JOINT DYNAMIC RESOURCE ALLOCATION AND WAVEFORM ADAPTATION IN COGNITIVE RADIO NETWORKS

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ABSTRACT

This paper discusses the issue of dynamic resource allocation (DRA) in the context of cognitive radio (CR) networks. We present a general framework adopting generalized transmitter and receiver signal-expansion functions, which allow us to join DRA with waveform adaptation, two procedures that are currently carried out separately. Moreover, the proposed DRA can handle many types of expansion functions or even combinations of different types of functions. An iterative game approach is adopted to perform multi-player DRA, and the best-response strategies of players are derived and characterized using convex optimization. To reduce the implementation costs of having too many active expansion functions after optimization, we also propose to combine DRA with sparsity constraints for dynamic function selection. Generally, it incurs little rate-performance loss since the effective resources required by a CR are in fact sparse.

***Index Terms*—** cognitive radio, dynamic resource allocation, game theory, waveform adaptation, sparsity

1. INTRODUCTION

In cognitive networks, radios dynamically decide the allocation of available radio resources to improve the overall utilization efficiency, also known as dynamic resource allocation (DRA) [1]. In view of the competitive nature of a resource-constrained multi-user system, DRA can be carried out in a distributed fashion using multi-player games [2, 3]. In that case, every radio will iteratively sense the available resources and adjust its own usage of these resources accordingly. Such resources could for instance be represented by transmitter and receiver signal-expansion functions, which can be judiciously chosen to enable various radio platforms, such as frequency, time, or code division multiplexing (FDM, TDM, CDM). While an OFDM platform with carriers has been generally considered for CRs [4], many platforms are TDM- or CDM-based and make use of different types of expansion functions such as pulses, codes, or wavelets.

This paper presents a general DRA approach that allows us to handle and even combine all kinds of expansion functions. This signal expansion framework also enables us to combine DRA with waveform adaptation [7, 8], two operations that are generally carried out separately. The joint treatment is essential in implementing practical iterative DRA games that require dynamic interference sensing.

Finally, when some fixed type of expansion functions is used to represent the dynamic resource opportunities in the wideband regime, the optimal number of active expansion functions could be huge, leading to highly complex iterative games with slow convergence. To solve this problem, we will incorporate some proper sparsity constraints in the DRA games to limit the number of active expansion functions. The dynamic function selection strategy, coupled with a set of redundant expansion functions as the selection base, may reduce the implementation costs at little performance loss, with improved convergence in the iterative games. Simulation results are provided to illustrate the proposed techniques.

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2. PER-USER TRANSMISSION MODEL

Consider a wireless network with Q active CR users seeking radio resources, where each CR refers to a pair of one transmitter and one receiver. In this paper, the resources are represented by means of a set of transmitter and receiver functions/filters $\{\psi_k(t)\}_{k=0}^{K-1}$ and $\{\phi_k(t)\}_{k=0}^{K-1}$. Through block transmission, CR q transmits a $K \times 1$ coded data vector $\mathbf{u}_q = \mathbf{F}_q \mathbf{s}_q$ in each block, where $\mathbf{s}_q$ consists of K *i.i.d.* information symbols $\{s_{q,k}\}_{k=0}^{K-1}$, and $\mathbf{F}_q$ is a square linear precoding matrix. Inter-block interference (IBI) can for instance be avoided by the use of a cyclic prefix, as we will illustrate later. The data $u_{q,k}$ are communicated using the transmitter expansion function $\psi_k(t)$, yielding the transmitted waveform $u_q(t) = \sum_k u_{q,k}\psi_k(t)$.

The CR sends $u_q(t)$ over a dispersive channel with impulse response $g_q(t)$, and preprocesses it at the receiver using the receiver functions $\{\phi_k(t)\}$ to collect a block of K data samples $\mathbf{x}_q := [x_{q,0}, \ldots, x_{q,K-1}]^T$. Each CR pair is assumed to be synchronized, but different CRs do not have to be synchronized among one another. Hence, the discrete-time input-output relationship is

$$\mathbf{x}_q = \mathbf{H}_q \mathbf{u}_q + \mathbf{v}_q \tag{1}$$

where $\mathbf{H}_q$ is the $K \times K$ aggregate channel matrix with its (k, l)-th element given by $h_{q,k,l} := \int g_q(t) \star \psi_k(t) \star \phi_l^*(-t)dt$, $\forall k, l$, and $\mathbf{v}_q$ is the $K \times 1$ filtered noise vector with $v_{q,l} = \int v_q(t) \star \phi_l^*(-t)dt$.

