Distributed Space-Time Cooperative Systems with Regenerative Relays

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***Abstract*— This paper addresses some of the opportunities and the challenges in the design of multi-hop systems that utilize cooperation with one or two intermediate regenerative relays to provide high-quality communication between a source and a destination. We discuss the limitations of using a distributed Alamouti scheme in the relay channel and the additional complexity required to overcome its loss of diversity. As an alternative to the distributed Alamouti scheme, we propose and analyze two Error Aware Distributed Space-Time (EADST) systems built around the Alamouti code. First, using a bit error rate based relay selection approach, we design an EADST system with one and two regenerative relays that rely on feedback from the destination and we show that the proposed system improves on the distributed Alamouti scheme. In addition, we prove that the proposed one relay EADST system collects the full diversity of the distributed MISO channel. Second, we introduce an EADST system without feedback in which the relaying energies depend on the error probabilities at the relays. Numerical results show that both EADST systems perform close to the error probability lower bound obtained by considering error-free reception at the relays.**

***Index Terms*— Distributed space-time coding, cooperative systems, regenerative relays.**

I. Introduction

STRONG shadowing, shielding, and or other longer term link degradations can be major detriments to the connectivity of parts of a wireless communication system. All the deleterious effects of the channel become more severe with the potential operation at higher frequencies, which may be needed for transceiver compactness and desired higher data rates. One possible solution is to use cooperating relay nodes as tetherless multiple antennas to effect distributed spatial diversity and low-power connectivity. Depending on the nature and complexity of the system, the nodes can serve as simple amplify-and-forward relays (i.e., non-regenerative relays) as in [1]–[3] or become sophisticated proxies that can carry out detection, storage, regeneration and coding, and aid in routing. In either case, the virtual arrays provide a structure that can be exploited as an *extended MIMO system* [4]–[6].

The initial work on the relay channel goes back as far as [7], where the capacity of *the degraded* relay channel has been solved. The capacity bounds proposed in [7] have been extended to a more practical cooperative system that considers the half-duplex constraint at the relay [8], [9]. Extensions to an n node network have been proposed in [10] where a transport capacity of $\Theta(n)$ bit-meters per second is shown to be achievable. The majority of the research on the relay channel has been concentrated on information theoretic bounds [5], [11]–[13] and channel coded transmissions [14]. Performance of the uncoded non-regenerative cooperative systems has also been reported in [15]–[17]. However, after the work of [4], which reignited the interest in cooperative systems, little attention has been given to uncoded schemes with regenerative relays, and it is in part due to the difficulties in modeling the errors at the relays.

Using distributed space-time coding over the relay channel was proposed independently in [3], [18], [19] and the effect of errors at the relays for a regenerative Distributed Space-Time Coding (DSTC) setup was analyzed in [6], [20]–[22]. In contrast with [6], which considers a suboptimum distributed Alamouti scheme, the work in [21] proposes the Maximum-Likelihood (ML) receiver for a DSTC system using both distributed Alamouti and Orthogonal Frequency Division Multiplexing. In addition, [21] shows how to allocate power between the source and the relay depending on the error probability at the relay. The work of [20] complements the results in [21] by establishing performance-complexity tradeoffs between a regenerative and a non-regenerative distributed Alamouti system. Unlike non-regenerative relays, regenerative relays do not naturally induce diversity in the system. It has been observed that due to errors at the relays the systems with distributed antennas using channel coding and/or distributed space-time coding lose diversity when compared to a one-hop MIMO system with the same number of antennas [22], [23]. In particular, the regenerative distributed Alamouti system cannot take full advantage of the diversity offered by the relay channel.

To overcome the loss of diversity of the distributed Alamouti system we propose two Error Aware Distributed Space-Time (EADST) schemes: a quasi-optimum scheme that requires feedback from the destination to the relays and an ad-hoc scheme, which dispenses with the feedback by taking advantage of the relative distance between source, relays, and the destination. The EADST system with feedback is designed to induce and collect full diversity in a distributed MISO channel by allowing feedback from the destination and error probability feedforward from the relays. Both EADST systems can be utilized in cellular systems as well as in multi-hop networks without centralized control. The paper is organized as follows. In Section II we describe the distributed Alamouti system with one relay. In Section III we show

Manuscript received November 14, 2004; revised August 4, 2005 and April 4, 2006; accepted May 23, 2006. The associate editor coordinating the review of this paper and approving it for publication was T. Duman.

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Digital Object Identifier 10.1109/TWC.2006.04750.

that the distributed Alamouti scheme loses diversity in the relay channel. Using a bit error rate (BER) based two-path selection combining approach, we propose an EADST system with feedback that collects the full diversity of the distributed MISO channel. We also propose an EADST system without feedback that uses error rate dependent transmission energies at the relays. In Section IV we extend the results of Section III to a system with two relays. We analyze the performance of the proposed systems in Section V by assuming that each link is Rayleigh distributed. Section VI is reserved for conclusions.

Let us introduce the following notational conventions. The bold upper (lower) case denotes a matrix (column vector). Superscript T stands for transpose, and * for conjugate. The signum function is defined as $\mathrm{sgn}(x) := |x|/x$, for $x \neq 0$, and $\mathrm{sgn}(x) := 1$ for $x = 0$. The Q-function is $Q(z) := \int_z^\infty \exp(-x^2/2)dx/\sqrt{2\pi}$. $\Re\{x\}$ stands for the real part of a complex number x and $E[\cdot]$ denotes the expected value operator.

II. System Model

In this section we describe the distributed Alamouti scheme, which is the backbone of the proposed EADST schemes. We consider a multiuser interference-free wireless communication system that uses wireless relay stations. The relays have no data symbols of their own to transmit; their goal is to improve the quality of the link between the source and the destination. The relay stations cannot transmit and receive simultaneously using the same channel resources because the signals received by the relay would be affected by strong interference from the relay's own transmitter (i.e., self-interference). In order to avoid self-interference a cooperative system requires two orthogonal subspaces for the signals received and transmitted by the relays (a.k.a the relay's half-duplex constraint). If the relays use two orthogonal signal subspaces, they can also eliminate multiple-relay-interference, which is the interference collected by one relay from all the other active relays in the system. For brevity of argument, we consider two different frequency bands, i.e., band A and band B for transmitting and receiving signals at the relays. More precisely, the relays monitor only band A on which they receive the information signal from the active source, and transmit in band B to the destination. All the radios in the system use *one antenna* per transceiver.

Through a relay discovery process and protocol, which is not the focus of this paper, it is assumed that the source has access to one fixed relay station R_1. The source uses energy E per symbol to communicate with the destination. During the generic time slot i the source broadcasts in band A to the relay and the destination the data block $\sqrt{\varepsilon_A}\boldsymbol{s}[i] = \sqrt{\varepsilon_A}[s[2i], s[2i+1]]^{\mathrm{T}}$, where $\varepsilon_A = \rho E \leqslant E$ is the transmitted bit energy in band A. The data symbols, $\{s[n]\}_n$, are drawn from a BPSK constellation with unit energy and are assumed independent and identically distributed. Relay R_1, which monitors frequency band A, receives

$$\boldsymbol{r}_1[i] = h_{sr_1}\sqrt{\varepsilon_A}\boldsymbol{s}[i] + \boldsymbol{z}_1[i], \tag{1}$$

where h_{sr_1} is the slow varying fading channel between the source and relay R_1, and $\boldsymbol{z}_1[i]$ is the noise vector with each entry being a complex circular Gaussian random variable with variance $N_0/2$ per dimension. Provided that the relay acquires the channel h_{sr_1} perfectly, the decision vector for $\boldsymbol{s}[i]$ with maximum likelihood decoding is $\boldsymbol{x}_1[i] = \boldsymbol{r}_1[i]/\big(\sqrt{\varepsilon_A}h_{sr_1}\big)$. The relay R_1 quantizes $\boldsymbol{x}_1[i]$ in order to obtain an estimate of $\boldsymbol{s}[i]$, which can be written as $\hat{\boldsymbol{s}}[i] = [\hat{s}[2i], \hat{s}[2i+1]]^{\mathrm{T}} = 2\,\mathrm{sgn}\big(\boldsymbol{x}_1[i]\big) - 1$. The probability of error at the relay R_1 is $P_{r_1} := Q\big(\sqrt{2\varepsilon_A|h_{sr_1}|^2/N_0}\big)$. During the next time slot, i.e, time slot $i+1$, the source and the relay transmit in band B using an Alamouti-type space-time code. The source transmits $\boldsymbol{u}_0[i+1] = \sqrt{\varepsilon_B}\boldsymbol{s}[i]$, which is the same as the block transmitted in band A during the previous time slot except for the transmit energy $\varepsilon_B = E - \varepsilon_A$. The source transmits continuously in band A as well as in band B. The transmissions from the source in band B are a delayed version of its transmissions in band A. The relay R_1 transmits $\boldsymbol{u}_1[i+1] = \sqrt{\alpha_{r_1}}\big[\hat{s}^*[2i+1], -\hat{s}^*[2i]\big]^{\mathrm{T}}$, where the transmit energy at the relay is $\alpha_{r_1} = \rho_{r_1}E \leqslant E_{r_1}$, E_{r_1} is the maximum transmit symbol energy at the relay, and ρ_{r_1} is not necessarily less than 1. While transmitting $\boldsymbol{u}_1[i+1]$, the relay receives $\boldsymbol{r}_1[i+1]$ from the source to update the information symbols needed for the relay-source cooperation in the next time-slot. One can very well use the standard Alamouti code pairs in [24] to mitigate inter-symbol interference (ISI). However, repetition coding offers extra flexibility (when for example channel coding is considered [14]), and shifts most of the additional complexity associated with cooperation to the relay since the source can be unaware of the cooperation strategy selected by the destination.

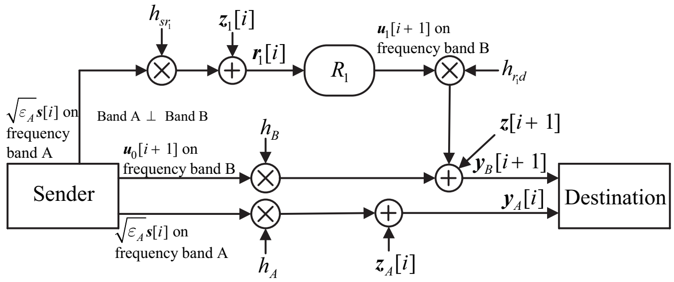

Fig. 1. Discrete-time equivalent relay channel with the half-duplex constraint. The signals $\boldsymbol{y}_A[i]$ and $\boldsymbol{y}_B[i+1]$ are received at the destination on the non-overlapping frequency bands A and B, respectively.

