LOW-COMPLEXITY BLOCK TURBO EQUALIZATION FOR OFDM SYSTEMS IN TIME-VARYING CHANNELS

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ABSTRACT

We propose a low-complexity block turbo equalizer for orthogonal frequency-division multiplexing (OFDM) systems in time-varying channels. The complexity of the proposed algorithm is linear in the number of subcarriers by exploiting the band structure of the frequency-domain channel matrix. The presented block turbo equalizer is based on a soft minimum mean squared error (MMSE) block linear equalizer (BLE).

Index Terms— Turbo equalization, OFDM, intercarrier interference, time-varying channels

1. INTRODUCTION

OFDM is one of the most important modulation schemes for wireless communications. OFDM can eliminate intersymbol interference (ISI) introduced by a frequency-selective channel by turning it into a set of parallel frequency-flat channels, and therefore renders simple one-tap equalization for each subcarrier [1]. However, high mobility causes Doppler shifts which give rise to a time-selective or time-varying channel and destroy the orthogonality among subcarriers. The related intercarrier interference (ICI) severely degrades the performance of the one-tap equalizer.

Recently, several low-complexity equalization algorithms have been proposed to combat these time-varying distortions [4, 9, 6, 5, 7]. All these methods exploit the banded character of the frequency-domain channel matrix to reach a complexity that is only linear in the number of subcarriers. In addition, simple time-domain receiver windowing can be used to enforce the banded assumption and improve the performance of the equalizer [9, 7]. One of the most promising approaches is the iterative MMSE serial linear equalizer (SLE) [9]. This iterative approach is inspired by turbo equalization [2, 3], where soft information is used in an iterative fashion to improve the bit error rate (BER) performance. However, it has been shown that the first step of this approach, i.e., the non-iterative MMSE SLE, is outperformed by the non-iterative MMSE block linear equalizer (BLE) of [5, 7] when receiver windowing is adopted, although their complexities are comparable. Hence, it is expected that an iterative version of the MMSE BLE would also perform better than the iterative MMSE SLE in case a receiver window is present. Such a block turbo equalizer will be presented in this paper, and it will be compared with the serial turbo equalizer.

Note that we only consider uncoded OFDM systems in this work, but it is clear that the performance can be further improved by incorporating error correction codes.

Notation: We use upper (lower) bold face letters to denote matrices (column vectors). $(\cdot)^T$ and $(\cdot)^H$ represent transpose and complex conjugate transpose (Hermitian). $[\mathbf{A}]_{m,n}$ indicates the entry in the mth row and nth column of $\mathbf{A}$. We use the symbol $\circ$ to denote the Hadamard (element-wise) product. $\mathrm{E}(\cdot)$ stands for the statistical expectation. $\mathrm{diag}(\mathbf{a})$ is a diagonal matrix with the vector $\mathbf{a}$ on the diagonal. The covariance matrix is defined as $\mathrm{Cov}(\mathbf{x}, \mathbf{y}) = \mathrm{E}(\mathbf{x}\mathbf{y}^H) - \mathrm{E}(\mathbf{x})\mathrm{E}(\mathbf{y}^H)$. Finally, $\mathbf{I}_N$ denotes the $N \times N$ identity matrix and $\mathbf{F}$ denotes the unitary DFT matrix.

2. SYSTEM MODEL

We consider a single-user OFDM system with N subcarriers, over a channel that is time- and frequency-selective. We assume that the bits at the transmitter are grouped and mapped into complex symbols in an uncoded fashion. For simplicity, we only consider quaternary phase-shift keying (QPSK) with a symbol alphabet $\mathcal{B}$ as shown in Table 1. Extensions to other constellations are straightforward [3]. Assuming the

k	1	2	3	4
$\alpha_{k,1}, \alpha_{k,2}$	00	10	01	11
α_k	$\frac{(+1+i)}{\sqrt{2}}$	$\frac{(-1+i)}{\sqrt{2}}$	$\frac{(+1-i)}{\sqrt{2}}$	$\frac{(-1-i)}{\sqrt{2}}$