The above setup incorporates FDM, TDM, as well as CDM scenarios. For example, in a baseband digital implementation, the set of transmitter and receiver functions can be chosen as:

$$\psi_k(t) = \frac{1}{\sqrt{K+N}} \sum_{n=0}^{K+N-1} c_{k,<n-N>_K}\, p(t - nT), \tag{2a}$$

$$\phi_l(t) = \frac{1}{\sqrt{K}} \sum_{n=N}^{K+N-1} c_{k,n-N}\, p(t - nT), \tag{2b}$$

where $\{c_{k,n}\}_{k,n}$ represent the digital modulation/demodulation coefficients, $< n >_K$ denotes the remainder after dividing n by K, and $p(t)$ is the normalized pulse used at the DAC and ADC. It is assumed that $p(t)$ has a span $[0, T)$ and an essential bandwidth $[-B/2, B/2)$ ($B = 1/T$). Here, the considered range is $t \in [0, (K + N)T)$, where NT is an upper bound on the length of any channel $g_q(t)$. We have assumed that the transmit functions $\psi_k(t)$ include a cyclic prefix of length NT, and that the receive functions $\phi_k(t)$ remove this cyclic prefix. That way IBI is avoided. In FDM, we could take $c_{k,n} = e^{j2\pi kn/K}$, whereas in TDM, we could take $c_{k,n} = \sqrt{K}\delta_{k-n}$. Note that the above described FDM scheme actually corresponds to OFDM (orthogonal frequency division multiplexing), whereas the TDM scheme actually corresponds to SCCP (single carrier with a cyclic prefix). CDM covers intermediate schemes, where the functions $\{\psi_k(t)\}_k$ and $\{\phi_k(t)\}_k$ could for example be related to spreading codes or wavelets. Also combinations of the above waveforms can be considered, as we will discuss later on.

DRA in a CR network concerns the spectrum utilization efficiency, measured for instance by the system capacity. DRA can be carried out through the linear precoder $\mathbf{F}_q$ and a length-K amplitude scaling vector $\mathbf{a}_q$, whose k-th element is $a_{q,k} = \sqrt{E(|s_{q,k}|^2)}$. Given the signal expansion structure and assuming uncorrelated interferences on the different waveforms $\{\phi_k(t)\}_k$, the per-user capacity formula is given by

$$C(\mathbf{a}_q, \mathbf{F}_q) = \frac{1}{K}\log_2\left|\mathbf{I}_K + \text{diag}(\mathbf{a}_q)\mathbf{F}_q^H\mathbf{B}_q\mathbf{F}_q\text{diag}(\mathbf{a}_q)\right| \quad (3)$$

where $\mathbf{B}_q := \mathbf{H}_q^H\mathbf{R}_q^{-1}\mathbf{H}_q$ and $\mathbf{R}_q := E(\mathbf{v}_q\mathbf{v}_q^H)$ is the interference covariance matrix. We have omitted the impact of data detection, since the MMSE receiver is known to be capacity-preserving [9]. We assume that the knowledge of $\mathbf{H}_q$ and $\mathbf{R}_q$, and thus $\mathbf{B}_q$, is known to CR q, using some channel and interference estimation techniques [9].

In CR applications, the spectral shapes of transmitted waveforms need to comply with some design and regulatory requirements. For CR q, the PSD of the transmitted signal $u_q(t)$ is given by

$$S_q(f;\mathbf{a}_q,\mathbf{F}_q) = \textstyle\sum_{k=0}^{K-1} a_{q,k}^2 \left|\sum_{i=0}^{K-1}[\mathbf{F}_q]_{i,k}\ \psi_i(f)\right|^2. \quad (4)$$

Hence, the average power constraint can be expressed as

$$\int S_q(f;\mathbf{a}_q,\mathbf{F}_q)df = tr\Big(\text{diag}(\mathbf{a}_q)\mathbf{F}_q^H\mathbf{S}_\psi\mathbf{F}_q\text{diag}(\mathbf{a}_q)\Big) \le P_{q,\max} \quad (5)$$

where $P_{q,\max}$ is the power upper limit, $\mathbf{S}_\psi$ is the $K \times K$ matrix with $[\mathbf{S}_\psi]_{k,l} = \int \psi_k(f)\psi_l^*(f)df$, and $tr(\cdot)$ denotes trace.