The synchronization of the relay and the source has been achieved during the call setup procedure, which is not discussed in this paper. We further assume that the transmissions in band B from the relay and the source reach the destination at the same time. In this case we can write the signals received at the destination in bands A and B as

$$\boldsymbol{y}_A[i] = \big[y_A[2i], y_A[2i+1]\big]^{\mathrm{T}} = h_A\sqrt{\varepsilon_A}\boldsymbol{s}[i] + \boldsymbol{z}_A[i], \tag{2}$$

$$\begin{aligned}\boldsymbol{y}_B[i+1] &= \big[y_B[2(i+1)], y_B[2(i+1)+1]\big]^{\mathrm{T}} \\ &= h_{r_1d}\boldsymbol{u}_1[i+1] + h_B\boldsymbol{u}_0[i+1] + \boldsymbol{z}[i+1].\end{aligned} \tag{3}$$

Notice from Fig. 1, which depicts the discrete-time equivalent relay channel, that $\boldsymbol{y}_A[i]$ is received in band A and $\boldsymbol{y}_B[i+1]$ is received in band B. We assume that $\boldsymbol{z}_A[i]$ and $\boldsymbol{z}[i] := [z[2i], z[2i+1]]^{\mathrm{T}}$ are mutually independent noise vectors with each entry being a complex circular Gaussian random variable with zero mean and variance $N_0/2$ per dimension. We also assume that the effect of the slowly time-varying flat fading is captured by the independent random variables h_{sr_1}, h_{r_1d}, h_A, and h_B. We choose a double subscript notation for the links to and from the relay. The first subscript specifies the the transmitter and the second subscript identifies the receiver. For example, h_{r_1d} denotes the channel between relay R_1 and the destination.

III. System Design

The main focus of this paper is to expose the limitations of the distributed Alamouti system and to propose EADST systems based on the Alamouti codes. The first step, which motivates the proposed EADST design, is to analyze the performance of the distributed Alamouti system and determine its diversity order. Before we proceed with the bit error rate analysis, we need to specify the receiver at the destination. For conciseness we take $i = 0$, and without loss of generality we write (3) as

$$\boldsymbol{y}_B[1] = \big[y_B[2], y_B[3]\big]^{\mathrm{T}} = h_B\sqrt{\varepsilon_B}\Big[s[0],\ s[1]\Big]^{\mathrm{T}} + h_{r_1d}\sqrt{\alpha_{r_1}}\Big[\theta_{r_1}[1]s^*[1],\ -\theta_{r_1}[0]s^*[0]\Big]^{\mathrm{T}} + \boldsymbol{z}[1], \tag{4}$$

where $\theta_{r_1}[n] \in \{-1, 1\}$ is a random process that characterizes the error event at the relay. More precisely, if an error occurs at relay R_1 at time n, then $\theta_{r_1}[n] = -1$ and the error probability is $\Pr(\theta_{r_1}[n] = -1) = 1 - \Pr(\theta_{r_1}[n] = 1) = P_{r_1}$. If we process $\tilde{\boldsymbol{y}}_B[1] := [y_B[2], y_B^*[3]]^{\mathrm{T}}$ with the 2×2 orthogonal matrix $\boldsymbol{G}_1 := [\sqrt{\varepsilon_B}h_B^*, -\sqrt{\alpha_{r_1}}h_{r_1d}; \sqrt{\alpha_{r_1}}h_{r_1d}^*, \sqrt{\varepsilon_B}h_B]$, we obtain $[x[2], x[3]]^{\mathrm{T}} := \boldsymbol{G}_1\tilde{\boldsymbol{y}}_B$. Recall that since the Alamouti receiver in [24] removes the ISI introduced by the MISO channel without enhancing the noise, it achieves the same error performance as a system that uses orthogonal transmissions and has half the bandwidth efficiency of the Alamouti system. To show that this property is not shared by the distributed Alamouti system, we reference the orthogonal transmissions cooperative (OTC) system with one relay proposed in [1], which plays the same role in the relay channel as the system with orthogonal transmissions plays in the MISO channel. An OTC system can be readily obtained if all radios transmit on orthogonal channels. For example, the source transmits $s[i]$ on bands A and B, while the relay uses a third frequency band, C, to transmit an estimate of $s[i]$. The destination optimality combines the 3 orthogonal transmissions using a maximum likelihood (ML) receiver similar to the one proposed in [1]. By inspecting

$$\begin{aligned} x[2] =& \big(\alpha_{r_1}|h_{r_1d}|^2\theta_1[0] + \varepsilon_B|h_B|^2\big)s[0] \\ &+ \sqrt{\alpha_{r_1}\varepsilon_B}h_{r_1d}h_B^*\big(\theta_1[1] - 1\big)s[1] \\ &+ \sqrt{\varepsilon_B}h_B^*z[2] - \sqrt{\alpha_{r_1}}h_{r_1d}z^*[3] \end{aligned} \tag{5}$$

we can see that if the relay introduces errors, $\boldsymbol{G}_1$ cannot remove the ISI. Since the OTC system is not affected by ISI, it outperforms the distributed Alamouti system. Nevertheless, if the quality of the channel between the source and the relay increases, which translates to less errors at the relay, we expect a diminishing performance gap between the OTC system and the distributed Alamouti system. Notice that the OTC system uses 50% more bandwidth than the distributed Alamouti system and the comparison between the two systems is unfair. However, we prefer not to compensate for the bandwidth overexpansion in the OTC case because we want to compare the BER of both the distributed Alamouti and the proposed EADST designs against the BER of the OTC, which constitutes a performance limit for DSTC systems (without feedback).

To maintain the simplicity of an Alamouti-type system, the destination only uses $x[2]$ in order to estimate $s[0]$, even though discarding $x[3]$ is suboptimal since information about $s[0]$ is also contained in $x[3]$. Note, however, that when P_{r_1} is low, $\theta_{r_1}[1] = 1$ with high probability and the ISI is eliminated most of the time. Of course, if $\theta_{r_1}[n] = 1$, $\forall n$ (i.e., no errors at the relays), then the system in (5) reverts to the standard Alamouti system. Because detecting $s[1]$ from $x[3]$ is similar to detecting $s[0]$ from $x[2]$, it is enough to focus only on $s[0]$.

A. Unknown P_{r_1} at the destination

If in the distributed Alamouti system the destination cannot make use of the statistics of $\theta_{r_1}[n]$, there is little to be done in order to come up with an effective receiver at the destination. Even though the relay does not detect all the symbols correctly, the destination is forced to assume, that the relay does not make any errors, i.e., $\theta_{r_1}[n] = 1$, for all n, which is the most plausible assumption given the limited amount of information available at the destination. The destination takes into account both (2) and (5), and decides that

$$s[0] = 1 \text{ only if } t := x[2] + h_A^*\sqrt{\varepsilon_A}\, y_A[0] \geqslant 0. \tag{6}$$

Note that the receiver in (6) is the maximum ratio combiner (MRC) receiver only if the detection at the relay is error-free. The bit error probability (BEP) for the detection criterion in (6) is $P(\alpha_{r_1}) = \Pr\big(\Re\{t\} < 0 | s[0] = 1\big)$, which after simple manipulations leads to

$$\begin{aligned} P(\alpha_{r_1}) =& (1-P_{r_1})^2\, Q\left(\frac{\alpha_s}{\sqrt{N_1}}\right) + (1-P_{r_1})P_{r_1}Q\left(\frac{\alpha_d}{\sqrt{N_1}}\right) \\ &+ 0.5(1-P_{r_1})P_{r_1}\left[Q\left(\frac{\alpha_s+\beta_{1B}}{\sqrt{N_1}}\right) + Q\left(\frac{\alpha_s-\beta_{1B}}{\sqrt{N_1}}\right)\right] \\ &+ 0.5P_{r_1}^2\left[Q\left(\frac{\alpha_d+\beta_{1B}}{\sqrt{N_1}}\right) + Q\left(\frac{\alpha_d-\beta_{1B}}{\sqrt{N_1}}\right)\right], \end{aligned} \tag{7}$$

where $\alpha_s := \alpha_{r_1}|h_{r_1d}|^2 + \varepsilon_B|h_B|^2 + \varepsilon_A|h_A|^2$, $\alpha_d := -\alpha_{r_1}|h_{r_1d}|^2 + \varepsilon_B|h_B|^2 + \varepsilon_A|h_A|^2$, $\beta_{1B} := 2\sqrt{\alpha_{r_1}\varepsilon_B}\,\Re\{h_{r_1d}h_B^*\}$ and $N_1 := \alpha_s N_0/2$. Derivation of (7) is not reproduced at this point. We will see later that (7) can be easily obtained as a specialization of (18). The main motivation for the paper comes from the following proposition:

Proposition 1. *The diversity gain of a system with $\bar{P}(\alpha_{r_1}) := E[P(\alpha_{r_1})]$, where the expectation is taken over the channels h_{sr_1}, h_{r_1d}, h_A, h_B is determined only by the distribution of h_{sr_1}, which is the channel between the source and relay R_1.* The proof presented in Appendix I allows us to conclude that the distributed Alamouti space-time system does not collect the full diversity of the MISO channel. Furthermore, the receiver in (6) destroys the diversity of the repetition code used at the source. For example, with Rayleigh fading channels the repetition coding will induce diversity order 2 in point-to-point transmissions if h_A is not fully correlated with h_B. However, the slope of $\bar{P}(\alpha_{r_1})$ becomes -1 as the SNR at the destination increases (see Appendix I). We can intuitively understand the diversity behavior of the distributed Alamouti system if we model the error at the relay as a 180 degree phase error in the estimation of h_{r_1d}. Phase errors lead to a non-coherent combination of channel coefficients, and therefore, to loss of diversity.

B. Known P_{r_1} at the destination

In this subsection we present the main result of the paper: the design of two simple DSTC systems with better error performance than the distributed Alamouti system. We have seen in the previous subsection that the diversity performance of the distributed Alamouti system is poor without knowledge of P_{r_1} at the destination[1]. If the destination knows P_{r_1}, it can use, as proposed in [21], a maximum likelihood (ML) receiver to process $\boldsymbol{y}_A[i]$ and $\boldsymbol{y}_B[i]$. Similar to the previous suboptimum receiver, the ML receiver does not collect the full diversity of the relay channel. When using the Alamouti codes over the relay channel the result is an inter-symbol interference (ISI) channel. To collect the full diversity of the ISI channel the receiver needs full (deterministic) information about all channel taps and not just statistical information, which is the case when P_{r_1} is made available to the destination. Even though the ML receiver has the potential to collect diversity, the Alamouti code does not enable the diversity of the relay channel.

