that OFDM signals exhibit well known spectral correlation properties [18]. The IEEE 802.22 WRAN standard is intended for cognitive radio-based reuse of spectrum that is allocated to digital TV broadcasts. Cyclostationarity-based detectors for detection of the OFDM-based digital TV-signals for the IEEE 802.22 WRAN standard were proposed e.g. in [19], [20]. Another cyclostationary-based detector of OFDM-signals based on multiple cyclic frequencies was proposed in [21]. We will return to the detection of OFDM signals in Section V-B.

B. Autocorrelation

Many communication signals contain redundancy, introduced for example to facilitate synchronization, by channel coding or to circumvent intersymbol interference. This redundancy occurs as non-zero average autocorrelation at some time lag T. For example, consider an OFDM signal with a cyclic prefix of length N_c and informative data of length N_d. Then, the average autocorrelation of the OFDM signal is non-zero at time lag N_d, owing to the fact that some of the data is repeated in the cyclic prefix of each OFDM symbol. In the sequel of this section we assume that the signal is an OFDM signal, although the described detectors are valid for all signals that show a non-zero average autocorrelation at some known time lag. Assume that the signal $\mathbf{y}$ contains $N \triangleq K(N_c+N_d)+N_d$

samples. As a preparation, let

$$r_i \triangleq y_i^* y_{i+N_d},\ i = 0, \ldots, K(N_c + N_d) - 1. \tag{8}$$

Furthermore, we know that if $E[r_i] \neq 0$, then $E\left[r_{i+k(N_c+N_d)}\right] \neq 0$, $k = 1, \ldots, K-1$, and analogously if $E[r_i] = 0$, then $E\left[r_{i+k(N_c+N_d)}\right] = 0$. That is, r_i and $r_{i+k(N_c+N_d)}$ will have identical statistics, and be independent (since the noise and signals are independent). Thus, we define

$$R_i \triangleq \frac{1}{K} \sum_{k=0}^{K-1} r_{i+k(N_c+N_d)},\ i = 0, \ldots, N_c + N_d - 1.$$

A detector that exploits the autocorrelation property of OFDM signals was proposed in [22]. The detector of [22] uses the test statistic

$$\max_{\tau \in \{0,\ldots,N_c+N_d-1\}} \left| \sum_{i=\tau}^{\tau+N_c-1} r_i \right|. \tag{9}$$

The variable τ can be viewed as the synchronization mismatch, or equivalently the time when the first sample is observed. The statistic (9) only takes one OFDM symbol at a time into account. A slight generalization of this test statistic, that uses the whole signal and not only one symbol, is to sum the variables R_i instead of r_i. Then, the test is

$$\Lambda_{\max \mathrm{ac}} \triangleq \max_{\tau \in \{0,\ldots,N_c+N_d-1\}} \left| \sum_{i=\tau}^{\tau+N_c-1} R_i \right| \underset{H_0}{\overset{H_1}{\gtrless}} \eta_{\max \mathrm{ac}}.$$

Another autocorrelation-based detector was proposed in [23]. This detector uses the empirical mean of the autocorrelation normalized by the received power, as test statistic. More precisely, the test proposed in [23] is

$$\Lambda_{\overline{\mathrm{ac}}} \triangleq \frac{\frac{1}{N-N_d} \sum_{i=0}^{N-N_d-1} \mathrm{Re}(r_i)}{\frac{1}{N} \sum_{i=0}^{N-1} |y_i|^2} \underset{H_0}{\overset{H_1}{\gtrless}} \eta_{\overline{\mathrm{ac}}}.$$

The detector proposed in [22] requires knowledge about the noise variance to set the decision threshold, while the detector proposed in [23] does not require any knowledge about the noise variance.

C. Covariance Matrix Eigenvalues

In this section, we will explain methods that use the eigenvalue structure of the signal covariance matrix. The concept of using the covariance matrix eigenvalues for detection is currently an ongoing research area. There are several variations on this theme, and we describe a selection of recent methods in order to convey the key concepts behind the approach.