Table 1. QPSK symbol alphabet

channel delay spread is smaller than the OFDM cyclic prefix (CP) length L, we can focus on a single OFDM symbol. After removing the CP at the receiver, the input-output relation of the OFDM system can be expressed as

$$\mathbf{y}^{'} = \mathbf{H}^{'}\mathbf{F}^H\mathbf{s} + \mathbf{n}^{'} \quad (1)$$

where $\mathbf{y}^{'}$ and $\mathbf{n}^{'}$ are the $N \times 1$ received vector and noise vector, respectively, $\mathbf{H}^{'}$ is the $N \times N$ time-domain channel

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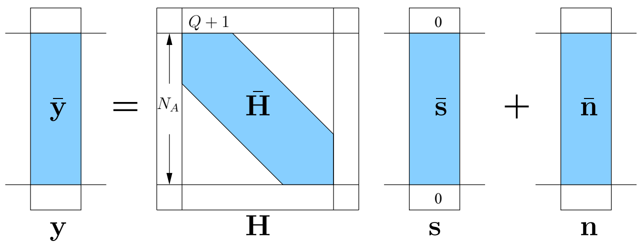

Fig. 1. System input-output relation after removing the guard intervals

matrix, and $\mathbf{s}$ is the $N \times 1$ OFDM symbol. For simplicity, we assume that $\mathbf{n}^{'}$ is a circularly symmetric zero-mean white complex Gaussian noise vector with covariance $E\{\mathbf{n}^{'}\mathbf{n}^{'H}\} = \sigma_n^2\mathbf{I}_N$.

Before the FFT operation at the receiver, a time-domain receiver window is often used to make the frequency-domain channel matrix more banded, thereby improving the equalization performance [9, 7]. In that case, the output after the FFT operation can be written as

$$\mathbf{y} = \mathbf{FWH}^{'}\mathbf{F}^H\mathbf{s} + \mathbf{FWn}^{'} = \mathbf{Hs} + \mathbf{n} \tag{2}$$

where $\mathbf{y} = \mathbf{FWy}^{'}$, $\mathbf{n} = \mathbf{FWn}^{'}$, $\mathbf{H} = \mathbf{FWH}^{'}\mathbf{F}^H$, and $\mathbf{W} = \mathrm{diag}(\mathbf{w})$, with $\mathbf{w}$ being the time-domain receiver window. Note that for classical OFDM (i.e., unwindowed), we have $\mathbf{W} = \mathbf{I}_N$.

When the channel is time-invariant, the time-domain channel matrix $\mathbf{H}^{'}$ is a circulant matrix and the frequency-domain channel matrix $\mathbf{H}$ (with $\mathbf{W} = \mathbf{I}_N$) is a diagonal matrix which makes the traditional simple OFDM one-tap equalizer possible. However, in a time-varying channel, the frequency-domain channel matrix $\mathbf{H}$ becomes a non-diagonal matrix giving rise to ICI. Fortunately, $\mathbf{H}$ (with $\mathbf{W} = \mathbf{I}_N$) is almost banded with the most significant elements around the main diagonal. This allows for low-complexity equalization architectures as proposed in [4, 9, 6, 5, 7]. With an appropriate window design $\mathbf{W}$, the banded character of $\mathbf{H} = \mathbf{FWH}^{'}\mathbf{F}^H$ can even be enforced, leading to an improved performance [9, 7].