We further impose a cognitive spectral mask $S_c(f)$ that accounts for government spectral regulations and cognition-based frequency notch masks for interference control. The spectral mask constraint is

$$S_q(f;\mathbf{a}_q,\mathbf{F}_q) \le S_c(f), \qquad \forall f, q. \quad (6)$$

3. MULTI-USER DRA GAMES

From a global network perspective, the objective of DRA is to determine the collective actions $\{(\mathbf{a}_q,\mathbf{F}_q)\}_{q=0}^{Q-1}$ that maximize the sum-rate of all users, that is,

$$\max_{\{\mathbf{a}_q\succeq\mathbf{0}\}_q,\{\mathbf{F}_q\}_q} \sum_{q=0}^{Q-1} C(\mathbf{a}_q,\mathbf{F}_q), \qquad s.t.\ (5),(6),\ \forall q. \quad (7)$$

Here we do not optimize the expansion functions, but nevertheless jointly adapt the waveforms $S_q(f;\mathbf{a}_q,\mathbf{F}_q)$ during DRA on $(\mathbf{a}_q,\mathbf{F}_q)$.

However, the formulation (7) leads to a centralized non-convex optimization problem with NP-hard complexity [3]. Furthermore, it requires knowledge of all the channel information $\{\mathbf{R}_q,\mathbf{H}_q\}_q$, which can be infeasible to obtain even for a central spectrum controller such as a base station. For any-to-any connections, it is more appropriate to perform decentralized DRA, for which the game-theoretic approach is well motivated due to its distributed nature.

3.1. Distributed Game Formulation and Implementation

In a DRA game, CRs are game players, each of which seeks to maximize a capacity-related utility function by taking allocation actions on $(\mathbf{a}_q,\mathbf{F}_q)$ from its own set of permissible strategies. A standard non-cooperative game can be formulated as

$$\max_{\mathbf{a}_q\succeq\mathbf{0},\mathbf{F}_q} C(\mathbf{a}_q,\mathbf{F}_q), \qquad s.t.\ (5),(6). \quad (8)$$

On a per-user basis, (8) presents a decentralized DRA formulation for deciding $(\mathbf{a}_q,\mathbf{F}_q)$, without knowledge of other users' allocation $\{(\mathbf{a}_r,\mathbf{F}_r)\}_{r\neq q}$. Nevertheless, the interference $\mathbf{R}_q$ needs to be sensed, while the sensing task needs to be carried out when other users are transmitting using their allocation $\{(\mathbf{a}_r,\mathbf{F}_r)\}_{r\neq q}$. The intricacy among sensing, transmission and distributed DRA suggests a repeated game approach, in which players repeat the optimization in (8) multiple rounds, until reaching steady-state DRA decisions, if existent. Steps in an iterative game are summarized below.

- S1) In current round, choose the order for CRs to take actions, in a sequential, simultaneous, or asynchronous fashion [11];
- S2) For CR q that is in order to take its action,
  - (a) senses the channel and interference $\mathbf{H}_q$ and $\mathbf{R}_q$,
  - (b) finds the current best response strategy $(\mathbf{a}_q^\star,\mathbf{F}_q^\star)$ that optimizes (8), which we will elaborate in Section 4,
  - (c) adapts its transmission by implementing $(\mathbf{a}_q^\star,\mathbf{F}_q^\star)$ on the signal expansion functions;
- S3) iterate to the next round, until convergence.

This game procedure joins the two tasks of DRA optimization (in S2(b)) and online waveform adaptation (in S2(c)), thanks to the enabling signal expansion framework we adopt. By doing so, it is feasible for CRs to perform dynamic sensing of the *aggregate interference* (in S2(a)). In contrast, existing DRA literature mainly focuses on direct optimization of the power spectrum $S_q(f)$ based on proper spectrum efficiency criteria [2, 3], while the waveform design literature investigates analog or digital pulse shaping techniques to comply with the allocated power spectrum $S_q(f)$ [7, 8]. The separate approach to DRA and waveform design requires that each CR estimates and exchanges the information of *all the received interfering channels* (with the exception of OFDM systems which is a special case herein), incurring considerable overhead in signaling, communication and computation. Our DRA approach offers a truly distributed framework in which the allocation actions are optimized under a practical transmitter implementation structure.