Assume that the signal $\mathbf{x}$ is zero-mean Gaussian. More specifically $\mathbf{x} \sim \mathcal{C}N(\mathbf{0}, \mathbf{R}_x)$, where $\mathbf{R}_x$ is the $N \times N$ covariance matrix of $\mathbf{x}$. Then, the hypothesis test (1) can be written

$$\begin{aligned} H_0 &: \mathbf{y} \sim \mathcal{C}N\left(\mathbf{0}, \sigma^2 \mathbf{I}\right), \\ H_1 &: \mathbf{y} \sim \mathcal{C}N\left(\mathbf{0}, \mathbf{R}_x + \sigma^2 \mathbf{I}\right). \end{aligned} \tag{10}$$

Let $\mathbf{R}_y$ be the covariance matrix of the received vector $\mathbf{y}$, $\mathbf{R}_y \triangleq E\left[\mathbf{y}\mathbf{y}^H\right]$. Typically $\mathbf{x}$ is correlated, so that $\mathbf{R}_x$ has large eigenvalue spread. This is the case for example in a typical MIMO system [24], or for an OFDM signal. Then, under H_0, all eigenvalues of $\mathbf{R}_y$ are equal to σ^2. However, under H_1 the eigenvalues of $\mathbf{R}_y$ are equal to $\delta_i + \sigma^2$, $i = 0, \ldots, N-1$, where δ_i are the eigenvalues of $\mathbf{R}_x$. Thus, if we sort the eigenvalues of $\mathbf{R}_y$ in descending order, there will be a significant difference between the "smallest" eigenvalues and the "largest" ones. Detectors exploiting the eigenvalue spread of the signal covariance matrix were proposed e.g. in [24], [25], [28], and will be briefly described in the following.

Consider K vectors $\mathbf{y}[k]$ received in a sequence. Define the sample covariance matrix

$$\widehat{\mathbf{R}_y} \triangleq \frac{1}{K} \sum_{k=0}^{K-1} \mathbf{y}[k]\mathbf{y}[k]^H.$$

Let λ_i, $i = 0, \ldots, N-1$, be the eigenvalues of $\widehat{\mathbf{R}_y}$. There were two eigenvalue-based detectors proposed in [24], that uses the test statistics

$$\frac{\max_i \lambda_i}{\min_j \lambda_j} \tag{11}$$

and

$$\frac{\frac{1}{N} \sum_{i=0}^{N-1} \lambda_i}{\min_i \lambda_i} \tag{12}$$

respectively. These eigenvalue-based detectors were shown to perform well when the signal to be detected is highly correlated.

A similar test statistic was recently proposed in [25]:

$$\frac{\max_i \lambda_i}{\frac{1}{N} \sum_{i=0}^{N-1} \lambda_i}. \tag{13}$$

It was shown in [25] that, under some circumstances, the detector (13) outperforms the detector (11) in terms of asymptotic error exponents. Similar tests were recently also proposed in [26], [27].

It was shown in [28], that the generalized likelihood ratio (GLR) for the hypothesis test (10) when all parameters (σ^2 and $\mathbf{R}_x$) are completely unknown, so that $\mathbf{R}_y$ under H_1 is treated as an unstructured and unknown positive definite matrix, is

$$\frac{\frac{1}{N} \sum_{i=0}^{N-1} \lambda_i}{\left(\prod_{i=0}^{N-1} \lambda_i\right)^{1/N}}. \tag{14}$$

This test is equivalent to the sphericity test of [29]. The sphericity test of [29] tests if the covariance matrix of a multivariate normal distribution is proportional to the identity matrix, or equivalently if all the eigenvalues of the sample covariance matrix are equal or not. The GLR detector (14) and the max/min-ratio detector (11) were compared in [28]. Simulations of a MIMO system where the number of transmit antennas was larger than the number of receive antennas, and the signal was assumed to be Gaussian, showed that the max/min-ratio (11) performs almost as well as the GLR (14).

VI. Cooperative Detection

One way of reducing the receiver sensitivity requirements is by using cooperative sensing. The concept of cooperative sensing is to use multiple sensors and combine their measurements to one common decision. This is in essence a way of getting diversity gains.