As in [5, 7], we assume that the OFDM symbol $\mathbf{s}$ is constructed as $\mathbf{s} = [\mathbf{0}^T_{N_V/2\times 1}, \bar{\mathbf{s}}^T, \mathbf{0}^T_{N_V/2\times 1}]^T$, where the $N_V/2\times 1$ vectors $\mathbf{0}$ represent guard bands and the $N_A \times 1$ vector $\bar{\mathbf{s}}$ is the actual data vector (note that $N = N_A + N_V$). Moreover, we remove the first and last $N_V/2$ entries of $\mathbf{y}$ and only focus on the N_A middle entries. Hence, introducing the matrix $\mathbf{S} = [\mathbf{0}_{N_A\times N_V/2}, \mathbf{I}_{N_A}, \mathbf{0}_{N_A\times N_V/2}]$, which selects the $N_A \times 1$ middle block out of an $N \times 1$ vector, we transform (2) into

$$\bar{\mathbf{y}} = \bar{\mathbf{H}}\bar{\mathbf{s}} + \bar{\mathbf{n}} \tag{3}$$

where $\bar{\mathbf{y}} = \mathbf{Sy}$, $\bar{\mathbf{n}} = \mathbf{Sn}$, and $\bar{\mathbf{H}} = \mathbf{SHS}^H$, with the latter representing the $N_A \times N_A$ middle block of the frequency-domain channel matrix $\mathbf{H}$ as shown in Fig. 1. $\bar{\mathbf{H}}$ is further approximated by its banded version

$$\mathbf{B} = \bar{\mathbf{H}} \circ \mathbf{\Theta}_Q \tag{4}$$

where $\mathbf{\Theta}_Q$ is the $N_A \times N_A$ Toeplitz matrix with entries defined as $[\mathbf{\Theta}_Q]_{m,n} = 1$ for $|m-n| \le Q$ and $[\mathbf{\Theta}_Q]_{m,n} = 0$ for $|m-n| > Q$. The bandwidth parameter Q is used to control how many off-diagonal elements should be included to give a good approximation of the banded frequency-domain channel matrix. Tuning Q allows for a trade-off between equalizer complexity and performance. Q is usually chosen much smaller than the number of subcarriers N, e.g., $1 \le Q \le 4$.

3. LOW-COMPLEXITY BLOCK TURBO EQUALIZATION

In this section, we derive a low-complexity block turbo equalizer for the system defined in the previous section. The receiver is assumed to have perfect channel state information (CSI) and the transmitter has no access to CSI. In practice, the techniques developed in [10] can be used to estimate the channel.

The transmission system groups $2N_A$ bits to form an OFDM symbol $\bar{\mathbf{s}} = [s_1, s_2, \ldots, s_{N_A}]^T$, where $s_i \in \{\alpha_k\}$ is a QPSK symbol and $(s_{i,1}, s_{i,2}) \in \{(\alpha_{k,1}, \alpha_{k,2})\}$ are the related bits (see Table 1). The information bits are assumed to be independent and identically distributed (i.i.d.).

Compared to the iterative SLE [9], which is updated from subcarrier to subcarrier in a circular fashion, the proposed iterative BLE remains fixed for the entire OFDM symbol and can thus only be updated from OFDM symbol to OFDM symbol. The linear MMSE estimate of the transmitted OFDM symbol is given by

$$\hat{\bar{\mathbf{s}}} = \mathbf{m} + \mathbf{G}^H(\bar{\mathbf{y}} - \mathbf{Bm}) \tag{5}$$

$$\mathbf{G} = (\mathbf{BVB}^H + \mathbf{R}_{\bar{\mathbf{n}}})^{-1}\mathbf{BV} \tag{6}$$

where $\mathbf{m} = [m_1, m_2, \ldots, m_{N_A}]^T$ and $\mathbf{V} = \mathrm{diag}([v_1, v_2, \ldots, v_{N_A}])$, with $m_i = \mathrm{E}\{\hat{s_i}\}$ and $v_i = \mathrm{Cov}(\hat{s}_i, \hat{s}_i)$ defined as the mean and variance of the ith subcarrier that was estimated in a previous iteration. Further, $\mathbf{R}_{\bar{\mathbf{n}}} = \mathrm{E}\{\bar{\mathbf{n}}\bar{\mathbf{n}}^H\} = \sigma_n^2\mathbf{SFWW}^H\mathbf{F}^H\mathbf{S}^H$ represents the frequency-domain noise covariance matrix. In the first iteration, where no *a priori* information is available, we take $m_i = 0$ and $v_i = 1$, and the equalizer becomes $\mathbf{G} = (\mathbf{BB}^H + \mathbf{R}_{\bar{\mathbf{n}}})^{-1}\mathbf{B}$, which is the same as the non-iterative MMSE BLE of [5, 7].