3.2. Game Characterization

In game theory, it is essential to characterize the properties of steady state Nash Equilibriums (NEs) in order to assess the game outcomes. Relevant issues include the existence, optimality, uniqueness of NEs, and whether a game implementation converges to the NEs.

The noncooperative game in (8) can be shown as a convex optimization problem. The utility is continuously quasi-concave, while the action space defined by the power and mask constraints is a non-empty compact convex set. As such, the Glicksberg-Fan fixed point theorem ensures the *existence* of NEs by pure strategies [11].

In Section 4, we will show that the matrix-valued problem in (8) can be transformed into a vector-valued convex problem with a diagonalized channel structure. As such, it resembles a standard OFDM-based game based on orthogonal carriers. For synchronous OFDM systems that appear in DSL applications, sufficient conditions for uniqueness have been delineated under the power and mask constraints [4, 6]. For the mask-constrained asynchronous case we consider here, the *uniqueness* of the NE is still an open problem [5].

4. BEST RESPONSES IN DRA GAMES

This section solves for the best responses to the per-user optimization problem in (8). The characteristic of the best response strategies affects the NE properties of the corresponding game.

4.1. Convex Formulation and Water-filling Interpretation

To deal with the pulse-shaping autocorrelation matrix $\mathbf{S}_\psi$ in (5), we define $\bar{\mathbf{F}}_q := \mathbf{\Lambda}_s^{1/2}\mathbf{U}_s^H\mathbf{F}_q$, where $\mathbf{U}_s$ and $\mathbf{\Lambda}_s$ are the eigenvector and eigenvalue matrices of $\mathbf{S}_\psi$ respectively. Rewriting the power constraint as $tr(\text{diag}(\mathbf{a}_q)\bar{\mathbf{F}}_q^H\bar{\mathbf{F}}_q\text{diag}(\mathbf{a}_q)) \leq P_{q,\max}$, and using the Hadamard inequality, we deduce that the determinant in (3) is maximized when $\bar{\mathbf{F}}_q$ diagonalizes $\bar{\mathbf{B}}_q := \mathbf{\Lambda}_s^{-1/2}\mathbf{U}_s^H\mathbf{B}_q\mathbf{U}_s\mathbf{\Lambda}_s^{-1/2}$. Let $\mathbf{U}_q$ and $\mathbf{\Lambda}_q$ denote the eigenvector and eigenvalue matrices of $\bar{\mathbf{B}}_q$, respectively. This suggests setting $\bar{\mathbf{F}}_q = \mathbf{U}_q$ and $\mathbf{F}_q := \mathbf{U}_s\mathbf{\Lambda}_s^{1/2}\mathbf{U}_q$, which yields

$$C(\mathbf{a}_q) := \max_{\mathbf{F}_q} C(\mathbf{a}_q, \mathbf{F}_q) = \frac{1}{K}\log_2\left|\mathbf{I}_K + \mathbf{\Lambda}_q\text{diag}(\mathbf{a}_q)^2\right|$$
$$= \tfrac{1}{K}\textstyle\sum_{k=0}^{K-1}\log_2(1 + a_{q,k}^2\lambda_{q,k}), \quad \lambda_{q,k} := [\mathbf{\Lambda}_q]_{k,k}. \quad (9)$$

Let us define an $K \times 1$ power loading vector $\mathbf{p}_q : p_{q,k} := a_{q,k}^2$. Since $\mathbf{F}_q^H\mathbf{S}_\psi\mathbf{F}_q = \bar{\mathbf{F}}_q^H\bar{\mathbf{F}}_q = \mathbf{I}$, the power constraint in (5) becomes

$$\mathbf{1}^T\mathbf{p}_q \leq P_{q,\max}. \quad (10)$$

Meanwhile, the transmitted PSD in (4) can be re-written as $S_q(f;\mathbf{p}_q) = \mathbf{z}_q^T(f)\mathbf{p}_q$, where $[\mathbf{z}_q(f)]_k := \left|\sum_{i=0}^{K-1}[\mathbf{F}_q]_{i,k}\psi_i(f)\right|^2$. To render the number of spectral mask constraints finite, we sample $S_q(f;\mathbf{p}_q)$ uniformly in frequency at N points $F_N := \{f_1,\ldots,f_N\}$, $N \geq K$, and replace (6) by

$$\mathbf{Z}_{q,N}^T\mathbf{p}_q \leq \mathbf{S}_{c,N}, \qquad \mathbf{S}_{c,N} := [S_c(f_1),\ldots,S_c(f_N)]^T, \quad (11)$$

where $\mathbf{Z}_{q,N} := [\mathbf{z}_q(f_1),\ldots,\mathbf{z}_q(f_N)]$ is an $N \times K$ matrix.