It is essential to modify the distributed transmitter in order to improve on the previously proposed Alamouti schemes. The general idea is to modulate the transmit power at the relay in response to variations of the channel between the source and relay. We will show later that we can recover the diversity lost by the distributed Alamouti system if we judiciously design the relay's amplification energy α_{r_1}. Before we describe the EADST system and the algorithm used to recover the diversity, we outline the reasoning behind the proposed algorithm. We start by focusing on the amplification energy α_{r_1} that maximizes the signal to interference and noise ratio (SINR) at the destination under the constraint that $\alpha_{r_1} \leqslant E_{r_1}$. In our quest for a useful SINR function, we model the errors at the relay as the additive errors $e_1[0] := \hat{s}[0]-s[0]$ and $e_1[1] := \hat{s}[1] - s[1]$ instead of using the multiplicative errors $\theta_{r_1}[0]$ and $\theta_{r_1}[1]$. Note that for $n \in \{0,1\}$, $e_1[n] = 2$ with probability P_{r_1} if $s[n] = -1$, $e_1[n] = -2$ with probability P_{r_1} if $s[n] = 1$, and $e_1[n] = 0$ with probability $1 - P_{r_1}$. We rewrite (5) as

$$x[2] = \big(\alpha_{r_1}|h_{r_1d}|^2+\varepsilon_B|h_B|^2\big)(s[0]+C_1e_1[0]) + \sqrt{\alpha_{r_1}\varepsilon_B}h_{r_1d}h_B^*e_1[1]+\sqrt{\varepsilon_B}h_B^*z[2]-\sqrt{\alpha_{r_1}}h_{r_1d}z^*[3], \tag{8}$$

where $C_1 := \alpha_{r_1}|h_{r_1d}|^2/(\alpha_{r_1}|h_{r_1d}|^2 + \varepsilon_B|h_B|^2)$. To decorrelate the information symbol $s[0]$ and the noise $e_1[0]$ in (8) we use an artifice similar to [1]. If we add $e := C_1E\big[e_1[0]\big|s[0]\big]$ to $s[0]$ and subtract e from $C_1e_1[0]$ it turns out that $\tilde{s}[0] := s[0] + e$ and $\tilde{e}_1[0] := C_1e_1[0] - e$ satisfy $E\big[\tilde{s}[0]\tilde{e}_1[0]\big] = 0$. We write (8) in terms of $\tilde{s}[0]$ and $\tilde{e}_1[0]$ to obtain

$$\begin{aligned} x[2] =& \big(\alpha_{r_1}|h_{r_1d}|^2+\varepsilon_B|h_B|^2\big)\tilde{s}[0] \\ &+(\alpha_{r_1}|h_{r_1d}|^2+\varepsilon_B|h_B|^2)\tilde{e}_1[0]+\sqrt{\alpha_{r_1}\varepsilon_B}h_{r_1d}h_B^*e_1[1] \\ &+\sqrt{\varepsilon_B}h_B^*z[2]-\sqrt{\alpha_{r_1}}h_{r_1d}z^*[3]. \end{aligned} \tag{9}$$

We can look at (9) as the input-output equation for a system where the information symbol $\tilde{s}[0]$, drawn from the constant-magnitude constellation $\{-(1-2C_1P_{r_1}), 1-2C_1P_{r_1})\}$, is received in noise that is uncorrelated with $\tilde{s}[0]$. Even though the noise is not independent of $\tilde{s}[0]$ to prompt an inverse proportional relationship between the BER of a threshold receiver and its SINR, compensating for the noise correlation turns out to be instrumental in deriving the proposed EADST system. Because $e = -2C_1P_{r_1}$ if $s[0] = 1$ and $e = 2C_1P_{r_1}$ if $s[0] = -1$, we find $\tilde{s}[0] = (1 - 2C_1P_{r_1})s[0]$, and $Var\big[\tilde{e}_1[0]\big] = Var\big[\tilde{e}_1[0]\big|s[0]=1\big] = 4C_1^2P_{r_1}(1-P_{r_1})$. The SINR in (9) is

$$\begin{aligned}\xi_1(\alpha_{r_1}) =& (1-2C_1P_{r_1})^2(\alpha_{r_1}\gamma_{r_1d}+\varepsilon_B\gamma_B)^2\Big/\Big[4C_1^2P_{r_1}(1-P_{r_1}) \\ &\cdot(\alpha_{r_1}\gamma_{r_1d}+\varepsilon_B\gamma_B)^2+4P_{r_1}\alpha_{r_1}\gamma_{r_1d}\varepsilon_B\gamma_B+\alpha_{r_1}\gamma_{r_1d}+\varepsilon_B\gamma_B\Big],\end{aligned} \tag{10}$$

where $\gamma_{r_1d} := |h_{r_1d}|^2/N_0$, is the quality of the channel between the relay and the destination, and $\gamma_B := |h_B|^2/N_0$ is the quality of the channel between the source and the destination. We substitute $C_1 = \alpha_{r_1}\gamma_{r_1d}/(\alpha_{r_1}\gamma_{r_1d}+\varepsilon_B\gamma_B)$ in (10) to obtain

$$\begin{aligned}\xi_1(\alpha_{r_1}) =& \big[(1-2P_{r_1})\alpha_{r_1}\gamma_{r_1d}+\varepsilon_B\gamma_B\big]^2\Big/\Big[4P_{r_1}(1 \\ &-P_{r_1})\alpha_{r_1}^2\gamma_{r_1d}^2+4P_{r_1}\alpha_{r_1}\gamma_{r_1d}\varepsilon_B\gamma_B+\alpha_{r_1}\gamma_{r_1d}+\varepsilon_B\gamma_B\Big],\end{aligned} \tag{11}$$

which could be interpreted as a correlation compensated SINR for a DSTC system. The following proposition establishes the maximum of (11).

Proposition 2. *The global optimum of the problem* $\max\{\xi_1(\alpha_{r_1})\}$ *subject to* $0 \leqslant \alpha_{r_1} \leqslant E_{r_1}$ *is* $\xi_1^{\text{opt}} = \max\{\xi_1(0), \xi_1(E_{r_1})\}$.

The proof is presented in Appendix II.

We infer from Proposition 2 that a two-state relay (which can be switched off or transmit at full power) is a simple answer to our initial question, i.e., how to adjust the gain at the relay in order to mitigate the loss of diversity in the distributed Alamouti scheme. The role of Proposition 2 is to reduce the acceptable choices for α_{r_1} from the interval $[0, E_{r_1}]$ to the set $\{0, E_{r_1}\}$. The maximum SINR approach identifies the two choices for the amplification at the relay, but it is not used to specify when the switching takes place. The relay switches between $\alpha_{r_1} = 0$ and $\alpha_{r_1} = E_{r_1}$ based on *the error probability at the destination*. If we implement the decision rule in (6), the error probability is $P(\alpha_{r_1})$ in (7). We propose the following amplification at the relay

$$\alpha_{r_1}^{(1)} := \underset{\alpha_{r_1}\in\{0,E_{r_1}\}}{\arg\min}\ P(\alpha_{r_1}). \tag{12}$$

[1]Note that if we consider path loss and shadowing into the channel model, the coding gain obtained using the scheme in Subsection III-A can overcome its diversity performance.

The EADST system with the relay amplification given in (12) is similar to a BER based two-path selection combining scheme. Notice that instead of selecting among the antennas located at the destination such as in the classical selection combining scheme, the scheme is designed to switch between antennas that are located at the relay and the source. The result in (12) is not guaranteed to achieve the minimum error probability for the detector in (6); finding a closed form solution to the problem $\min_{\alpha_{r_1} \geqslant 0} \{P(\alpha_{r_1})\}$ subject to $\alpha_{r_1} \leqslant E_{r_1}$ is not an easy task. However, we establish the following result, which we prove in Appendix III.

Proposition 3. *The diversity gain of the EADST system with α_{r_1} as in* (12) *is $t_A + t_B + min\{t_{sr_1}, t_{r_1d}\}$, where t_A, t_B, t_{sr_1}, and t_{r_1d} are the diversity orders of the channels h_A, h_B, h_{sr_1}, and h_{r_1d}, respectively.*

The result of Proposition 3 is to be contrasted with $\bar{P}(\alpha_{r_1})$, which achieves at most the diversity of the channel h_{sr_1} (see Appendix I).

Instead of the decision rule in (6), one could use the ML receiver of [21] or refine the detector in (6) by judiciously weighting $x[2]$ and $y_A[0]$ before combining them. However, by specializing to Rayleigh fading channels, we illustrate later on that the effect of relay errors on the system's BER is practically eliminated when using (12) under certain conditions to be clarified in Section V. It is important to emphasize at this point that the scope of the paper is not to analyze optimal receivers, but to propose receivers that achieve a good balance between complexity and performance.

Designing an EADST system with α_{r_1} as in (12) requires solving two problems. First, the destination has to know P_{r_1} (or equivalently, $\varepsilon_A|h_{sr_1}|$) in order to compute $P(\alpha_{r_1})$ and determine which α_{r_1} to use. Second, α_{r_1} has to be transmitted from the destination to the relay. To solve the first problem we propose the following scheme. The source transmits pilot symbols on channel A to facilitate the estimation of $|h_{sr_1}|$ at the relay and $|h_A|$ at the destination. Without utilizing space-time coding the relay only retransmits on channel B the first 2 out of every 3 pilots received on channel A. The relay powers down for the duration of the third pilot. To avoid interference with pilots sent by the relay the source sends pilot symbols on channel B only when the relay is powered down. In every group of 3 pilots the relay regenerates the first pilot symbol and amplifies-and-forwards (see [1]) the second pilot received. The regenerated pilots at the relay are used by the destination to estimate the channel between the relay and the destination, i.e., h_{r_1d}. The amplified pilots carry information about the channel between the source and the relay, and consequently, $\varepsilon_A|h_{sr_1}|h_{r_1d}$ can be estimated at the destination. Knowing h_{r_1d}, the destination can estimate $\varepsilon_A|h_{sr_1}|$. Notice that the estimate of $\varepsilon_A|h_{sr_1}|$ depends on the relay noise when in non-regenerative mode. However, in this paper we are not concerned about channel estimation errors. We assume perfect channel estimation for all links.

In Section V we illustrate for fading distributions of interest that most of the time $P(E_{r_1}) > P(0)$. Consequently, there is no need to use a binary digital modulation scheme to transmit α_{r_1} to the relay. To minimize bandwidth and power consumption the destination should only send a pulse that switches off the relay. In most cases we expect the transmission rate on the feedback channel to be a small percentage of the data symbol rate of the DSTC system.