In turbo equalization, the mean m_i and variance v_i are computed based on soft information from the previous iteration. This soft information is generally represented by means of the log-likelihood ratio (LLR). The *a priori*, *a posteriori* and extrinsic LLRs are defined as

$$L(s_{i,j}) = \ln\frac{P(s_{i,j}=0)}{P(s_{i,j}=1)} \quad (7)$$

$$L(s_{i,j}|\hat{s}_i) = \ln\frac{P(s_{i,j}=0|\hat{s}_i)}{P(s_{i,j}=1|\hat{s}_i)} \quad (8)$$

$$\begin{aligned} L_e(s_{i,j}) &= L(s_{i,j}|\hat{s}_i) - L(s_{i,j}) \\ &= \ln\frac{\sum\limits_{\alpha_k:\alpha_{k,j}=0} p(\hat{s}_i|s_i=\alpha_k)p(s_{i,j'}=\alpha_{k,j'})}{\sum\limits_{\alpha_k:\alpha_{k,j}=1} p(\hat{s}_i|s_i=\alpha_k)p(s_{i,j'}=\alpha_{k,j'})} \end{aligned} \quad (9)$$

where $j, j' = 1, 2$ and $j \neq j'$. We make the same simplification as in [3] by assuming that the *a posteriori* LLR is calculated only with respect to $\hat{s}_i$ rather than to the entire estimated OFDM symbol (MAP equalizers). Notice that $\mathbf{G}$ is a biased MMSE equalizer operating on a single block, which means that we can not always assume that $m_i = 0$ and $v_i = 1$ when estimating the ith subcarrier as in [2, 9], and thus we can not use only extrinsic information. In other words, the extrinsic LLR $L_e(s_{i,j})$ is not completely independent of the *a priori* LLR $L(s_{i,j})$.

The probability density function (PDF) $p(\hat{s}_i|s_i=\alpha_k)$ is assumed to be Gaussian with mean $\mu_{i,k}$ and variance $\sigma_{i,k}$. This assumption is extensively used in turbo equalization to simplify the calculation (see e.g. [2]). Hence $p(\hat{s}_i|s_i=\alpha_k)$ can be written as

$$p(\hat{s}_i|s_i=\alpha_k) = \frac{1}{(2\pi\sigma_{i,k}^2)^{1/2}} \cdot e^{-|\hat{s}_i-\mu_{i,k}|^2/\sigma_{i,k}^2} \quad (10)$$

$$\begin{aligned} \mu_{i,k} &= \mathrm{E}(\hat{s}_i|s_i=\alpha_k) \\ &= m_i + \mathbf{g}_i^H\mathbf{b}_i(\alpha_k - m_i) \\ &= m_i + v_i t_i(\alpha_k - m_i) \end{aligned} \quad (11)$$

$$\begin{aligned} \sigma_{i,k} &= \mathrm{Cov}(\hat{s}_i, \hat{s}_i|s_i=\alpha_k) \\ &= \mathbf{g_i}^H\mathrm{Cov}(\bar{\mathbf{y}}_i, \bar{\mathbf{y}}_i|s_i=\alpha_k)\mathbf{g}_i \\ &= v_i^2 t_i(1 - v_i t_i) \end{aligned} \quad (12)$$

$$t_i = \mathbf{b}_i^H(\mathbf{BVB}^H + \mathbf{R}_{\bar{\mathbf{n}}})^{-1}\mathbf{b}_i \quad (13)$$

with $\mathbf{b}_i$ and $\mathbf{g}_i$ representing the ith column of $\mathbf{B}$ and $\mathbf{G}$, respectively. The extrinsic information L_e can be calculated as

$$L_e(s_{i,1}) = \frac{\sqrt{8}\mathrm{Re}(\hat{s}_i)}{v_i(1 - v_i t_i)} \quad (14)$$

$$L_e(s_{i,2}) = \frac{\sqrt{8}\mathrm{Im}(\hat{s}_i)}{v_i(1 - v_i t_i)} \quad (15)$$

