ADAPTIVE FEEDBACK REDUCTION FOR PRECODED SPATIAL MULTIPLEXING MIMO SYSTEMS

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ABSTRACT

In this paper we extend feedback compression schemes for precoded spatial multiplexing multiple-input multiple-output (MIMO) systems to adapt to changing channel characteristics. The presented schemes use the past feedback to estimate the conditional probability of a precoder occurrence given the last feedback at runtime. These probabilities are then used to losslessly compress the feedback.

1. INTRODUCTION

In the last years precoded spatial multiplexing [1] emerged as a promising scheme to provide high data rates. Further, it makes spatial multiplexing more robust to rank deficient channels, and it allows for simpler receiver architectures.

Determining the linear precoding matrix requires channel state information (CSI), which is in general just available at the receiver. Instead of quantizing and feeding back the full CSI, it is more efficient [2] to calculate the precoder matrix at the receiver side, and then to feedback the quantized precoder matrix.

The available data-rate on the limited feedback link can be better exploited by taking the temporal correlation into account. The temporal correlation between the quantized precoders is modeled as a Markov chain in [3]. The feedback rate is reduced by not sending feedback if the actual state, i.e., the actual precoder, is identical to the previous state. The transition to a different state is encoded using a fixed-length code. This approach is extended in [4] where the least probable precoders are ignored, which leads to a further reduction of the feedback rate since a shorter fixed-length code can now be used to encode the states. However, ignoring these low-probability states also leads to a performance degradation. Another approach is presented in [5] where data compression techniques are used to losslessly compress the feedback. The selected compression scheme depends if we consider a dedicated feedback link or a non-dedicated one. In [6] another lossless method is presented where the selected codebook depends on the previous feedback.

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In this paper, we modify the approach of [5] to include an adaptation mechanism to changing channel characteristics.

Notation: Vectors are designated with lowercase boldface letters, and matrices with capital boldface letters. Further, $\mathbf{A}^H$ denotes the conjugate transpose of the matrix $\mathbf{A}$, and $\mathbf{A}^{-1}$ the inverse. The cardinal number of the set $\mathcal{A}$ is denoted $|\mathcal{A}|$. $\|\mathbf{A}\|$ is the Frobenius norm of $\mathbf{A}$. The matrix $\bar{\mathbf{A}}$ consists of the first N_S columns of the matrix $\mathbf{A}$. $\mathcal{U}_{m\times n}$ denotes the set of unitary $m \times n$ matrices. Finally, $E(\cdot)$ represents expectation, and $P(\cdot)$ probability.

2. SYSTEM MODEL

We assume a narrowband linear precoded spatial multiplexing MIMO system, with N_T transmit and N_R receive antennas. The system transmits $N_S \leq \min(N_T, N_R)$ symbol streams. The input-output relation is

$$\mathbf{y} = \mathbf{HFs} + \boldsymbol{\nu}, \quad (1)$$

where $\mathbf{y} \in \mathbb{C}^{N_R\times 1}$ is the received vector, $\mathbf{H} \in \mathbb{C}^{N_R\times N_T}$ the channel matrix, $\mathbf{F} \in \mathbb{C}^{N_T\times N_S}$ the precoder matrix, $\mathbf{s} \in \mathbb{C}^{N_S\times 1}$ the data symbol vector, and $\boldsymbol{\nu} \in \mathbb{C}^{N_R\times 1}$ the noise vector. The elements of the noise vector $\boldsymbol{\nu}$ are i.i.d. and complex Gaussian distributed with zero mean and variance 1. The elements of the data symbol vector $\mathbf{s}$ are from an alphabet $\mathcal{A}$ with zero mean and variance 1. The elements of the channel matrix $\mathbf{H}$ are i.i.d. and complex Gaussian distributed with zero mean and variance P, and every element is distributed according to Jakes' model [7]. In order to reduce the feedback [8], the precoder $\mathbf{F}$ is limited to be unitary, $\mathbf{F} \in \mathbf{U}_{N_T\times N_S}$. Thus, the signal-to-noise ratio (SNR) per transmit antenna is given by P.

We write the singular value decomposition (SVD) of the channel $\mathbf{H}$ as $\mathbf{H} = \mathbf{U\Sigma V}^H$. For several performance criteria [8] the optimal precoder $\mathbf{F}$ consists of the first N_S columns of the right unitary matrix $\mathbf{V}$ calculated by the SVD of the channel matrix, thus $\mathbf{F} = \bar{\mathbf{V}}$. Even through a closed form expression of the BER optimal precoder is still unknown, it has been shown in [9] that, depending on the system parameters, the optimal precoder is either $\bar{\mathbf{V}}$ or $\bar{\mathbf{V}}\mathbf{M}$, where $\mathbf{M}$ is a unitary matrix with constant modulus entries, e.g., the Hadamard or the DFT matrix.