Because the feedback channel is bandwidth consuming, we also propose an EADST system that does not require transmissions from the destination to the relay. To achieve this goal we look for an α_{r_1} that is a continuous function of P_{r_1}. In order to obtain a more manageable SINR function we approximate $4P_{r_1}\alpha_{r_1}\gamma_{r_1d}\varepsilon_B\gamma_B$ and $\alpha_{r_1}\gamma_{r_1d}$ in (11) with $4P_{r_1}\frac{\alpha_{r_1}^2}{E_{r_1}}\gamma_{r_1d}\varepsilon_B\gamma_B$ and $\frac{\alpha_{r_1}^2}{E_{r_1}}\gamma_{r_1d}$, respectively. Note that the quadratic approximations coincide with the original function for $\alpha_{r_1} = 0$ and $\alpha_{r_1} = E_{r_1}$. If we take the derivative of the approximate SINR function we find that

$$\alpha_{r_1} = \frac{(1-2P_{r_1})E_{r_1}}{1+4P_{r_1}\left[\varepsilon_B\gamma_B + (1-P_{r_1})E_{r_1}\gamma_{r_1d}\right]} \leqslant E_{r_1} \quad (13)$$

achieves the unique maximum. Using the amplification in (13) may require even more feedback from the destination to the relay than our previous choice for α_{r_1}. However, unlike (12), Equation (13) can be simplified further. We propose using the average value of the channel SINRs instead of the instantaneous values, i.e., replace γ_{r_1d} and γ_B in (13) with $\bar{\gamma}_{r_1d}$ and $\bar{\gamma}_B$, respectively. Furthermore, because the relay only cooperates with nearby sources, the path loss from the relay to the destination is approximately equal to the path loss from the source to the destination, i.e, $\bar{\gamma}_{r_1d} \approx \bar{\gamma}_B$. In this case, we can choose

$$\alpha_{r_1}^{(2)} := \frac{(1-2P_{r_1})E_{r_1}}{1+4P_{r_1}\left[\varepsilon_B + (1-P_{r_1})E_{r_1}\right]\bar{\gamma}_{r_1d}}. \quad (14)$$

If R_1 is a fixed relay, $\bar{\gamma}_{r_1d}$ varies slowly in time and can be communicated to the relay during a calibration phase. Once the relay knows $\bar{\gamma}_{r_1d}$, there is no need for a feedback channel provided that channel h_{r_1d} and h_B are affected by similar shadowing and the relay only cooperates with nearby users. Note that the destination still needs to compute P_{r_1} in order to implement the space-time decoder. Even though the ad-hoc approach in (14) is expected to perform worse then (12), it is not prone to errors on the feedback channel.

We assumed in Section II that channels A and B are affected by independent fading. However, if bandwidth resources are limited, it might not be possible to insure the independence of h_A and h_B. In order to gain further insight into the performance of the EADST system we analyze the extreme case when the channels A and B are affected by identical fading, i.e., $h_A = h_B$. If we maximize the total SINR at the destination, i.e., $\xi_{1t}(\varepsilon_A, \varepsilon_B) := \xi_1(\alpha_{r_1}) + \varepsilon_A\gamma_A$, where $\gamma_A := |h_A|^2/N_0$, subject to the energy constraint $\varepsilon_A + \varepsilon_B = \varepsilon$, we obtain that $\varepsilon_B = 0$ whenever $h_A = h_B$. Consequently, based on the received SINR, we should not use an EADST scheme with one relay if $h_A = h_B$. The solution for this case requires that a second relay be included in the EADST scheme, which motivates the analysis in the next section.

Note that in general the maximum of $\xi_{1t}(\varepsilon_A, \varepsilon_B)$ subject to $\varepsilon_A + \varepsilon_B = \varepsilon$ is $\xi_{1t}^* = \max\{\xi_{1t}(\varepsilon, 0), \xi_{1t}(0, \varepsilon)\}$ irrespective of the system's parameters (i.e., the quality of the channels and the amplification at the relay). The practical usefulness of such a scheme is limited since it requires channel state information at the source, which is not assumed in the EADST setup.

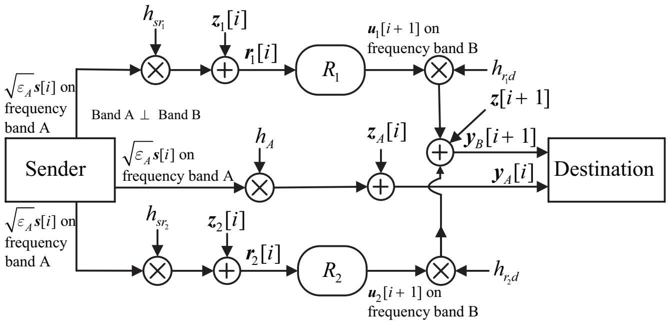

Fig. 2. Discrete-time equivalent channel with 2 relays.

IV. EADST Systems With Two Relays

If it is available, an additional fixed relay, R_2, should be considered in the EADST scheme. The overall approach remains the same as in the one relay case except that now R_1 and R_2 space-time encode the received symbols. The system's block diagram is presented in Fig. 2. The source is not involved in the distributed space-time coding scheme and it is not transmitting on channel B. To reflect those changes we modify (3) as follows:

$$\begin{aligned}\boldsymbol{y}_B[i+1] &= \left[y_B[2(i+1)], y_A[2(i+1)+1]\right]^{\mathrm{T}} \\ &= h_{r_1d}\boldsymbol{u}_1[i+1] + h_{r_2d}\boldsymbol{u}_2[i+1] + \boldsymbol{z}[i+1],\end{aligned} \tag{15}$$

where h_{r_2d} is the channel between R_2 and the destination, $\boldsymbol{u}_2[i] = \sqrt{\alpha_{r_2}}\hat{\boldsymbol{s}}[i]$, where $\hat{\boldsymbol{s}}[i] = \left[\hat{s}[2i], \hat{s}[2i+1]\right]^T = 2\mathrm{sgn}\left(\frac{\boldsymbol{r}_2[i]}{\sqrt{\varepsilon_A}h_{sr_2}}\right) - 1$, and h_{sr_2} is the channel between the source and R_2.

After processing (15) with the space-time receiver, i.e., construct $\tilde{\boldsymbol{y}}_B[1] := \left[y_B[2], y_B^*[3]\right]^T$ and apply the 2×2 matrix $\boldsymbol{G}_2 = [\sqrt{\alpha_{r_2}}h_{r_2d}^*, -\sqrt{\alpha_{r_1}}h_{r_1d}; \sqrt{\alpha_{r_1}}h_{r_1d}^*, \sqrt{\alpha_{r_2}}h_{r_2d}]$, we obtain

$$\begin{aligned}x[2] = &\left(\alpha_{r_1}|h_{r_1d}|^2 + \alpha_{r_2}|h_{r_2d}|^2\right)\left(s[0] + C_1e_1[0] + C_2e_2[0]\right) \\ &+ \sqrt{\alpha_{r_1}\alpha_{r_2}}h_{r_1d}h_{r_2d}^*\left(e_1[1] - e_2[1]\right) \\ &+ \sqrt{\alpha_{r_2}}h_{r_2d}^* z[2] - \sqrt{\alpha_{r_1}}h_{r_1d}z^*[3],\end{aligned}$$

where $e_q[n]$ is the detection error at relay R_q and $C_q := \alpha_{r_q}|h_{r_qd}|^2/(\alpha_{r_1}|h_{r_1d}|^2 + \alpha_{r_2}|h_{r_2d}|^2)$, $q \in \{1,2\}$. Recall from the previous section that $e_q[0] = 2$ with probability P_{r_q} if $s[0] = -1$, $e_q[0] = -2$ with probability P_{r_q} if $s[0] = 1$, and $e_q[0] = 0$ with probability $1 - P_{r_q}$. We assume that $e_1[n]$ and $e_2[n]$ are independent since they quantify the errors of different relays. To decorrelate the data symbols and the error noise we apply the same artifice as in the one relay case. If we redefine $\tilde{s}[0] := s[0] + e_1 + e_2$ and $\tilde{e}_q[0] := C_q e_q[0] - e_q$, where $e_q := E\left[C_q e_q[0]\middle|s[0]\right]$ we can write

$$\begin{aligned}x[2] = &\left(\alpha_{r_1}|h_{r_1d}|^2 + \alpha_{r_2}|h_{r_2d}|^2\right)\left(\tilde{s}[0] + \tilde{e}_1[0] + \tilde{e}_2[0]\right) \\ &+ \sqrt{\alpha_{r_1}\alpha_{r_2}}h_{r_1d}h_{r_2d}^*\left(e_1[1] - e_2[1]\right)s[1] \\ &+ \sqrt{\alpha_{r_2}}h_{r_2d}^* z[2] - \sqrt{\alpha_{r_1}}h_{r_1d}z^*[3],\end{aligned} \tag{16}$$

It is easy to show that $\tilde{s}[0]$ and $\tilde{e}_1[0] + \tilde{e}_2[0]$ are uncorrelated and that $\tilde{s}[0] = (1 - 2C_1P_{r_1} - 2C_2P_{r_2})s[0]$. With $\boldsymbol{\alpha} := [\alpha_{r_1}, \alpha_{r_2}]$ and $\gamma_{r_qd} := |h_{r_qd}|^2/N_0$, the SINR for (16) can be written as in (17). Let us constrain the maximum transmit power at the relay R_q to be E_{r_q}, $q \in \{1,2\}$. Similar to the one relay case the maximum of (17) is achieved on the boundaries of the feasible region. We establish the following proposition.

Proposition 4. *The global optimum of the problem* $\max\{\xi_2(\boldsymbol{\alpha})\}$ *subject to* $0 \leqslant \alpha_q \leqslant E_{r_q}$, $q \in \{1,2\}$, *is* $\xi_2^{\mathrm{opt}} = \max\left\{\xi_2\left([E_{r_1}, 0]\right), \xi_2\left([0, E_{r_2}]\right), \xi_2\left([E_{r_1}, E_{r_2}]\right)\right\}$.