The per-user DRA optimization problem in (8) can now be simplified by (9), (10) and (11) into

$$\max_{\mathbf{p}_q \succeq \mathbf{0}} \quad C(\mathbf{p}_q) = \frac{1}{K}\sum_{k=0}^{K-1}\log_2(1 + p_{q,k}\lambda_{q,k}) \quad (12a)$$
$$s.t. \quad \mathbf{1}^T\mathbf{p}_q \leq P_{q,\max} \quad (12b)$$
$$\mathbf{p}_q \leq \mathbf{p}_{q,\max}, \quad \mathbf{p}_{q,\max} := \mathbf{Z}_{q,N}^\dagger\mathbf{S}_{c,N}. \quad (12c)$$

Interestingly, the matrix-valued DRA problem in (8) with respect to $(\mathbf{a}_q, \mathbf{F}_q)$ has been reformulated into a vector-valued problem in (12) with respect to $\mathbf{p}_q$. The best response to (12) is the well-known water-filling scheme [2, 4, 6], that is,

$$\mathbf{F}_q^\star = \mathbf{U}_s\mathbf{\Lambda}_s^{1/2}\mathbf{U}_q, \qquad \mathbf{p}_q^\star : p_{q,k}^\star = \left[\mu_q - \lambda_{q,k}^{-1}\right]_0^{p_{q,\max}(k)} \quad (13)$$

where $[x]_a^b$ is the Euclidean projection of x onto $[a,b]$ and the water level μ_q is chosen to satisfy $\sum_{k=0}^{K-1}p_{q,k} = P_{q,\max}$ as in (12b) [4].

Remark 1: In [4], a noncooperative game is proposed using linear precoding strategies under both power and spectral mask constraints. The schemes therein require block-by-block synchronization among users, whereas this paper obviates this assumption. In [9], a filter-bank structure is proposed for transceiver optimization, whereas our expansion functions are not confined to be mutually orthogonal. The capability in subsuming various types of expansion functions offers added flexibility in re-shaping the dispersive channels. Even though these expansion functions are fixed in this paper for simplicity, we do allow for redundant non-orthogonal functions to explore the sparsity property in CR networks, as discussed next.

4.2. Reduced-cost DRA under Sparsity Constraints

In order to represent the optimal transmitted PSD over a very wide band, the required number of expansion functions can generally be very large. On the other hand, the effective resources needed for a CR to transmit reliably are in fact sparse compared with the total available resources in the wideband network. This observation suggests that (near-)optimal DRA may be carried out over a few selected expansion functions. Function selection also reduces hardware costs.

4.2.1. Function Selection Formulation

Suppose that each CR transmits data over M expansion functions, $M < K$. Functions are selected via a selection matrix $\mathbf{J}_q = \text{diag}(\mathbf{j}_q)$ where $\mathbf{j}_q \in \{0,1\}^K$ is a $K \times 1$ sequence indicating whether $\psi_{q,k}(t)$ is selected ('1') or not ('0'). Removing those all-zero columns in $\mathbf{J}_q$, we get $\tilde{\mathbf{J}}_q$ of size $K \times M$. DRA are performed on the M selected functions, via an $M \times 1$ loading vector $\tilde{\mathbf{a}}_q = \tilde{\mathbf{J}}_q^H\mathbf{a}_q$ and an $M \times M$ precoder $\tilde{\mathbf{F}}_q = \tilde{\mathbf{J}}_q^H\mathbf{F}_q\tilde{\mathbf{J}}_q$. The aggregated channel effect is captured in $\tilde{\mathbf{B}}_q = \tilde{\mathbf{J}}_q^H\mathbf{B}_q\tilde{\mathbf{J}}_q$, and the capacity formula in (3) is modified to

$$C(\mathbf{a}_q,\mathbf{F}_q,\mathbf{J}_q) = \frac{1}{M}\log_2\left|\mathbf{I}_M + \text{diag}(\tilde{\mathbf{a}}_q)\tilde{\mathbf{F}}_q^H\tilde{\mathbf{B}}_q\tilde{\mathbf{F}}_q\text{diag}(\tilde{\mathbf{a}}_q)\right|. \quad (14)$$

Replacing the utility function in (8) by (14), we reach a DRA game with dynamic function selection. It resembles the antenna selection problem in MIMO literature [12, 13], which is known to be NP-hard due to its combinatorial optimization nature. To bypass the difficulty, we resort to a suboptimal two-stage approach to separately treat the filter selection problem and the DRA problem, as follows.