The symbol estimate $\hat{s}_i$ can be used to update the soft information of s_i as

$$L_{new}(s_{i,j}) = L(s_{i,j}) + L_e(s_{i,j}) \quad (16)$$

$$m_i = \frac{\tanh(\frac{L_{new}(s_{i,1})}{2}) + i \cdot \tanh(\frac{L_{new}(s_{i,2})}{2})}{\sqrt{2}} \quad (17)$$

$$v_i = 1 - |m_i|^2 \quad (18)$$

The BLE calculates the estimate of the entire OFDM symbol $\{\hat{s}_i, i = 1, \ldots, N_A\}$ according to (5)-(6), and then the priors are updated using (16)-(18).

To calculate $\hat{\mathbf{s}}$ in (5) and t_i in (13), a matrix inverse operation $(\mathbf{BVB}^H + \mathbf{R}_{\bar{\mathbf{n}}})^{-1}$ is involved. The standard computation requires a complexity of $O(N_A^3)$, which is too large for a system with a large number of active subcarriers. However, [5, 7] exploits the banded structure of the approximated frequency-domain channel matrix $\mathbf{B}$ to compute the inverse using a banded LDL^H factorization

$$\mathbf{BVB}^H + \mathbf{R}_{\bar{\mathbf{n}}} = \mathbf{LDL}^H \quad (19)$$

which has a complexity of $O(N_A)$. This requires the frequency-domain noise covariance matrix $\mathbf{R}_{\bar{\mathbf{n}}}$ to be also banded. The minimum band approximation error-sum of exponentials window developed in [7] fulfills this requirement and is therefore used in this paper as the receiver window. Note that this window does not differ much from the maximum average signal to interference and noise ratio (SINR) window developed in [9], or any other standard window developed for filter design.

Applying (19) to compute $\hat{\mathbf{s}}$, we obtain

$$\hat{\mathbf{s}} = \mathbf{m} + \mathbf{VB}^H\mathbf{L}^{-H}\mathbf{D}^{-1}\mathbf{L}^{-1}(\bar{\mathbf{y}} - \mathbf{Bm}) \quad (20)$$

which requires two matrix-vector products involving a banded matrix, two matrix-vector products involving a diagonal matrix, and solving two triangular systems involving a banded matrix, leading to a total complexity of $O(N_A)$. Similarly, applying (19) to compute t_i, we obtain

$$t_i = \|\mathbf{D}^{-\frac{1}{2}}\mathbf{L}^{-1}\mathbf{b}_i\|^2 \quad (21)$$

which requires one matrix-vector product involving a diagonal matrix, and solving one triangular system involving a banded matrix, leading to a complexity of $O(N_A)$. However, this computation has to be done for $i = 1, \ldots, N_A$, which results in a total complexity of $O(N_A^2)$. Fortunately, this complexity can be lowered to $O(N_A)$ with only a minor performance loss as will be explained next.

Defining $\mathbf{x}_i = \mathbf{D}^{-\frac{1}{2}}\mathbf{L}^{-1}\mathbf{b}_i$ and stacking $\mathbf{x}_i$ for $i = 1, \ldots, N_A$, we basically have to solve

$$\mathbf{LD}^{\frac{1}{2}}\mathbf{X} = \mathbf{B} \quad (22)$$

Due to the specific banded structure of $\mathbf{L}$ and $\mathbf{B}$, $\mathbf{X}$ has a banded upper triangular part with bandwidth $2Q$ and a full lower triangular part. Hence, solving (22) by backsubstitution for $\mathbf{X}$ leads to a complexity of $O(N_A^2)$. However, it can be observed that the lower triangular part of $\mathbf{X}$ is approximately banded. Hence, we can approximate $\mathbf{X}$ by $\tilde{\mathbf{X}}$, which has a banded lower triangular part with bandwidth $\tilde{Q}$ (see Fig. 2). This means we have to solve (22) by backsubstitution only for $\tilde{\mathbf{X}}$ instead of $\mathbf{X}$, leading to a complexity that is only $O(N_A)$ instead of $O(N_A^2)$. Simulation results show that a $\tilde{Q}$ in the order of $2Q$ achieves a very good approximation.