We defer the proof to Appendix IV. In order to design a receiver similar to (12) we need to compute the error probability for the two-relay system. The receiver uses the same detection rule as in (6), i.e., $s[0] = 1$ has been transmitted only if $t_2 := x[2] + \sqrt{\varepsilon_A}h_A^* y_A[0] > 0$, where now $x[2]$ is given in (16). The error probability of the two-relay system, i.e., $P_2 = \Pr\left(\Re\{t_2\} < 0 | s[0] = 1\right)$, can be written as in (18), where $S := P_{r_1} + P_{r_2}$, $P := P_{r_1}P_{r_2}$, $\alpha_s := \alpha_{r_1}|h_{r_1d}|^2 + \alpha_{r_2}|h_{r_2d}|^2 + \varepsilon_A|h_A|^2$, $\alpha_{d_1} := -\alpha_{r_1}|h_{r_1d}|^2 + \alpha_{r_2}|h_{r_2d}|^2 + \varepsilon_A|h_A|^2$, $\alpha_{d_2} := \alpha_{r_1}|h_{r_1d}|^2 - \alpha_{r_2}|h_{r_2d}|^2 + \varepsilon_A|h_A|^2$, $\beta_{12} := 2\sqrt{\alpha_{r_1}\alpha_{r_2}}\Re\{h_{r_1d}^* h_{r_2d}\}$, and $N_1 := \alpha_s N_0/2$ (see Appendix V for a sketch of the proof). Notice that by taking $P_{r_2} = 0$, $h_{r_2d} = h_B$ and $\alpha_{r_2} = \varepsilon_B$ we obtain the error probability of the one-relay system presented in (7).

Instead of selecting the amplifications at the relays similar to (12), i.e., by minimizing $P_2(\boldsymbol{\alpha})$ in (18) over $\boldsymbol{\alpha} \in \{[E_{r_1}, 0], [0, E_{r_2}], [E_{r_1}, E_{r_2}]\}$, we propose the following approach. Recall that when $\boldsymbol{\alpha} = [E_{r_1}, 0]$ or $\boldsymbol{\alpha} = [0, E_{r_2}]$, there is no ISI in the system. The EADST system reverts to an OTC system since the signals from the source and the active relay R_q are received in different bands. More precisely, $y_A[0] = h_A\sqrt{\varepsilon_A}s[0] + z_A[0]$ is received in band A, while $y_B[2] = h_{r_qd}\sqrt{E_{r_q}}\theta_{r_q}[0]s[0] + z_q[2]$ is received in band B, where q is either 1 or 2 depending on which relay is active. If we choose the maximum SNR receiver of [1], i.e.,

$s[0] = 1$ has been transmitted only if

$$t_2^{(q)} := \frac{(1 - 2P_{r_q})h_{r_qd}^*\sqrt{E_{r_q}}}{4P_{r_q}(1 - P_{r_q})|h_{r_qd}|^2E_{r_q} + N_0}\,y_B[2] + h_A^*\sqrt{\varepsilon_A}\,y_A[0] \geqslant 0.$$

the probability of error is $P_2^{(q)}(\boldsymbol{\alpha}) := \Pr\left(\Re\{t_2^{(q)}\} < 0 | s[0] = 1\right)$. We select the amplifications at the relay as

$$\boldsymbol{\alpha}^{(1)} := \arg\min\left\{P_2^{(1)}([E_{r_1}, 0]), P_2^{(2)}([0, E_{r_2}]), P_2([E_{r_1}, E_{r_2}])\right\}. \tag{19}$$

Because we want to eliminate the feedback channel from the destination to the relays R_1 and R_2, we propose, similar to the one relay case (see (14)), the amplifications

$$\alpha_{r_q}^{(2)} := \frac{(1 - 2P_{r_q})E_{r_q}}{1 + 4P_{r_q}\left[E_{r_{\bar{q}}} + (1 - P_{r_q})E_{r_q}\right]\overline{\gamma}_{r_qd}}, \tag{20}$$

where $\bar{q} = 2$ if $q = 1$ and $\bar{q} = 1$ if $q = 2$. Recall from the one relay setup that for α_{r_q} in (20) to be effective we require fixed relays that are situated at the same distance from the destination. In this paper we have proposed suboptimal receivers for the one- and the two-relay setups. However, we will see in the next section that by using feedback from the destination to the relays, the proposed EADST schemes effectively mitigate the detrimental effects of errors at the relays.

V. Performance Analysis

In this section we analyze the performance of the EADST system with the relay amplifications given in (12) and (14)

$$\xi_2(\boldsymbol{\alpha})=\frac{[(1-2P_{r_1})\alpha_{r_1}\gamma_{r_1d}+(1-2P_{r_2})\alpha_{r_2}\gamma_{r_2d}]^2}{4P_{r_1}(1-P_{r_1})\gamma_{r_1d}^2\alpha_{r_1}^2+4P_{r_2}(1-P_{r_2})\gamma_{r_2d}^2\alpha_{r_2}^2+4(P_{r_1}+P_{r_2})\alpha_{r_1}\alpha_{r_2}\gamma_{r_1d}\gamma_{r_2d}+\alpha_{r_1}\gamma_{r_1d}+\alpha_{r_2}\gamma_{r_2d}} \quad (17)$$

$$\begin{aligned}P_2(\boldsymbol{\alpha})=&(1-S+2P)\left[(1-S+P)Q\left(\frac{\alpha_s}{\sqrt{N_1}}\right)+(P_{r_1}-P)Q\left(\frac{\alpha_{d_1}}{\sqrt{N_1}}\right)+(P_{r_2}-P)Q\left(\frac{\alpha_{d_2}}{\sqrt{N_1}}\right)\right]\\&+.5(S-2P)\left\{(1-S+2P)\left[Q\left(\frac{\alpha_s-\beta_{12}}{\sqrt{N_1}}\right)+Q\left(\frac{\alpha_s+\beta_{12}}{\sqrt{N_1}}\right)\right]\right.\\&\left.+(P_{r_1}-P)\left[Q\left(\frac{\alpha_{d_1}+\beta_{12}}{\sqrt{N_1}}\right)+Q\left(\frac{\alpha_{d_1}-\beta_{12}}{\sqrt{N_1}}\right)\right]+(P_{r_2}-P)\left[Q\left(\frac{\alpha_{d_2}+\beta_{12}}{\sqrt{N_1}}\right)+Q\left(\frac{\alpha_{d_2}-\beta_{12}}{\sqrt{N_1}}\right)\right]\right\}\end{aligned} \quad (18)$$

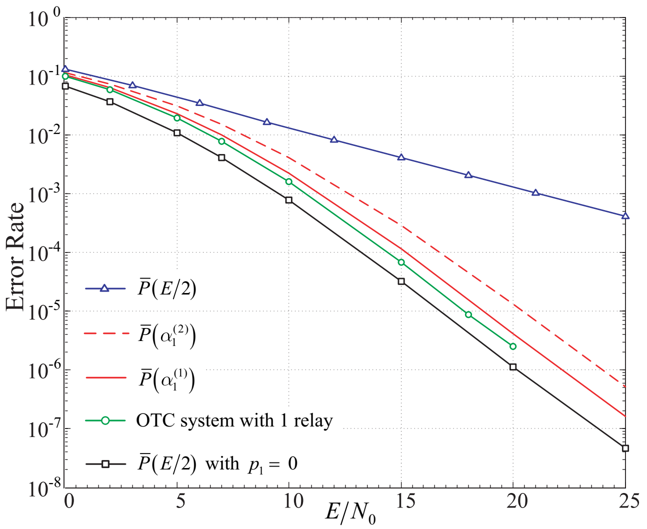

Fig. 3. Error performance of the EADST systems with 1 relay; comparison with the OTC system for equally balanced channels.

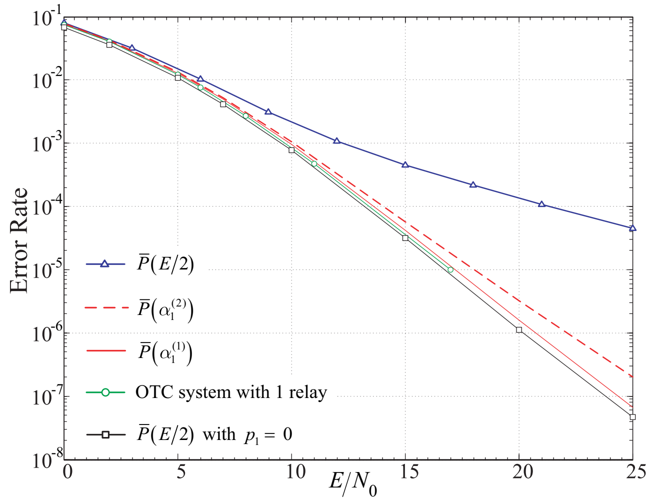

Fig. 4. Error performance of the EADST systems with 1 relay; comparison with the OTC system for unequally balanced channels.

for the one relay system, and (19) and (20) for the two relay system. We compare the error performance of the EADST systems with the error performance of the OTC system. The error performance of each system is obtained by averaging its error probability over 10^4 channel realizations.

Example 1: one relay and identically distributed fading channels. We consider an EADST system with one relay. We assume that all the channels in the system are affected by Rayleigh fading and their average power is equal to one, i.e., $E[|h_A|^2] = E[|h_B|^2] = E[|h_{sr_1}|^2] = E[|h_{r_1d}|^2] = 1$. We select the transmit energies at the source $\varepsilon_A = \varepsilon_B = E/2$, and the maximum amplification at the relay equal to $E_{r_1} = E/2$. We plot in Fig. 3, $\bar{P}(E_{r_1})$, which is the error performance of the distributed Alamouti system without knowledge of P_{r_1} at the destination. As predicted in Appendix I the diversity slope of $\bar{P}(E_{r_1})$ is -1. We can also see from Fig. 3 that there is a large performance gap between $\bar{P}(E_{r_1})$ and the performance of the distributed Alamouti with a perfect relay, (i.e., $\bar{P}(E_{r_1})$ in (7) with $P_{r_1} = 0$). Recall from our discussion at the beginning of Section III that the OTC system with one relay offers a better lower bound on the EADST designs since it takes into account the errors at the relay. When we compare in Fig. 3 the error performance of an EADST system that uses the relay amplification in (12), i.e., $\bar{P}\left(\alpha_{r_1}^{(1)}\right)$, with the error performance of the OTC we observe that the difference is less than 2dB at 10^{-5}. If no feedback channel is present in the EADST system then one should expect a degradation in performance. We can see from Fig. 3 that if the relay amplifies the regenerated symbols with $\alpha_{r_1}^{(2)}$ it looses about 3 dB. Nevertheless, $\bar{P}\left(\alpha_{r_1}^{(2)}\right)$ shows considerable improvement when compared to $\bar{P}(E_{r_1})$.