- s1) decide $\mathbf{j}_q$ using fast antenna selection schemes, e.g., [12, 13];
- s2) find the best response to (14) on the selected filters, using the formulation in (12) and the solution in (13).

These two stages are carried out in Step S2(b) in Section 3.

4.2.2. Sparsity-Constrained DRA Formulation

Selecting M functions is equivalent to setting $(K - M)$ elements of the allocation vector $\mathbf{a}_q$ to be zeros, that is, $||\mathbf{a}_q||_0 = M$. When $M \ll K$, $\mathbf{a}_q$ becomes a sparse vector, which can be treated under the framework of compressive sampling [10]. A general-form sparsity measure of $\mathbf{a}_q$ is its l-norm $||\mathbf{a}_q||_l$, where $0 \leq l < 2$. We now tackle the function selection problem by introducing sparsity constraints.

In the absence of linear precoding, function selection boils down to limiting the l-norm of $\mathbf{a}_q$ by an upper bound $L_{q,\max}^{(l)}$. Adding this sparsity constraint to (8), we merge DRA with function selection:

$$\begin{aligned} \max_{\mathbf{a}_q \succeq \mathbf{0}} \quad & C(\mathbf{a}_q, \mathbf{F}_q = \mathbf{I}_K) \\ s.t. \quad & (5), (6); \\ & ||\mathbf{a}_q||_l \leq L_{q,\max}^{(l)}. \end{aligned} \quad (15)$$

When $l = 0$, the parameter $L_{q,\max}^{(0)}$ directly reflects the number of functions selected, but (15) is nonconvex and difficult to solve. When $l \in [1, 2)$, (15) is a convex problem that permits well-behaved numerical algorithms. However, the parameter $L_{q,\max}^{(l)}$ is more difficult to choose in order to produce exact sparsity.

When linear precoding is present ($\mathbf{F}_q \neq \mathbf{I}_K$), we note from (9) that the linear precoder $\bar{\mathbf{F}}_q$ serves to diagonalize the channel $\bar{\mathbf{B}}_q$, while the ensuing power loading in (12) is determined by the channel eigenvalues $\mathbf{\Lambda}_q$. This observation suggests to perform function selection by finding a primary minor channel matrix $\tilde{\mathbf{B}}_q$ with the best eigenvalue quality measured by $|\mathbf{I}_M + \tilde{\mathbf{B}}_q| = \left|\mathbf{I}_K + \mathbf{J}_q^H\bar{\mathbf{B}}_q\mathbf{J}_q\right|$, i.e.,

$$\max_{\mathbf{j}_q \succeq \mathbf{0}} \quad \log_2 \left| \mathbf{I}_K + \mathbf{J}_q^H \bar{\mathbf{B}}_q \mathbf{J}_q \right| \tag{16a}$$

$$s.t. \quad ||\mathbf{j}_q||_l \leq L_{q,\max}^{(l)} \tag{16b}$$

$$j_{p,k}^2 - j_{p,k} = 0, \quad k = 0, \dots, K-1 \tag{16c}$$

$$\mathbf{e}_k^T \mathbf{J}_q \mathbf{e}_k = j_{q,k}, \quad k = 0, \dots, K-1 \tag{16d}$$

$$\mathbf{e}_k^T \mathbf{J}_q \mathbf{e}_l = 0, \qquad \forall k \neq l \tag{16e}$$

where $\mathbf{e}_k$ denotes the k-th column of the identity matrix $\mathbf{I}_K$, $\forall k$. Here, (16c) is imposed to enforce $j_{q,k} \in \{0, 1\}$, and (16d) and (16e) are used together to express the relationship $\mathbf{J}_q = \text{diag}(\mathbf{j}_q)$ as the intersection of a set of convex functions in $\mathbf{j}_q$ and $\mathbf{J}_q$. Relaxing l to be $l = 1$, (16) becomes a convex problem that obviates undesired combinatorial search. The optimized selection decision $\mathbf{j}_q$ replaces the first stage s1) in Section 4.2.1. Afterwards, DRA is carried out on the M functions using (14), in which the linear precoder $\tilde{\mathbf{B}}_q$ can diagonalize the channel to simplify (14) to a water-filling scheme.