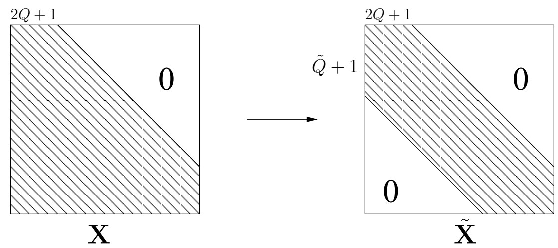

Fig. 2. Approximation of $\mathbf{X}$

4. SIMULATION RESULTS

In this section, the proposed low-complexity algorithm is examined and compared by simulations. We consider an OFDM system with $N = 128$ and $N_A = 96$. The maximum channel delay spread and the CP length are the same and equal to $L = 32$. The channel is assumed to be Rayleigh distributed with an exponential power delay profile, and Jakes' Doppler spectrum. We consider a high mobility case where the normalized Doppler frequency is $f_d/\Delta f = 0.15$ with f_d the maximum Doppler frequency shift and $\Delta f = 1/T$ the subcarrier spacing. The time-domain receiver window is designed for $Q = 2$.

Fig. 3 compares the BER performance of the iterative MMSE BLE and the iterative MMSE SLE [9] for different numbers of iterations. The simulation results show that the block turbo equalizer outperforms the serial turbo equalizer, even after a large number of iterations. This is mainly due to the windowing operation and corroborates our initial expectation. Further, we observe that both methods converge slowly after two iterations, and do not get close to the matched filter bound (MFB) at high SNR. All the banded equalizers have an error floor due to the band approximation error of the channel.

Fig. 4 compares the BER performance of the proposed iterative MMSE BLE with $Q' = 5$ and the iterative MMSE-SLE [9] for different numbers of iterations. The simulation results show that the band approximation of the lower triangular part of $\mathbf{X}$ does not incur a big performance loss.

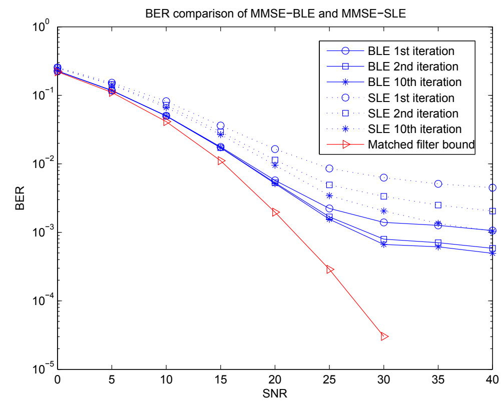

Fig. 3. BER comparison between BLE and SLE

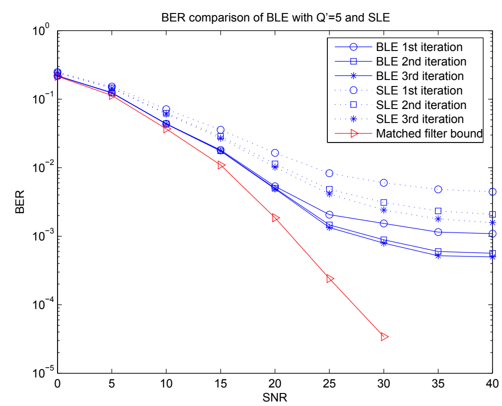

Fig. 4. BER comparison of low-complexity block turbo equalizer and SLE

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