Example 2: one relay and unbalanced channels. In general, it is expected from a relay to cooperate only with nearby sources (e.g., mobile users crossing the coverage area of the relay). Consequently, the channel from the source to the relay is on average better than the channel from the relay to the destination. For example, if the source is twice closer to the relay than the destination and if we consider a path loss coefficient of $\log_2(10) \approx 3.32$, we obtain that $E[|h_{sr_1}|^2]/E[|h_{r_1d}|^2] = 10$. To find the effect of the new setup on the performance of the EADST system, we select the same channel parameters as in Example 1 with the exception of h_{sr_1}, which has $E[|h_{sr_1}|^2] = 10$. When the quality of channel h_{sr_1} increases, the relay makes less errors, and $\bar{P}\left(\alpha_{r_1}^{(1)}\right)$ and $\bar{P}\left(\alpha_{r_1}^{(2)}\right)$ come closer to the error performance of the OTC system than in Example 1. We see from Fig. 4 that $\bar{P}\left(\alpha_{r_1}^{(1)}\right)$ is almost indistinguishable from the error performance of the OTC system. Even though $\bar{P}(E_{r_1})$ seems to follow the lower bounds at low SINR, it is performing poorly at high SINR due to its diversity problem. Notice that $\bar{P}\left(\alpha_{r_1}^{(2)}\right)$ is losing

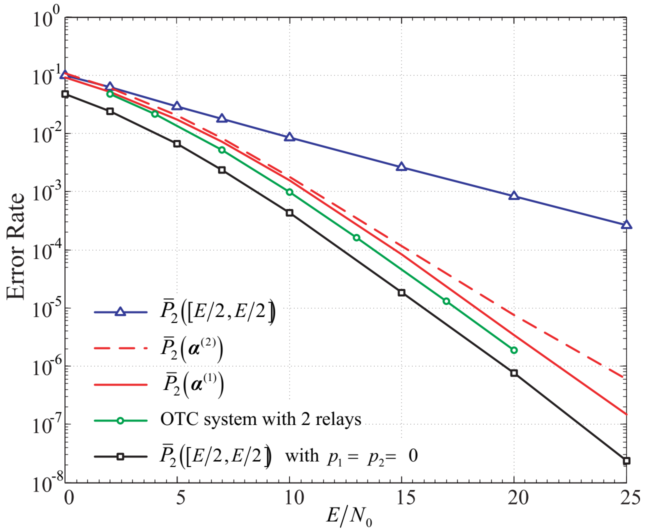

Fig. 5. Error performance of the EADST systems with 2 relays and equally balanced channels.

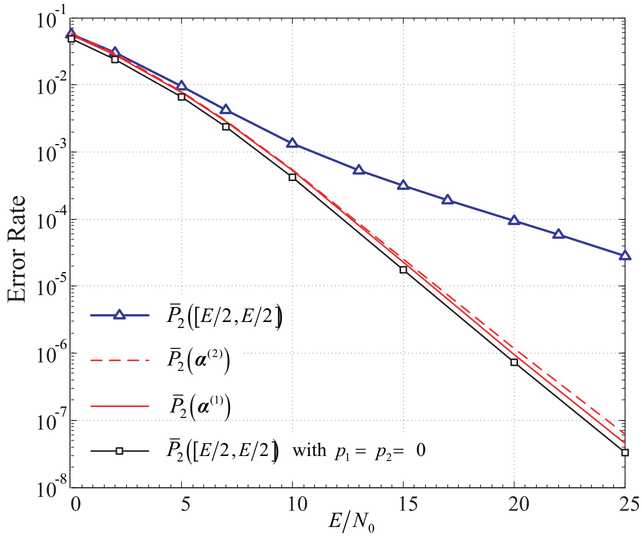

Fig. 6. Error performance for the EADST systems with 2 relays and unequally balanced channels.

diversity too. However, it is a slower process and it happens at a higher SINR.

Example 3: two relays with balanced and unbalanced channels. In this example we do the same comparison as above for a two relay scenario. We take $\varepsilon_A = E$ since the source does not transmit on channel B. We select identically distributed unit power Rayleigh fading channels, i.e., $E[|h_A|^2] = E[|h_{sr_q}|^2] = E[|h_{r_q d}|^2] = 1$, $q \in \{1,2\}$, and we plot in Fig. 5, $\bar{P}_2([E/2,E/2])$, $\bar{P}_2(\boldsymbol{\alpha}^{(2)})$, the performance of the EADST system with the amplifications in (19), the performance of the OTC system with two relays, and $\bar{P}_2([E/2,E/2])$ with $P_{r_1} = P_{r_2} = 0$. In order to simulate the error performance of the OTC system with two relays we use the ML receiver:

$$s[0] = 1 \text{ only if } a + \log\Big\{\big[(1-S-P) + (P_{r_1}-P)e^{-b} + (P_{r_2}-P)e^{-c} + Pe^{-b-c}\big] \Big/ \big[(1-S-P)e^{-b-c} + (P_{r_1}-P)e^{-c} + (P_{r_2}-P)e^{-b} + P\big]\Big\} > 0, \quad (21)$$

where $a := 4\Re\{\sqrt{\varepsilon_A} h_A^* y_A\}/N_0$, $b := 4\Re\{\sqrt{\alpha_{r_1}} h_{r_1 d}^* y_1\}/N_0$, $c := 4\Re\{\sqrt{\alpha_{r_2}} h_{r_2 d}^* y_2\}/N_0$. The signals y_A, y_1, and, y_2 contain information about $s[0]$, and they are received on orthogonal channels from the source, R_1, and R_2, respectively. We observe in Fig. 5 the same behavior as in the one relay case, i.e, $\bar{P}_2(\boldsymbol{\alpha}^{(1)})$ and $\bar{P}_2(\boldsymbol{\alpha}^{(2)})$ are close to the lower bound provided by the OTC system, while $\bar{P}_2([E/2,E/2])$ is considerably worse than even $\bar{P}_2(\boldsymbol{\alpha}^{(2)})$. Even when we increase the quality of channels $\{h_{sr_q}\}_{q=1}^2$ by selecting $E[|h_{sr_q}|^2] = 10$, for $i \in \{1,2\}$, we observe little improvement when plotting $\bar{P}_2([E/2,E/2])$ in Fig. 6. We conjecture that in order to successfully implement a DSTC system with regenerative relays the destination has to provide some feedback to the relays or take advantage of the spatial localization of the source and relays. Note, however, that even though the distributed Alamouti scheme performs poorly when compared to the proposed schemes, it is the least complex system since it does not require $\{P_{r_q}\}_{q=1}^2$ at the destination.

Example 4: robustness to imperfect channel estimates. We analyze the robustness of the proposed EADST with feedback by modeling all channel estimates as in [25], [26], i.e., $\hat{h} = h + e$, where h is the true channel and e is the estimation error. If we assume that h is Gaussian with zero mean and we use an MMSE channel estimator, then e is a zero mean Gaussian random variable. With an ideal low pass Doppler spectrum it is possible to show that the variance of the estimation error is $\sigma_e^2 \approx (2Lf_dT)N_0/E$, where $L-1$ is the number of information symbols between any two pilot symbols, f_d is the Doppler spread, T is the symbol duration, and E/N_0 is the symbol SNR (see [26]). If we consider a Doppler spread of 200Hz, which is representative of a mobile going over 100km/h, and with a symbol rate of 200ksymbols/s, the normalized Doppler spread is $f_dT = 10^{-3}$. If we pick $L = 50$ (see [26] for other representative alternatives) we obtain $\sigma_e^2 = 0.1N_0/E$. Because we consider the channel powers to be normalized to 1, the signal-to-noise ratio for the channel state information (CSI) in dB is $SNR_{CSI} = E/N_0 + 10$ dB.

In our simulations we consider that all links have the same SNR_{CSI}. For $SNR_{CSI} = E/N_0 + 10$ dB and for the same channel parameters as in Example 2, we plot in Fig. 7 the average bit error rate of the proposed EADST system, i.e., $\bar{P}(\alpha_1^{(1)})$, versus the average bit error rate of a distributed Alamouti with ideal relays. We observe little degradation in performance when we compare Fig. 7 with Fig. 4. Satisfactory results are obtained even if we drop the CSI SNR to $SNR_{CSI} = E/N_0 + 5$dB. Note, however, that as SNR_{CSI} decreases, the gap between the performance of the proposed system and the distributed Alamouti with ideal relays increases. The reason is that in addition to errors in designing the matched filter receiver, the proposed system has errors at the relay, and errors in computing the bit error rates for the selection combining approach. In spite of the additional error sources we expect the EADST system to perform well for mobile speeds below 100km/h. Robustness to imperfect CSI is a result of the binary decision process for the amplification at the relay. Similar to the 2×1 Alamouti

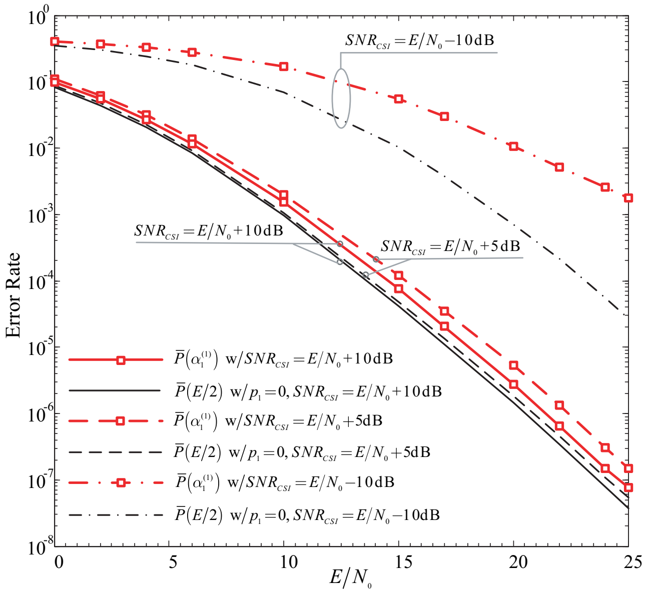

Fig. 7. Comparison between the error performance of the EADST system with one relay and the ideal DSTC system with $p_1 = 0$ for the case of imperfect channel estimates.

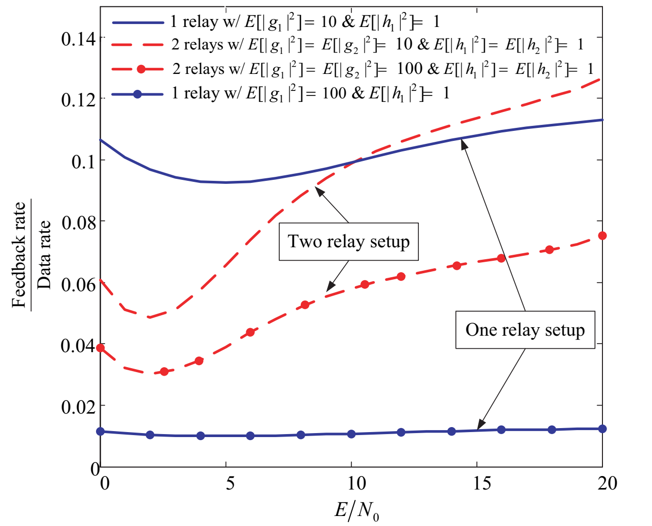

Fig. 8. Feedback rate over information symbol rate for one and two relay systems.

space-time coding system for collocated antennas, the EADST system loses diversity with a fixed SNR_{CSI}.