A. Soft Combining

Assume that there are M sensors. Then, the hypothesis test (1) becomes

$$H_0: \mathbf{y}_m = \mathbf{w}_m,\ m = 0, \ldots, M-1,$$
$$H_1: \mathbf{y}_m = \mathbf{x}_m + \mathbf{w}_m,\ m = 0, \ldots, M-1.$$

Assume that the received signals at all sensors are independent. Let $\mathbf{z} = \left(\mathbf{y}_0^T \mathbf{y}_1^T \ldots \mathbf{y}_{M-1}^T\right)^T$. Then, the log-likelihood ratio is

$$\begin{aligned}\Lambda_{\text{coop}} &\triangleq \log\left(\frac{P(\mathbf{z}|H_1)}{P(\mathbf{z}|H_0)}\right) = \log\left(\prod_{m=0}^{M-1} \frac{P(\mathbf{y}_m|H_1)}{P(\mathbf{y}_m|H_0)}\right) \\ &= \sum_{m=0}^{M-1} \log\left(\frac{P(\mathbf{y}_m|H_1)}{P(\mathbf{y}_m|H_0)}\right) = \sum_{m=0}^{M-1} \Lambda^{(m)},\end{aligned} \tag{15}$$

where $\Lambda^{(m)} \triangleq \log\left(\frac{P(\mathbf{y}_m|H_1)}{P(\mathbf{y}_m|H_0)}\right)$ is the log-likelihood ratio for the mth sensor. That is, if the received signals for all sensors are independent, the optimal fusion rule is to sum the log-likelihood ratios.

Consider the case when the noise vectors $\mathbf{w}_m$ are independent $\mathbf{w}_m \sim \mathcal{C}N(\mathbf{0}, \sigma_m^2 \mathbf{I})$, and the signal vectors $\mathbf{x}_m$ are independent $\mathbf{x}_m \sim \mathcal{C}N(\mathbf{0}, \gamma_m^2 \mathbf{I})$. Then, the log-likelihood ratio (15) is written

$$\Lambda_{\text{ce}} = \sum_{m=0}^{M-1} \log\left(\frac{\frac{1}{\pi^N(\sigma_m^2+\gamma_m^2)^N} \exp(-\frac{\|\mathbf{y}_m\|^2}{\sigma_m^2+\gamma_m^2})}{\frac{1}{\pi^N \sigma_m^{2N}} \exp(-\frac{\|\mathbf{y}_m\|^2}{\sigma_m^2})}\right).$$

Removing all constants that are independent of $\mathbf{z}$ yields

$$\Lambda_{\text{ce}} = \sum_{m=0}^{M-1} \|\mathbf{y}_m\|^2 \frac{\gamma_m^2}{\sigma_m^2\left(\sigma_m^2+\gamma_m^2\right)}. \tag{16}$$

The statistic $\|\mathbf{y}_m\|^2$ is the soft decision from an energy detector at the mth sensor, as shown in (2). Thus, the optimal cooperative detection scheme is to use energy detection for the individual sensors, and combine the soft decisions by the weighted sum (16). This result was also shown in [30], for the case when $\sigma_m^2 = 1$, and thus γ_m^2 is equivalent to the SNR experienced by the mth sensor. Clearly, if both the noise power and signal power are equal for all sensors, we can ignore the weight factor and just sum the soft decisions. The cooperative gain under that assumption was analyzed in [31]. It was shown in [31], that correlation between the sensors severely decreases the cooperation gain. The main source of correlation between users is shadow fading. Multipath fading is uncorrelated at very small distances, on the scale of half a wavelength, and can easily be avoided. Hence the correlation is mainly distance dependent, and the cooperation gains are limited by the distance separation of the cognitive users. From a detection perspective a large distance separation between users is desired. However, if cognitive users should be able to communicate without disturbing the primary system they must be sufficiently near to one another. Thus, there is a distance trade off between detection performance and cognitive communication. The effect of untrusted users was also analyzed. The conclusion of [31] is that if one out of M sensors is untrustworthy, the sensitivity of each individual sensor must be as good as that achieved with M trusted users.

B. Hard Combining

So far we have considered optimal cooperative detection. That is, all users transmit soft decisions to a fusion center, which combines the soft values to one common decision. This is equivalent to the case where the fusion center has access to the received data for all sensors, and performs optimal detection based on all data. This requires potentially a huge amount of data to be transmitted to the fusion center. The other extreme case of cooperative detection is that each sensor takes its own decision, and transmits only a binary value to the fusion center. Then, the fusion center combines the hard decisions to one common decision.