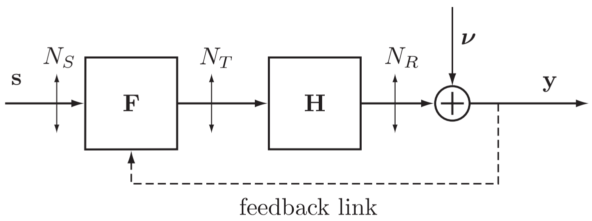

Fig. 1. System model.

We assume that the data is transmitted in frames, and that the receiver can perfectly estimate the channel at the start of each frame. The optimal precoder for this channel is calculated, quantized, and sent to the transmitter over the delay- and error-free feedback link. We distinguish between a dedicated and a non-dedicated feedback channel. The dedicated channel is only used for transmitting the feedback to the transmitter. The non-dedicated channel is also used to transmit data. In the latter case, the transmitter has to distinguish between when the codeword ends and when the data starts.

3. FEEDBACK QUANTIZATION

Due to the limited feedback rate the optimal precoder needs to be quantized before it can be fed back to the transmitter. The quantized version of the precoder is picked from a codebook $\mathcal{C}$ using a selection criterion.

3.1. Codebook Design

The codebook design consists of finding the K codebook entries $\mathbf{F}_i$ and the corresponding channel regions $\mathcal{R}_i$ which minimize the average distortion measure d over all possible channels

$$\mathcal{C} = \underset{\{\mathcal{R}_i,\mathbf{F}_i\}|\mathcal{R}_i\subset\mathbb{C}^{N_R\times N_T},\mathbf{F}_i\in\mathcal{U}_{N_T\times N_S}}{\arg\min} \sum_i E(d(\mathbf{H},\mathbf{F}_i|\mathbf{H}\in\mathcal{R}_i))P(\mathbf{H}\in\mathcal{R}_i). \quad (2)$$

Unfortunately, there exists no close form solution to find such codebooks. They are, in general, found with iterative algorithms, e.g., the generalized Lloyd algorithm [10].

Lately, there have been a large number of possible distortion measures presented, e.g., based on subspace distances [8], matrix norms [11], capacity loss [12], or BER [9]. However, simulations in [6] show that choosing a good selection criterion is more important for the overall performance than the type of distortion measure.

3.2. Selection Criteria

The selection criterion is used to select the best codebook entry based on the optimal precoder. Usually, the same distance is used for the distortion measure and for the selection criterion. The optimal selection criterion is the BER selection criterion [9], which picks the precoder that minimizes the BER for the effective channel $\mathbf{HF}$.

4. FEEDBACK REDUCTION

There exist two major methods to reduce the average feedback rate in time correlated channels without sacrificing performance. Both methods exploit the correlation of consecutive precoders due to the time correlation of the channel.

The first method, presented in [6], selects the precoder codebook based on the previous feedback. This method requires the storage of a large number of codebooks at the transmitter and receiver. The extension of this method to changing channel characteristics is currently under investigation and not the scope of this paper.

The second method, called the bitword approach [5], uses source encoding methods with the transition probabilities between the past feedback and the actual feedback to compress the feedback. We present an extension of this method to adapt it to changing channel characteristics.

4.1. Bitword Approach

This method [5] uses the same precoder codebook $\mathcal{C}$ at every time instant, but it adapts the bitword $w_{i,j}$ which represents the current precoder $\mathbf{F}$ being $\mathbf{F}_i$, assuming the previous precoder $\mathbf{F}'$ was $\mathbf{F}_j$. The transition probabilities are denoted as $P_{i,j} = P(\mathbf{F} = \mathbf{F}_i|\mathbf{F}' = \mathbf{F}_j)$. The bitwords are selected to minimize

$$\sum_{i=1}^{K} l(w_{i,j})P_{i,j}, \quad (3)$$

with $l(w_{i,j})$ being the length of the bitword $w_{i,j}$. The selected encoding scheme depends on the feedback link. If we consider a non-dedicated feedback link, then the selected code must be prefix-free, or the receiver cannot decide where the bitword ends and where the data starts. A simple example of such a code is the Huffman code. For the dedicated channel, however, the code can be non-prefix-free (NPF), i.e., it does not need to satisfy the prefix condition, since the end of the transmission also means the end of the codeword. An example for a prefix-free and a NPF code can be seen in Table 1. The performance of the Huffman code strongly depends on an accurate knowledge of the transition probabilities, unlike the NPF codes, where the performance just depends on the knowledge of the correct order of the transition probabilities.