Example 5: amount of feedback from the destination to the relays. In this example we analyze the amount of feedback required by the relays. We let the destination transmit a pulse whenever it has to *switch off* the relays. For the one and two relay setups we plot in Fig. 8 the feedback rate as a percentage of the information symbol rate for two cases: 1) $E[|h_{sr_q}|^2] = 10$, $E[|h_{r_qd}|^2] = 1$ and 2) $E[|h_{sr_q}|^2] = 100$, $E[|h_{r_qd}|^2] = 1$, $q \in \{1, 2\}$. The rest of the channels are the same as in Example 1. We observe that the feedback rate is between 1% and 10% of the feedforward data rate depending on the setup; increasing the quality of the channel between the source and the relay translates to a lower feedback rate. Notice that at low SINR the EADST system uses both relays most of the time. As the SINR increases, the feedback becomes more important. Intuitively, that explains why the slope of the error probability of an EADST system without feedback degrades when increasing the SINR.

VI. Conclusions

In this paper we have analyzed regenerative DSTC systems that use the Alamouti space-time code. We have shown that the distributed Alamouti coding design loses diversity in the relay channel. We have proposed a novel EADST scheme with one relay that is able to achieve full diversity by switching between cooperation using the Alamouti design and one-hop transmissions from the source to the destination based on the minimum error probability at the destination. We have also proposed a feedback-free EADST system, which improves on the distributed Alamouti design by allowing the amplification at the relay to depend on the relay's own error rate. We have shown that the design guidelines for a one relay system can be extended to a two relay system and we have illustrated through simulations that both the one and the two relay schemes perform close to the error probability lower bound obtained by considering error-free relays.

Appendix I

Proof of Proposition 1. We show that the channel between the source and the relay (i.e., h_{sr_1}) limits the diversity slope of $\bar{P}(\alpha_{r_1})$. Notice that $P^{(l)} := (1-P_{r_1})P_{r_1}Q(\alpha_d/\sqrt{N_1})$ is a lower bound on $P(\alpha_{r_1})$. If we define $\zeta_1 := 2\rho_{r_1}|h_{r_1d}|^2$ and $\zeta_T := 2(\rho|h_A|^2 + (1-\rho)|h_B|^2)$, we can write $P^{(l)} = (1-P_{r_1})P_{r_1}Q\left(\sqrt{\frac{E}{N_0}}\frac{\zeta_T-\zeta_1}{\sqrt{\zeta_T+\zeta_1}}\right)$. Because $\frac{\zeta_T-\zeta_1}{\sqrt{\zeta_T+\zeta_1}} < \sqrt{\zeta_T}-\sqrt{0.5\zeta_1}$ for $\zeta_T < \zeta_1$, the average error probability is

$$\bar{P}(\alpha_{r_1}) > E\left[(1-P_{r_1})P_{r_1}Q\left(\sqrt{\frac{E}{N_0}}\frac{\zeta_T-\zeta_1}{\sqrt{\zeta_T+\zeta_1}}\right)\right] > E[(1-P_{r_1})P_{r_1}] \cdot \int_0^\infty\left[\int_u^\infty Q\left(\sqrt{\frac{E}{N_0}}(\sqrt{u}-\sqrt{0.5v})\right)p_{\zeta_1}(v)dv\right]p_{\zeta_T}(u)du,$$

where $p_{\zeta_1}(v)$ is the PDF of ζ_1 and $p_{\zeta_T}(u)$ is the PDF of ζ_T. Furthermore, since $Q\left(\sqrt{\frac{E}{N_0}}(\sqrt{u}-\sqrt{0.5v})\right) \geqslant 1 - \exp\left[-\frac{E}{N_0}(\sqrt{u}-\sqrt{0.5v})^2\right]$ when $v \geqslant 2u$,

$$\bar{P}(\alpha_{r_1}) > E[(1-P_{r_1})P_{r_1}] \cdot \int_0^\infty\left[\int_{2u}^\infty\left(1-e^{-\frac{E}{N_0}(\sqrt{u}-\sqrt{0.5v})^2}\right)p_{\zeta_1}(v)dv\right]p_{\zeta_T}(u)du. \tag{22}$$

There are 4 terms in (22), but only $I := -E[P_{r_1}]\int_0^\infty\int_{2u}^\infty e^{-\frac{E}{N_0}(\sqrt{u}-\sqrt{0.5v})^2}p_{\zeta_1}(v)p_{\zeta_T}(u)dvdu$ and $J = E[P_{r_1}]\int_0^\infty\left(\int_{2u}^\infty p_{\zeta_1}(v)dv\right)p_{\zeta_T}(u)du$ should be analyzed at high E/N_0. If we let $\zeta := (\sqrt{\zeta_T}-\sqrt{0.5\zeta_1})^2$, we can write $I := -E[P_{r_1}]\int_0^\infty e^{-\frac{E}{N_0}u}p_\zeta(u)du$, where $p_\zeta(u)$ is the PDF of ζ conditioned on $\zeta_1 > 2\zeta_T$. If $p_\zeta(u)$ is continuous and admits a Taylor series expansion about zero, i.e., $p_\zeta(u) = u^{t-1}p_\zeta^{(t-1)}(0)/(t-1)! + o(u^t)$, where $p_\zeta^{(t-1)}(u)$ is the $(t-1)$th order derivative of $p_\zeta(u)$ and t is the smallest integer greater or equal to 1 for which $p_\zeta^{(t-1)}(0) \neq 0$, we can always find a finite $\delta_\zeta > 1$ such that

$u^{t-1}\delta_\zeta p_\zeta^{(t-1)}(0)/(t-1)! \geqslant p_\zeta(u)$ for all $u \geqslant 0$. We obtain that $I \geqslant I_L := -E[P_{r_1}]\frac{\delta_\zeta p_\zeta^{(t-1)}(0)}{(t-1)!}\int_0^\infty e^{-\frac{E}{N_0}u}u^{t-1}du = -E[P_{r_1}]\left(\frac{E}{N_0}\right)^{-t}\delta_\zeta p_\zeta^{(t-1)}(0)$. Note that I_L converges to zero as fast as $E[P_{r_1}]\left(\frac{E}{N_0}\right)^{-t}$ when E/N_0 increases. Because $J_U := 1-\int_0^\infty\left(\int_{2u}^\infty p_{\zeta_1}(v)dv\right)p_{\zeta_T}(u)du$ does not depend on E/N_0, $J = E[P_{r_1}]J_U$ is the dominant term at high SINR. The highest diversity achieved by the distributed Alamouti scheme is the diversity of the channel h_{sr_1} (and it is 1 if $|h_{sr_1}|$ is Rayleigh distributed).

Appendix II

Proof of Proposition 2. The sign of the derivative of $\xi_1(\alpha_{r_1})$ depends only on $f(\alpha_{r_1}) := \varepsilon_B\gamma_B f_0 + \alpha_{r_1}\gamma_{r_1 d}f_1$, where $f_0 := 1-4P_{r_1}(1+\varepsilon_B\gamma_B)$ and $f_1 := 1-2P_{r_1}(1+2\varepsilon_B\gamma_B)$. If $f_0 > 0$ and $f_1 \geqslant 0$ then the optimum is achieved for $\alpha_{r_1} = E_{r_1}$ since $\xi_1(\alpha_{r_1})$ is increasing with α_{r_1}. If $f_0 \leqslant 0$ and $f_1 \leqslant 0$ then the optimum is achieved for $\alpha_{r_1} = 0$ since $\xi_1(\alpha_{r_1})$ is decreasing with α_{r_1}. The case when $f_0 > 0$ and $f_1 < 0$ cannot be achieved. We show next that for $P_{r_1} \in [0, 1/2]$, $f_0 > 0$ implies $f_1 \geqslant 0$. If $f_0 > 0$, then $\varepsilon_B\gamma_B < \delta_p := \frac{1-4P_{r_1}}{4P_{r_1}}$. If we substitute $\varepsilon_B\gamma_B = \delta_p$ in f_1, we obtain the minimum $f_1 = 1-2P_{r_1}\left(1+2\frac{1-4P_{r_1}}{4P_{r_1}}\right) > 0$, which is what we had to prove. It remains to analyze the case when $f_0 \leqslant 0$ and $f_1 > 0$. However, in this case, the unique solution to $f(\alpha_{r_1}) = 0$ achieves the minimum instead of the maximum $\xi_1(\alpha_{r_1})$. The maximum is always achieved on the boundaries of the feasible region, i.e., $\xi_1^{\text{opt}} = \max\{\xi_1(0), \xi_1(E_{r_1})\}$, which concludes the proof[2].