In the following we will describe the AND, OR, and voting rules (cf. [32]) for combining of hard decisions. Assume that the individual statistics $\Lambda^{(m)}$ are quantized to one bit, such that $\Lambda^{(m)} = 0, 1$ is the hard decision from the mth sensor. Here, 1 means that a signal is detected and 0 means that the channel is available.

The **AND** rule decides that a signal is detected if *all* sensors have detected a signal. That is, the cooperative test using the AND rule decides on H_1 if

$$\sum_{m=0}^{M-1} \Lambda^{(m)} = M.$$

The **OR** rule decides on signal presence if *any* of the sensors reports signal detection. Hence, for the OR rule the cooperative test decides on H_1 if

$$\sum_{m=0}^{M-1} \Lambda^{(m)} \geq 1.$$

Finally, the **voting** rule decides that a signal is present if at least V of the M sensors have detected a signal, for $1 \leq V \leq M$. The test decides on H_1 if

$$\sum_{m=0}^{M-1} \Lambda^{(m)} \geq V.$$

Taking a majority decision is a special case of the voting rule, for $V = M/2$. The AND-logic and the OR-logic are clearly also special cases of the voting rule for $V = M$ and $V = 1$ respectively.

C. Energy Efficiency

As the number of cooperating users grows, the energy consumption of the cognitive radio network increases, but the performance generally saturates. Hence, techniques have been developed to improve the energy efficiency in cognitive radio networks. Throughout this subsection, we will assume that the different cognitive radios take conditionally independent observations, and we will use $P_{\rm D}$ and $P_{\rm FA}$ to denote the global probability of detection and false alarm, respectively.

A first simple technique to save energy is on-off sensing or *sleeping*, where every cognitive radio will randomly turn off its sensing device with a probability μ, the sleeping rate. This can be applied in many different settings and we will come back to it later on.

Another popular approach is *censoring* [33]. In such a system a cognitive radio m will only send a sensing result if it is deemed informative, and it will censor those sensing results that are uninformative. In [34], it has been shown that the optimal local decision rule under a global communication constraint is a censored local log-likelihood ratio $\Lambda^{(m)}$ where the censoring region consists of a single interval. More specifically, a radio will not send anything when $\eta_1^{(m)} \leq \Lambda^{(m)} < \eta_2^{(m)}$ and it will send $\Lambda^{(m)}$ otherwise. Note that in the Bayesian framework, optimality is in terms of the global probability of error $P_{\rm E} = Pr(H_0)P_{\rm FA} + Pr(H_1)(1-P_{\rm D})$, and the communication rate constraint is

$$\text{Pr}(H_0)\sum_{m=0}^{M-1}\text{Pr}(\Lambda^{(m)} \text{ is sent } |H_0) + \text{Pr}(H_1)\sum_{m=0}^{M-1}\text{Pr}(\Lambda^{(m)} \text{ is sent } |H_1) \leq \kappa. \quad (17)$$

Whereas in the Neyman-Pearson framework, optimality is in terms of the global probability of detection $P_{\rm D}$ subject to a global probability of false alarm constraint $P_{\rm FA} \leq \alpha$, and the communication rate constraint is

$$\sum_{m=0}^{M-1}\text{Pr}(\Lambda^{(m)} \text{ is sent } |H_0) \leq \kappa.$$

Furthermore, it has been proven in [34] that if the communication rate constraint κ is sufficiently small and either $\text{Pr}(H_1)$ (in the Bayesian framework) or the probability of false alarm constraint α (in the Neyman-Pearson framework) is small enough, then the optimal lower threshold $\eta_1^{(m)}$ is given by $\eta_1^{(m)} = 0$. For the Neyman-Pearson framework, this result has been generalized in [35] to a communication rate constraint *per radio*:

$$\text{Pr}(\Lambda^{(m)} \text{ is sent } |H_0) \leq \kappa_m,$$

in which case the upper threshold $\eta_2^{(m)}$ can be directly determined from κ_m and no joint optimization of the set of upper thresholds $\{\eta_2^{(m)}\}_{m=0}^{M-1}$ is required.