Remark 2: It is interesting to observe that our DRA problem based on the signal expansion framework resembles the multiuser MIMO problems. Expansion functions play the roles of transmit and receive antennas, corroborated by the capacity formula (3) that applies to both problems. Hence, the literature on multiuser MIMO can benefit our work and from our work as well. Nevertheless, our design focus is to perform efficient resource allocation rather than harvesting antenna diversity and multiplexing gains. As such, we may use a large number of expansion functions to induce redundancy in resource representation, followed by dynamic function selection to allocate resources efficiently at reduced implementation costs. In this sense, our theme departs from that of multiuser MIMO problems. Besides, channel estimation is an easier task in our problem, allowing possibly compressed sensing at reduced sampling rates.

5. SIMULATIONS

Consider a Q-user peer-to-peer CR network. Each channel link experiences frequency-selective fading modeled by an N_t-tap tapped delay line, where each tap coefficient is complex Gaussian with zero-mean and unit variance. The link power gain is denoted by a scalar $\rho_{rq} > 0, \forall r, q \in [1, Q]$, which captures both the path loss and the fading power. Subsequently, the interference covariance matrix $\mathbf{R}_q$ is given by the covariance of the aggregated interference (from all the $Q-1$ received interference channels) plus noise. We set $Q = 4$, $P_{q,\max} = 20$ (with reference to unit noise variance), $\rho_{qq} = 1, \forall q$, and $\rho_{rq} = 8$ for $\forall r \neq q$. FDM subcarriers are used as the transmitter and receiver functions, with $K = 32$ subcarriers.

To demonstrate the inherent sparsity in the CR context, we compare three DRA techniques: *i*) DRA via (8) without function selection; *ii*) DRA via (14) performed over a fixed selection of $L_{q,\max}^{(0)} = M$ functions; and, *iii*) DRA with dynamic function selection via (15) under sparsity constraints on the l-norm, for $l = 0.2$ and $l = 1$.

Fig. 1(a) depicts the sum capacity of all users versus the sparsity parameter $L_{q,\max}^{(l)}$, averaged over 100 sets of channel realizations. When the sparsity constraints in *ii*) and *iii*) are loose, all DRA designs converge to the same $C(\mathbf{p}_q^\star)$ of the sparsity-unconstrained design *i*), indicated by the rightmost region in Fig. 1(a). In design *ii*), as M decreases, the sparsity constraints becomes tighter, and the resulting capacity exhibits a noticeable gap from that of *i*). This indicates that a fixed function selection cannot efficient allocate resources in dynamic channel environments. In contrast, the sparsity-constrained DRA designs in *iii*) result in average capacities close to that in *i*), because *iii*) optimally selects active expansion functions where the effective resources lie on dynamically.

In terms of the sparsity metric, the l_0-norm constraint directly controls the exact sparsity, but incurs combinatorial computational load. When l is close to 0, e.g., $l = 0.2$, the optimal solution closely approximates a sparse representation. However, $l < 1$ results in a nonlinear concave constraint and is thus subject to convergence issues. For $l \geq 1$, the sparsity constraint becomes convex. However, as l increases, the resulting allocation tends to be less sparse. The assessment on the average complexity is corroborated in Fig. 1(b).

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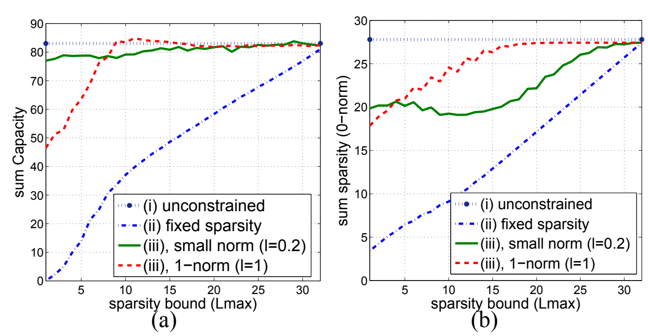

Fig. 1. Sparsity-constrained DRA: (a) average sum capacity, (b) average sum complexity measured by l_0-norm.