Appendix III

Proof of Proposition 3. When $\alpha_{r_1} = 0$ the relay does not transmit any information and if $\gamma_A := |h_A|^2/N_0$, then $P(0) = Q\left(\sqrt{2(\varepsilon_A\gamma_A+\varepsilon_B\gamma_B)}\right)$, which can be trivially upper bounded as $P(0) < 4Q\left(\sqrt{2(\varepsilon_A\gamma_A+\varepsilon_B\gamma_B)}\right)$. Finding a useful bound for $P(E_{r_1})$ is only a bit more laborious. The first term of $P(E_{r_1})$ is $(1-P_{r_1})^2Q\left(\alpha_s/\sqrt{N_1}\right) = (1-2P_{r_1}+P_{r_1}^2)Q\left(\sqrt{2\alpha_s/N_0}\right) < (1-P_{r_1}+P_{r_1}^2)Q\left(\sqrt{2E_{r_1}\gamma_{r_1 d}}\right)$, where in order to establish the inequality we used $\alpha_s \leqslant E_{r_1}|h_{r_1 d}|^2$. Using a similar approach and the fact that $Q(x) < 1$, the second, third and fourth terms in $P(E_{r_1})$ can be upper bounded by $\left(P_{r_1} - P_{r_1}^2\right)\left[1-Q\left(E_{r_1}|h_{r_1 d}|^2/\sqrt{N_1}\right)\right]$, $\left(P_{r_1} - P_{r_1}^2\right)\left[Q\left(\sqrt{2E_{r_1}\gamma_{r_1 d}}\right)+1\right]$, and $P_{r_1}^2\left[1-Q\left(E_{r_1}|h_{r_1 d}|^2/\sqrt{N_1}\right)+1\right]$ respectively. After canceling out opposite terms we obtain $P(E_{r_1}) < Q(\sqrt{2E_{r_1}\gamma_{r_1 d}}) + 2P_{r_1}$. Using the definition of P_{r_1}, i.e., $P_{r_1} = Q\left(\sqrt{2\varepsilon_A\gamma_{sr_1}}\right)$, we can easily establish that $P(E_{r_1}) < 4Q\left(\sqrt{2\min\{\varepsilon_A\gamma_{sr_1}, E_{r_1}\gamma_{r_1 d}\}}\right)$. Hence, $\min\{P(0), P(E_{r_1})\} < 4Q\left(\sqrt{2\max\{\varepsilon_A\gamma_A+\varepsilon_B\gamma_B, \min\{E_{r_1}\gamma_{r_1 d}, \varepsilon_A\gamma_{sr_1}\}\}}\right)$. Note that the last bound is 4 times the probability of error of a system with one-hop BPSK transmissions and SINR $\xi_t := \max\left\{\varepsilon_A\gamma_A+\varepsilon_B\gamma_B, \min\{E_{r_1}\gamma_{r_1 d}, \varepsilon_A\gamma_{sr_1}\}\right\}$. Using the high SINR approximation developed in [27] for the symbol error rate of one-hop systems, we can show that the diversity gain of a system with SINR $\min\{E_{r_1}\gamma_{r_1 d}, \varepsilon_A\gamma_{sr_1}\}$ is $\min\{t_{sr_1}, t_{r_1 d}\}$. Our conclusion follows from the fact ξ_t can be interpreted as the SINR at the output of a selection combiner preceded by a maximum ratio combiner (see also Proposition 4 and its corollary in [27]).

[2]As a result of the analysis in Appendix II we can also claim that $\xi_1(\alpha_{r_1})$ is quasi-convex for $0 \leqslant \alpha_{r_1} \leqslant E_{r_1}$ irrespective of the channel conditions.

Appendix IV

Proof of Proposition 4. We start the analysis by proving the following lemma.

Lemma 1. *Let $\mathcal{D}_f$ be a closed set in the Euclidean N-space. If a function $f : \mathcal{D}_f \to \mathbb{R}$ is differentiable and has no stationary points in the interior of $\mathcal{D}_f$ (i.e., the largest open set included in $\mathcal{D}_f$), then any local maximum or minimum of $f(\boldsymbol{x})$ is achieved on the boundaries of $\mathcal{D}_f$.*

We prove Lemma 1 by contradiction. We assume that there is a non-stationary point $\boldsymbol{x}_0$ in the interior of $\mathcal{D}_f$ that achieves a minimum or a maximum of $f(\boldsymbol{x}_0)$. If $\nabla f(\boldsymbol{x}_0) \neq 0$, then $f(x_0)$ can be increased or decreased by either moving $\varepsilon > 0$, which is small enough to guarantee feasibility, in the direction or in the opposite direction of the gradient $\nabla f(\boldsymbol{x}_0)$. Consequently, $\nabla f(\boldsymbol{x}_0)$ has to be zero and as a result, $\boldsymbol{x}_0$ is a stationary point, which is a contradiction. ■

The numerator in $\nabla\xi_2(\boldsymbol{\alpha})$ is zero for $\boldsymbol{\alpha}_1 = [0,0]$, $\boldsymbol{\alpha}_2 = \left[\frac{P_{r_1}-P_{r_2}}{8\gamma_{r_1 d}P_{r_1}P_{r_2}(1-P_{r_1}-P_{r_2})}, \frac{-(P_{r_1}-P_{r_2})}{8\gamma_{r_1 d}P_{r_1}P_{r_2}(1-P_{r_1}-P_{r_2})}\right]$ and $\boldsymbol{\alpha}_3 = \left[\frac{-(P_{r_1}-P_{r_2})(1-2P_{r_2})}{2\gamma_{r_1 d}[P_{r_1}^2+P_{r_2}^2-4P_{r_1}P_{r_2}(1-P_{r_1}-P_{r_2}+2P_{r_1}P_{r_2})]}, \frac{(P_{r_1}-P_{r_2})(1-2P_{r_2})}{2\gamma_{r_1 d}[P_{r_1}^2+P_{r_2}^2-4P_{r_1}P_{r_2}(1-P_{r_1}-P_{r_2}+2P_{r_1}P_{r_2})]}\right]$. Therefore, if $\{\boldsymbol{\alpha}_i\}_{i=1}^3$ are stationary points, they are the only stationary points of $\xi_2(\boldsymbol{\alpha})$. Because $\{\boldsymbol{\alpha}_i\}_{i=1}^3$ are not in the interior of $\mathcal{D}_\xi := \{[0, E_{r_1}] \times [0, E_{r_2}]\}$, which is the constraint set of the optimization problem, we conclude from Lemma 1 that the maximum of $\xi_2(\boldsymbol{\alpha})$ can only be achieved on the boundaries of $\mathcal{D}_\xi$. We analyze 4 cases in order to exclude most of the points on the boundaries. Before we proceed with the analysis let us exclude the point $[0,0]$ since it is evident that $\xi_2([0,0]) = 0$ is the minimum of $\xi_2(\boldsymbol{\alpha})$ when $\boldsymbol{\alpha} \in \mathcal{D}_\xi$.

Case 1. Let $\alpha_{r_1} \in (0, E_{r_1}]$ and $\alpha_{r_2} = 0$. The SINR in (17) becomes

$$\xi_2([\alpha_{r_1}, 0]) = \frac{(1-2P_{r_1})^2\gamma_{r_1 d}\alpha_{r_1}}{4P_{r_1}(1-P_{r_1})\gamma_{r_1 d}\alpha_{r_1}+1}, \tag{23}$$

which is an increasing function of α_{r_1}. The maximum of (23) is achieved for $\alpha_{r_1} = E_{r_1}$.

Case 2. Let $\alpha_{r_1} = 0$ and $\alpha_{r_2} \in (0, E_{r_2}]$. Using the same approach as in Case 1 we obtain a maximum when $\alpha_{r_2} = E_{r_2}$.

Case 3. Let $\alpha_{r_1} \in [0, E_{r_1}]$ and $\alpha_{r_2} = E_{r_2}$. We observe that the sign of the derivative of $\xi_2([\alpha_{r_1}, E_{r_2}])$ with respect to α_{r_1} depends only on $g(\alpha_{r_1}) = E_{r_2}\gamma_{r_2 d}g_0 + \alpha_{r_1}\gamma_{r_1 d}g_1$, where $g_0 := 1-4P_{r_1}+2P_{r_2}-4E_{r_2}\gamma_{r_2 d}\left(P_{r_2}-P_{r_1}-2P_{r_1}P_{r_2}+4P_{r_1}P_{r_2}^2\right)$ and $g_1 := 1-2P_{r_2}-4E_{r_2}\gamma_{r_2 d}\left(P_{r_2}-P_{r_1}+2P_{r_1}P_{r_2}-4P_{r_1}^2P_{r_2}\right)$. If $P_{r_2} \geqslant P_{r_1}+2P_{r_1}P_{r_2}-4P_{r_1}P_{r_2}^2$ then both g_0 and g_1 are positive, and consequently, $\xi_2([\alpha_{r_1}, E_{r_2}])$ is increasing with α_{r_1}. The maximum is achieved at $\alpha_{r_1} = E_{r_1}$. If $P_{r_2} < P_{r_1}+2P_{r_1}P_{r_2}-4P_{r_1}P_{r_2}^2$, we follow the same steps as in Appendix II. For the case when g_0 and g_1 have the same sign the maximum is achieved for $\alpha_{r_1} = E_{r_1}$ or

$\alpha_{r_1} = 0$ since $\xi_2([\alpha_{r_1}, E_{r_2}])$ is either increasing or decreasing. Note that g_1 is non-positive as long as $E_{r_2}\gamma_{r_2 d} \in D_\gamma := \left[\frac{1-2P_{r_1}}{4\left(P_{r_1}-P_{r_2}-2P_{r_1}P_{r_2}+4P_{r_1}^2 P_{r_2}\right)}, \infty\right]$. However, $E_{r_2 d}\gamma_{r_2 d} \in D_\gamma$ also implies $g_0 < 0$, and therefore, $g_0 > 0$ and $g_1 \leqslant 0$ is not a valid case. It remains to analyze $g_0 < 0$ and $g_1 > 0$. In this case the stationary point $\alpha_{r_1} = \frac{-E_{r_2}\gamma_{r_2 d} g_0}{\gamma_{r_1 d} g_1}$ attains the minimum of $\xi_2([\alpha_{r_1}, E_{r_2}])$. The maximum is reached for either $\alpha_{r_1} = 0$ or $\alpha_{r_1} = E_{r_1}$.

Case 4. The analysis for the case when $\alpha_{r_1} = E_{r_1}$ and $\alpha_{r_2} \in (0, E_{r_2}]$ is similar to Case 3.

Consequently, the only points that could reach the maximum irrespective of the system's parameters are $[E_{r_1}, 0]$, $[0, E_{r_2}]$, and $[E_{r_1}, E_{r_2}]$, which concludes the proof. ■

APPENDIX V

We condition the error probability of the two-relay system, i.e. P_2, on the error variables and on the data symbol $s[1]$ to obtain $P(\boldsymbol{i}) := \Pr\big(\Re\{t_2\} < 0 | s[0] = 1, \theta_1[0] = i_1, \theta_1[1] = i_2, \theta_2[0] = i_3, \theta_2[1] = i_4, s[1] = i_5\big)$, where $\boldsymbol{i} := [i_1, i_2, i_3, i_4, i_5] \in \{0,1\}^4 \times \{-1, 1\}$. Using the law of total probability we write

$$P_2(\boldsymbol{\alpha}) = \sum_{\boldsymbol{i}\in\{0,1\}^4\times\{-1,1\}} 0.5\Pr(\theta_1[0] = i_1)\Pr(\theta_1[1] = i_2) \cdot \Pr(\theta_2[0] = i_3)\Pr(\theta_2[1] = i_4)P(\boldsymbol{i}). \tag{24}$$

Since conditioned on $\{\theta_1[i]\}_{i=1}^2$, $\{\theta_2[i]\}_{i=1}^2$, and $s[1]$, $\Re\{t_2\}$ is a Gaussian random variable, we can write $P(\boldsymbol{i})$ in terms of the Q-function. After simple manipulations we obtain (18).

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