Next to communication rate constraints, it is also possible to consider the global cost of sensing and transmission, which is given by

$$C = \sum_{m=0}^{M-1} C_{{\rm s},m} + C_{{\rm t},m}\text{Pr}(\Lambda^{(m)} \text{ is sent}), \quad (18)$$

where $C_{{\rm s},m}$ and $C_{{\rm t},m}$ are respectively the cost of sensing and transmission for cognitive radio m. Note that both these costs can be different for every cognitive radio, since they could be using different hardware or software for sensing and transmission, and they might have different distances to the fusion center, meaning that they will have to use different transmission powers. Under a constraint on C, it can again be shown that the optimal local decision rule is a censored local log-likelihood ratio $\Lambda^{(m)}$ where the censoring region consists of a single interval, and this for the Bayesian as well as the Neyman-Pearson framework [36]. Furthermore, even if a digital transmission is considered, the optimal local decision rule is a quantized local log-likelihood ratio $\Lambda^{(m)}$, where every quantization level corresponds to a single interval and where one of the quantization levels is censored [36].

In [37], the above censoring approach has been adopted for energy detection and binary digital transmission. In other words, the local decision is based on the locally collected energy $\Lambda_{\rm e}^{(m)} = \|\mathbf{y}_m\|^2$, and the radio will not send anything when $\eta_{{\rm e},1}^{(m)} \leq \Lambda_{\rm e}^{(m)} < \eta_{{\rm e},2}^{(m)}$, and it will send a 0 when $\Lambda_{\rm e}^{(m)} < \eta_{{\rm e},1}^{(m)}$ and a 1 when $\Lambda_{\rm e}^{(m)} \geq \eta_{{\rm e},2}^{(m)}$. Assuming that the SNR γ_m^2/σ_m^2 is the same for all radios, and the OR combining rule is used, [37] then gives expressions for the communication rate, and the global probabilities of false alarm and detection. In [38] and [39], the idea of [37] has been combined with sleeping and the global cost of sensing and transmission has been optimized w.r.t. the sleeping rate μ and the thresholds $\eta_{{\rm e},1}^{(m)}$ and $\eta_{{\rm e},2}^{(m)}$, subject to a global probability of false alarm constraint $P_{\rm FA} \leq \alpha$ and a global probability of detection constraint $P_{\rm D} \geq \beta$. Note that the considered global cost of sensing and transmission is now given by (18) multiplied by $1-\mu$. The first interesting result from [38] and [39] is that the optimal lower threshold is again given by $\eta_{{\rm e},1}^{(m)} = 0$ if the feasible set is not empty. In [38], it is assumed that the system is highly underutilized, i.e., $\text{Pr}(H_0) \gg \text{Pr}(H_1)$ (similar results hold for $\text{Pr}(H_0) \ll \text{Pr}(H_1)$), whereas in [39], the more general case has been treated. Note that although [38] and [39] have considered the same thresholds $\eta_{{\rm e},1}^{(m)}$ and $\eta_{{\rm e},2}^{(m)}$ for all radios (optimal when all the SNRs γ_m^2/σ_m^2, sensing costs $C_{{\rm s},m}$, and transmission costs $C_{{\rm t},m}$ are the same), it can be generalized to the case they are not the same. To conclude, the results show that the optimal sleeping rate and censoring probabilities grow with the number of cognitive radios, but the censoring probabilities saturate fairly rapidly. Hence, the thresholds $\eta_{{\rm e},2}^{(m)}$ can be designed independent of the number of cognitive radios.

VII. Concluding Remarks

Due to space limitations, there are several lines of work that we were unable to treat in this survey. If the transmitted signals have an unknown structure, blind detection algorithms based on information theoretic criteria (Akaike, MDL) can be used [40]. If the spectral properties of the signal to be detected are known, but the signal has otherwise no usable features that can be efficiently exploited, then filterbank-based detectors may be preferable [41]. One special class of filterbanks that has certain optimality properties is that consisting of the Slepian windows [1], [42].

Finally, even though many of the spectrum sensing methods have similar performance, they may differ significantly in terms of implementation complexity. Much effort is currently being made to develop implementation-friendly algorithms and evaluate them in practical scenarios, see for example [43].

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