4.2. Adaptive Encoding

In general, the underlying channel characteristics, e.g., the Doppler spread, are unknown. This makes an exact determination of the transition probabilities, and thus of the precoder

Codebook	$P_{i,8}$	Huffman Code	NPF Code
$\mathbf{F}_8$	0.25	01	/
$\mathbf{F}_2$	0.20	11	0
$\mathbf{F}_7$	0.18	000	1
$\mathbf{F}_4$	0.16	001	00
$\mathbf{F}_3$	0.10	101	01
$\mathbf{F}_6$	0.08	1000	10
$\mathbf{F}_5$	0.02	10010	11
$\mathbf{F}_1$	0.01	10011	000

Table 1. Bitwords for a non-dedicated and a dedicated feedback link for the case that $\mathbf{F}' = \mathbf{F}_8$.

encoding, troublesome. A solution is to start the algorithm assuming that all the precoders are equiprobable and then to adapt the transition probabilities, and thus the encoding, at runtime. A simple approach to adapt the transition probabilities for Huffman encoding was presented in [10]. Assuming the actual precoder is $\mathbf{F}_k$ and the previous precoder is $\mathbf{F}_j$, then the transition probabilities at time instant t, denoted $P_{i,j}[t]$, are updated based on the actual precoder $\mathbf{F}_k$ and the previous transition probabilities $P_{i,j}[t-1]$ at time instant $t-1$ as

$$P_{k,j}[t] = \frac{(N-1)P_{k,j}[t-1]+1}{N} \quad (4)$$

$$P_{i,j}[t] = \frac{(N-1)P_{i,j}[t-1]}{N} \quad \text{for} \quad i \neq k \quad (5)$$

The factor N regulates how fast or how accurate the transition probabilities adapt. This trade-off between accuracy and speed is treated in more detail in Section 5. This adaptive estimation of the transition probabilities is done at both the transmitter and receiver side. Then, the transition probabilities are used with the selected code, e.g., Huffman code, to encode the feedback.

Please note that there exists several low-complexity implementations of the adaptive Huffman code, e.g., the Vitter's algorithm [13] or the FGK algorithm [14]. However, since the encoding scheme of the bitword is just updated once per frame, the complexity reduction achieved by using these algorithms is negligible.

5. SIMULATIONS

We consider a spatial multiplexing MIMO system where 2 data streams $N_S = 2$ are transmitted over 2 antennas $N_T = 2$. The number of receive antennas does not have an influence on the feedback rate, and can thus be neglected. The transmission is precoded with the unitary matrix $\mathbf{F} \in \mathcal{U}_{N_T \times N_S}$ from a codebook $\mathcal{C}$, with $|\mathcal{C}| = 16$ entries, and then transmitted over $N_T = 2$ antennas. The codebook is designed using the generalized Lloyd algorithm with the squared modified Frobenius norm distance as distortion measure [11], and the same distance is used as selection criterion. Note that the mentioned

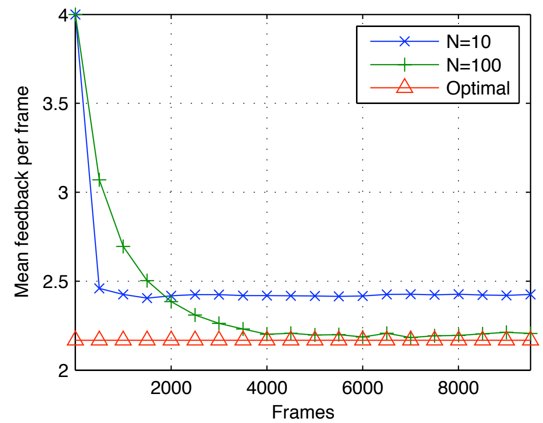

Fig. 2. Mean feedback rate per frame on a non-dedicated channel. $N_T = N_S = 2$, $|\mathcal{C}| = 16$, $f_D = 30Hz$, and $T_f = 10^{-2}s$.

schemes also work with different codebooks and different selection schemes.

We first consider a non-dedicated feedback channel. The frame rate is fixed at $T_f = 10^{-2}s$, and the Huffman code is used to compress the feedback. The algorithm is initialized with equal transition probabilities $P_{i,j}[0] = 1/|\mathcal{C}|$ for $\forall i, j$, which then adapt at runtime. The mean feedback per frame is depicted in Fig. 2 . We see how the selection of the parameter N influences the choice between having a fast and having an accurate system.

The next simulation, with the results depicted in Fig. 3, shows the performance of the same system with a dedicated feedback link. Here the systems adapts faster than in the previous simulation, even with a small N. A simple NPF code as in Table 1 does not require the exact transition probabilities, but just the correct order of the transition probabilities, i.e., from the highest probable precoder to the least probable precoder. This is why the choice of N does not have a big influence on the mean feedback rate.

6. CONCLUSIONS

In this paper we extended feedback compression schemes for precoded spatial multiplexing MIMO systems to adapt to unknown channel characteristics. We depicted the trade-off between the adaptation speed and adaption accuracy for dedicated and non-dedicated channels through simulations.

7. REFERENCES

[1] Anna Scaglione, Petre Stoica, Sergio Barbarossa, Georgios B. Giannakis, and Hemanth Sampath, "Optimal designs for space-time linear precoders and decoders,"

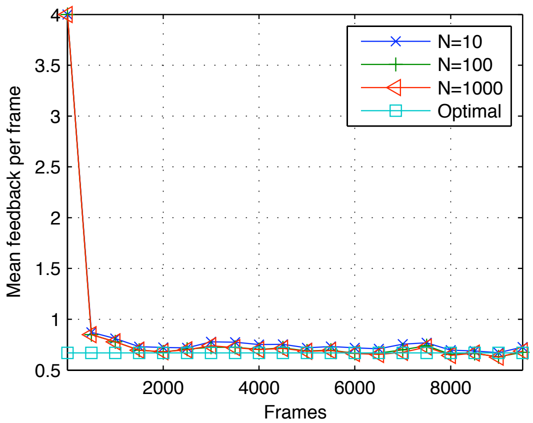

Fig. 3. Mean feedback rate per frame on a dedicated channel. $N_T = N_S = 2$, $|\mathcal{C}| = 16$, $f_D = 30Hz$, and $T_f = 10^{-2}s$.

IEEE Trans. Signal Processing, vol. 50, no. 5, pp. 1051–1064, May 2002.

[2] David J. Love and Robert W. Heath Jr., "Limited feedback precoding for spatial multiplexing systems using linear receivers," in *Proc. Military Communications Conference (MILCOM'03)*, Oct. 2003, vol. 1, pp. 627 – 632.

[3] Kaibin Huang, Bishwarup Mondal, Robert W. Heath Jr., and Jeffrey G. Andrews, "Markov models for limited feedback MIMO systems," in *Proc. IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP'06)*, May 2006, vol. 4, pp. IV–9–IV–12, unknown.

[4] Kaibin Huang, Bishwarup Mondal, Robert W. Heath Jr., and Jeffrey G. Andrews, "Multi-antenna limited feedback for temporally-correlated channels: Feedback compression," in *Proc. Global Telecommunications Conf. (GLOBECOM'06)*, Dec. 2006.

[5] Claude Simon and Geert Leus, "Feedback reduction for spatial multiplexing with linear precoding," in *Proc. 32nd International Conference on Acoustics, Speech, and Signal Processing (ICASSP'07)*, Honolulu, HI, USA, Apr. 2007, accepted for publication.

[6] Geert Leus and Claude Simon, "Quantized feedback and feedback reduction for precoded spatial multiplexing MIMO systems," in *Proc. International Symposium on Signal Processing and its Applications (ISSPA'07)*, Sharjah, U.A.E., Feb. 2007.

[7] William C. Jakes, Jr., *Microwave Mobile Communications*, John Wiley & Sons, 1974.

[8] David J. Love and Robert W. Heath Jr., "Limited feedback unitary precoding for spatial multiplexing systems," *IEEE Trans. Inform. Theory*, vol. 51, no. 8, pp. 2967–2976, Aug. 2005.

[9] Shengli Zhou and Baosheng Li, "BER criterion and codebook construction for finite-rate precoded spatial multiplexing with linear receivers," *IEEE Trans. Signal Processing*, vol. 54, no. 5, pp. 1653–1665, May 2006.

[10] Allen Gersho and Robert M. Gray, *Vector Quantization and Signal Compressing*, Kluwer Academic Publishers, 1995.

[11] Nadia Khaled, Bishwarup Mondal, Robert W. Heath Jr., Geert Leus, and Frederik Petré, "Quantized multi-mode precoding for spatial multiplexing MIMO-OFDM systems," in *Proc. IEEE 62nd Vehicular Technology Conference (VTC'05)*, Sept. 2005, vol. 2, pp. 867–871.

[12] June Chul Roh and Bhaskar D. Rao, "Design and analysis of MIMO spatial multiplexing systems with quantized feedback," *IEEE Trans. Signal Processing*, vol. 54, no. 8, pp. 2874–2886, Aug. 2006.

[13] Jeffrey Scott Vitter, "Design and analysis of dynamic Huffman codes," *Journal of the ACM*, vol. 4, no. 34, pp. 825–845, Oct. 1987.

[14] Donald E. Knuth, "Dynamic Huffman coding," *J. Algorithms*, vol. 6, no. 2, pp. 163–180, 1985.